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A NEW MODEL FOR COMPLEX DYNAMICS IN A DC GLOW DISCHARGE TUBE

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Abstract

Based on a detailed experimental characterization of the complex dynamics of a Ne discharge tube, a new phenomenological model governed by four differential equations is proposed. Such a model takes into account the competition between a nonlinear relaxation oscillator related with external electronic features and a forced harmonic oscillator accounting for the plasma eigenfrequency. From their interaction the quasi periodicity route to chaos emerges as confirmed by the experimental observations. In the control parameter space, stability diagrams for periodic oscillations of arbitrary period are computed using the isospike technique and compared with standard diagrams of Lyapunov exponents. Such diagrams reveal complex patterns of stability phases with extended regions of multistabilty.

Key words

Plasma physics, chaos, nonlinear dynamics.

1 Introduction

Complex chaotic dynamics of glow discharge plasma tubes has been the subject of numerous works and much is known about them [Braun, Lisboa, Francke, and Gallas, 1987][Ticos, Rosa, Pardo, Walkenstein, and Monti, 2000][Dinklage, Wilke, Bonhomme, and Atipo, 2000][Letellier, Dinklage, El-Naggar, Wilke, and Bonhomme, 2001]. This problem was first considered by van der Pol studying a relaxation oscillator consisting of a neon tube, connected to an RC circuit and exited by means of a battery [van der Pol and van der Mark, 1928]. Relaxation oscillators are of fundamental importance in biological systems including heartbeats, respiration, walking and hormone secretion [Wang, 2003] [Buzsáki, 2006].

Here a novel approach has been followed in exploring the dynamical behavior of a Ne plasma tube in the region of glow discharge. We consider the macroscopic volt-ampere characteristic of the discharge, regarding the whole plasma simply as a nonlinear two

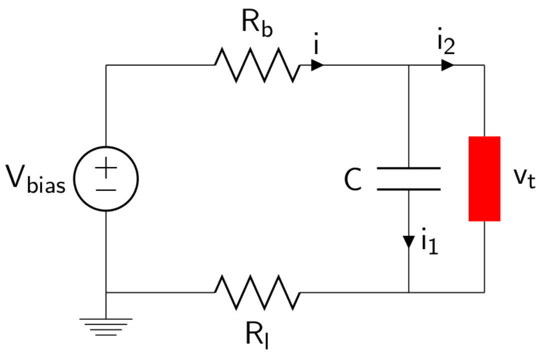

Figure 1. Experimental setup. The discharge tube is shown as a red element. V_{bias}, adjustable high voltage power supply; $R_b = 150\ k\Omega$, ballast resistor; $R_l = 1\ k\Omega$, load resistor; $C = 2.4\,nF$, capacitor.

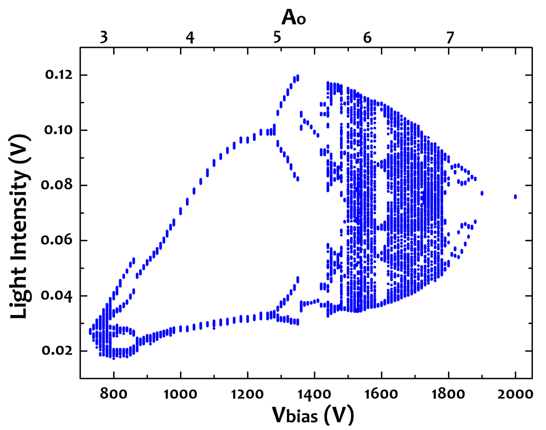

Figure 2. Experimental bifurcation diagram. Light intensity maxima versus V_{bias}.

terminal electrical component. Such an approach removes dependencies from charge spatial distribution and reduces the mathematical description to a nonlinear set of two coupled ordinary differential equations. The experimentally observed chaotic behavior induce us to introduce an additional oscillator associated with the plasma eigenfrequencies [Piel, 2010]. The model, governed by four autonomous differential equations, is used to compute stability diagrams for periodic oscillations of arbitrary period in the control parameter space of the discharge. Isospike contour plots [Freire and Gallas, 2011] and standard Lyapunov exponents have been evaluated revealing intricate mosaics of stability domains and multistability regions.

2 Experiment

The experimental setup is reported in Fig.1. The main element is a plasma tube, having a length of 50 cm and an internal diameter of 2.5 cm, filled with neon at low pressure. Such a device is of the type commonly used in commercial neon signs and other applications and presents identical electrodes that can be used as anode and cathode indifferently. An adjustable high voltage dc source, indicated as V_{bias}, allows us to excite and drive the tube in the glow discharge operation region. The V_{bias} plays the role of control parameter. The range of interest for V_{bias} is $730\ V < V_{bias} < 2000\ V$. Two distinct signals were recorded, the discharge voltage across the tube and the corresponding light emission. The last one is proportional to current flowing in the tube. The bifurcation diagram as function of V_{bias} is shown in Fig.2. This graph is obtained extracting the maxima values of recorded light signals when the capacitance C is set at 2.4 nF.

