Algorithms for command control of hypersonic aircraft at the climbing leg

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Abstract: The problem of forming sampling control with forecast (command control) for a hypersonic cruising aircraft at the climbing leg is considered. The motion is analyzed under the atmospheric density disturbance and deviations from aerodynamic characteristics of the aircraft. Two control algorithms are suggested: one-channel (by the angle of attack) and two-channel (by the angle of attack and fuel consumption per second).

Keywords: hypersonic aircraft, disturbed motion, automatic control with forecast, one- and two-channel sampling control.

1. INTRODUCTION

We consider a hypersonic aircraft with a combined power unit (CPU) in the form of steam and hydrogen rocket turbine engine. Automatic control of the aircraft at the climbing leg is analyzed with a view of fulfilling the preset terminal motion conditions under atmospheric and aerodynamic disturbance.

2. NOMINAL CONTROL

The system of differential equations describing the aircraft motion in the velocity coordinate system takes the form

$$\left.\begin{aligned}
\dot{V} = {} & \frac{I_{sp}(h,M)\beta}{m}\cos\alpha - C_{xa}(\alpha,M)\frac{\rho(h)V^2}{2m}S- \\
& -g\sin\theta, \\
\dot{\theta} = {} & \frac{1}{V}\left(\frac{I_{sp}(h,M)\beta}{m}\sin\alpha - g\cos\theta + \right. \\
& \left. + C_{ya}(\alpha,M)\frac{\rho(h)V^2}{2m}S\right) + \frac{V\cos\theta}{R+h}, \\
\dot{h} = {} & V\sin\theta, \\
\dot{m} = {} & -\beta.
\end{aligned}\right\} \tag{1}$$

Here V is velocity, θ is the trajectory inclination angle, h is height, α is the angle of attack, β is fuel consumption per second, M is Mach number, ρ is the atmospheric density, S is the wing area, g is free fall acceleration, R is the Earth radius, I_{sp} is specific impulse, C_{xa}, C_{ya} are, respectively, coefficients of drag force and aerodynamic lift.

Aerodynamic characteristics and height and velocity characteristics of CPU are assumed according to Nechayev (1996). Fuel consumption per second is assumed to be constant and equal to the maximal ($\beta_{max} = 76kg/sec$).

The following limits are imposed on the angle of attack:

$$\alpha_{min} \le \alpha \le \alpha_{max}, \tag{2}$$

where $\alpha_{min} = 0$, $\alpha_{max} = 10°$.

Initial motion conditions have the following values: $M_0 = 1.921$, $\theta_0 = 13.8°$, $h_0 = 11000m$, $m_0 = 290000kg$ (Nechayev, 1996).

Terminal motion conditions are determined by the power unit performance ($h_k = 30000m$, $M_k = 6$) (Nechayev, 1996) and the next leg of horizontal cruise flight ($\theta_k = 0$).

Two-stage program of the angle of attack control is used as nominal (Balakin et al., 2008):

$$\alpha = \begin{cases} \alpha_1, & t < t_p; \\ \alpha_2, & t \ge t_p. \end{cases} \tag{3}$$

Here α_1, α_2, t_p are constant parameters, determined when the terminal conditions are fulfilled (the trajectory inclination angle is θ_k, height is h_k and velocity is M_k) with minimal fuel consumption.

Using the Newton method and the gradient method, taking into account the constraints (2) in Balakin et al. (2008), parameter values of the program are defined: $\alpha_1 = 0.45°$, $\alpha_2 = 6.00°$, $t_p = 62.5sec$.

3. DISTURBED MOTION

The impact of random disturbance in the atmospheric density on the terminal motion conditions is investigated.

With non-disturbed motion the density is calculated from the law corresponding to standard atmosphere for the heights from 0 to $40000m$ (Sedunov, 1991):

$$\rho = \rho_0 \exp\left(-\frac{h}{H_1(h)}\right), \tag{4}$$

$$H_1(h) = H_{10} + H_{11}h + H_{12}h^2 + H_{13}h^3, \tag{5}$$

where $H_1(h)$ is the scale of height; $H_{10} = 10351.8m$; $H_{11} = -3.68512 \cdot 10^{-2}$; $H_{12} = -1.02368 \cdot 10^{-5}m^{-1}$; $H_{13} = 2.63363 \cdot 10^{-10}m^{-2}$ (Letov, 1969).

Two models of random disturbances of the atmospheric density are considered.

In the first model $\rho(h)$ is presented as stochastic variable, distributed according to the normal law with the expectation $\rho_{st}(h)$, calculated from the formula (4).