The onset of the glow dynamics is characterized by a period-1 solution ($V_{bias} \simeq 730\ V$) at a frequency of 670 Hz, meaning that we are dealing with a stationary regime characterized only by the relaxation frequency imposed by the external RC coupling. A slight increase of V_{bias} leads to the appearance of another frequency competing with the previous one. Such a frequency is the plasma eigenfrequency. From the nonlinear interaction between them a first region of chaos emerges followed by other extended periodic windows of period-2 solutions. After that large chaotic windows appear.

For the selected value of the capacitance C the ratio of the two competing frequencies is close to two but irrational. For this reason, the underlying route to chaos is the quasiperiodicy and not the period doubling as suggested in the bifurcation diagram. A more clear evidence of the transition to chaos is given by plotting the temporal behavior of light intensity (see Fig.3).

The nature of the transition to chaos was more clearly manifested when the capacitance C is increased to $4.8\ nF$. In such a case the ratio between the two competing frequency is around 1.13, the bifurcation diagrams do not show evidence of period-2 solutions and period doubling bifurcations [Pugliese, Meucci, Euzzor, Freire, and Gallas, 2015].

3 Model

The starting point to experimentally derive a phenomenological and macroscopic model of the discharge plasma is provided by the Kirchoffs laws applied to the circuit of Fig.1. Denoting by v_t and i_2 the discharge voltage and the current through the tube respectively, we derive the following equations

$$i = i_1 + i_2 \quad (1)$$

$$i_1 = C\frac{dv_t}{dt} \quad (2)$$

$$i_1 + i_2 = \frac{V_{bias} - v_t}{R_b} \quad (3)$$

$$\frac{dv_t}{dt} = \frac{1}{R_b C}\left(V_{bias} - v_t - R_b i_2\right) \quad (4)$$

$$L\frac{di_2}{dt} = v_t - G(i_2) \quad (5)$$

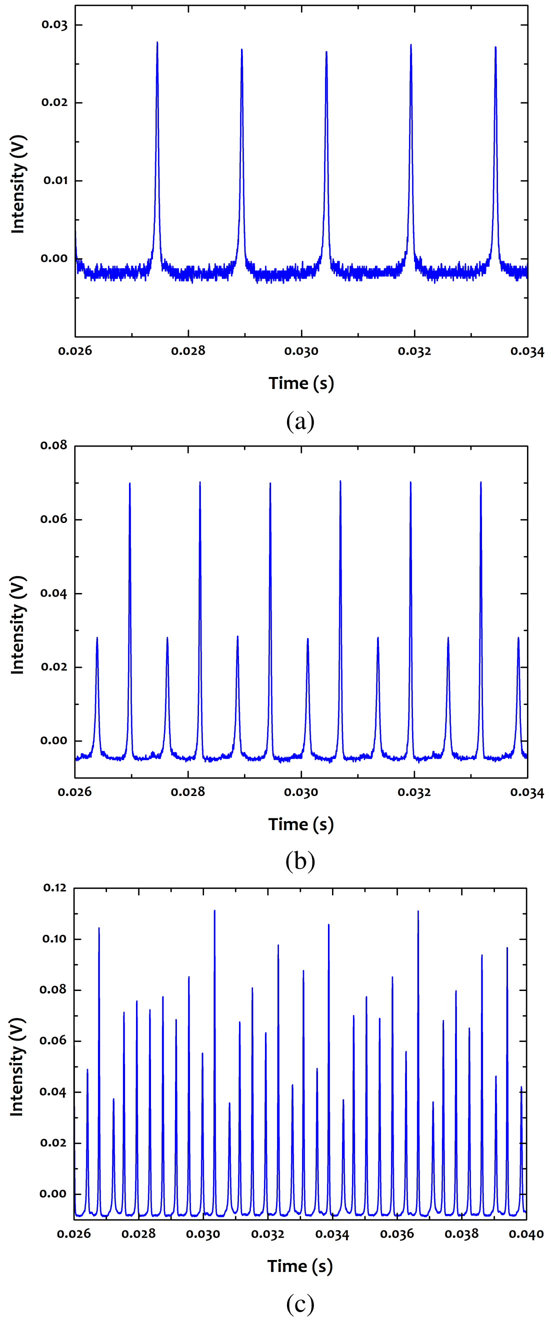

Figure 3. Temporal evolution of the light intensity for three values of the control parameter V_{bias}. (a) $V_{bias} = 730\ V$, periodic behavior of the relaxation oscillator; (b) $V_{bias} = 1000\ V$, two frequency periodic regime; (c) $V_{bias} = 1550\ V$, spike amplitude chaotic regime.

where we introduced the inductance L accounting for a real inductive effect [Raizer, Gurevich, and Mokrov,2006] and the voltage-current characteristic $G(i_2)$ of the tube. Proceeding with the dimensionless variables introduced in Ref. [Pugliese, Meucci, Euzzor, Freire, and Gallas, 2015], the dynamics of the relaxation oscillator is described by

$$\dot{y} = A_0 - A_1 y - A_2 x \tag{6}$$

$$\mu \dot{x} = y - g(x) \tag{7}$$

where $g(x)$ is the dimensionless characteristic curve with the following mathematical form

$$g(x) = y_c + a e^{-k_1 \cdot x} - (y_c + a) e^{-k_2 \cdot x} \tag{8}$$

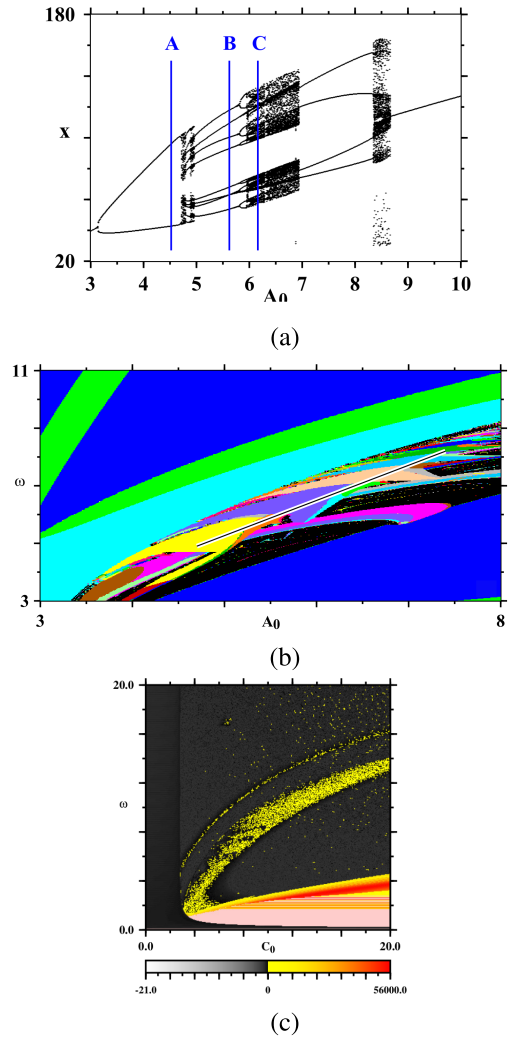

Figure 4. Numerical results obtained using the model defined by Eqs. (11)–(14). (a) bifurcation diagram of local maxima of x for $\gamma = 4.2$ and ω varying in the interval [6,7]; (b) isospike diagram counting the number of spikes in one period of the variable x; (c) the corresponding standard Lyapunov stability diagram.

As experimentally observed, the dynamics of the relaxation oscillator is strongly influenced by an intrinsic plasma oscillation. Such oscillation is taken into account by introducing a damped oscillator driven by the discharge current x as follows

$$\ddot{z} + \beta\dot{z} + \omega^2 z = \gamma x \tag{9}$$

where ω is the angular eigenfrequency. The new variable z introduces a modulation of the working point on the characteristic curve $g(x)$ of Eq. 8. This assumption is justified by the temporal behavior in the chaotic regime where the spikes are only characterized by their different amplitudes and not by the time of their occurrences. The actual electric characteristic is now given by

$$g(x, z) = y_c + (a+z)e^{-k_1 \cdot x} - (y_c + a + z)e^{-k_2 \cdot x} \tag{10}$$

where the parameters y_c, a, k_1 and k_2 are fixed by fitting the experimental data.
The complete model used to describe the dynamics is given by following set of first order differential equations

$$\dot{x} = \frac{1}{\mu}(y - g(x, z)) \tag{11}$$
$$\dot{y} = A_0 - A_1 y - A_2 x \tag{12}$$
$$\dot{z} = w \tag{13}$$
$$\dot{w} = -\beta w - \omega^2 z + \gamma x \tag{14}$$

The numerical bifurcation diagram, reported in Fig. 4 (a) shows a fair overall agreement with the experimental one. The model was also used to compute the stability diagrams using the isospike technique [Freire and Gallas, 2011] (b) and the Lyapunov exponents (c). The isospike diagram shows, for every point of the control parameter space, the number of spikes contained in one period of the regular oscillations. Each color corresponds to stable solutions of the x variable with a certain number of spikes per period. Black color represents chaos, i.e., parameters for which it was not possible to detect periodicity.

4 Conclusion

In this paper, we reported an experimental study of a low pressure Ne discharge tube showing evidence of chaos via quasiperiodicity. An accurate electrical and dynamical characterization allows to derive a phenomenological model able to reproduce the interaction between a relaxation oscillator and an intrinsic plasma oscillator. By computing the stability diagrams we characterized the complex organization of the stable solutions and the transition to chaos.

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