Standard deviation $\sigma_\rho(h)$ is determined by the relation (Shkolny and Maiboroda, 1973):

$$\sigma_\rho(h) = \sigma_{\rho_0} \cdot \exp(-0.15h), \tag{6}$$

where $\sigma_{\rho_0} = 50g/m^3$ is the mean value of standard deviation of air density on earth.

The normal distribution law of air density with the above-mentioned characteristics is obtained by means of linear transformation of the random quantity normal law ξ with zero expectation and variance equal to unity:

$$\rho(h) = \rho_{st}(h) + \xi \cdot \sigma(h). \tag{7}$$

In the sample obtained of equations system solution (1) for this disturbance model only 20% of terminal conditions are fulfilled with the preset accuracy (height $\varepsilon_h = 10m$, velocity $\varepsilon_M = 0.01M$, trajectory inclination angle $\varepsilon_\theta = 0.1°$).

For the second model, $\rho(h)$ is written in the form of spectral canonical decomposition:

$$\rho(h) = \rho_{st}(h) + \sigma_\rho(h) \sum_{\nu=1}^{n} \sigma_{\rho_\nu}(\gamma_\nu \cos \Omega_\nu h + \varepsilon_\nu \sin \Omega_\nu h). \tag{8}$$

where σ_{ρ_ν} is standard deviation of random coefficients; γ_ν, ε_ν are normally distributed random numbers with zero expectation and unity variance; Ω_ν are frequencies; $n = 11$ (Shkolny and Maiboroda, 1973).

Terminal motion conditions in the sample obtained for this model are never fulfilled with preset accuracy.

Using the disturbance model (8) leads to inferior results as compared to model (7). Therefore, only the atmosphere disturbance model (8) is considered later on.

Our investigation revealed that deterioration of aerodynamic characteristics of the aircraft (decrease of C_{ya} and increase of C_{xa}) by 1% only will result in the failure of terminal conditions in M and θ.

Simulation of disturbed motion revealed the existence of two types of trajectories. The first type has the preset terminal height when the trajectory inclination angle is positive and the velocity is insufficient. In the second type the terminal inclination angle is preset as zero, height and velocity are less than preset values.

4. ONE-CHANNEL COMMAND CONTROL

Let us consider one-channel control by the angle of attack to compensate for disturbance impact on terminal motion conditions.

The time corresponding to the climbing leg is divided into intervals. One step of control correction is performed at every interval. At every step the command control is formed from forecasting terminal motion conditions results, based on the known information which includes the values of the running phase coordinates, aerodynamic characteristics and fuel consumption per second in the power unit, standard atmosphere characteristics and previously formed control. The angle of attack at the current step is controlled by the program obtained at the previous step. The first step employs the nominal control program(3).

To form the algorithm of the command angle of attack, the following sequence is performed.

1. Forecasting aircraft motion is done by integrating differential equations (1) to fulfill one of the three terminal conditions: $h_k = 30000m$, $M_k = 6$, $\theta_k = 0$. The values of V, θ and h at the beginning of control step are used as initial conditions. The decision to correct control is made. If every terminal motion condition is fulfilled with preset accuracy, than the control program is not corrected. If even one of the terminal conditions is not fulfilled, then the control program is corrected.

2. Control is corrected by calculating new values of the program parameters of the angle of attack (3). In the interval between the initial motion and the moment t_p a two-point boundary problem of defining parameters t_p and α_2 in order to fulfill terminal conditions of height and trajectory inclination angle is solved. After the moment of switching t_p a one-point boundary problem is solved to define parameter α_2 to fulfill the terminal condition of height. The Newton method is used to solve boundary problems.

Let us introduce the following notations: $\mathbf{x} = \{\alpha_2, t_p\}$, $\mathbf{y} = \{h_k, \theta_k\}$.

The running values of program parameters of the angle of attack control make up initial estimates $\mathbf{x}_0 = \{\alpha_{20}, t_{p0}\}$. Thus, the next approximation for vector $\mathbf{x}$ is determined by the matrix equation:

$$\mathbf{y} - \mathbf{y}_j = J_j(\mathbf{x}_{j+1} - \mathbf{x}_j), \quad j = \overline{0, N}, \tag{9}$$

where

$$J = \begin{bmatrix} \dfrac{\partial h_k(\alpha_2, t_p)}{\partial \alpha_2} & \dfrac{\partial h_k(\alpha_2, t_p)}{\partial t_p} \\ \dfrac{\partial \theta_k(\alpha_2, t_p)}{\partial \alpha_2} & \dfrac{\partial \theta_k(\alpha_2, t_p)}{\partial t_p} \end{bmatrix} \text{ is Jacobi matrix;}$$

N is the number of iterations, necessary to meet the demand of convergence.

Because Cauchy problem has numerical solution, Jacobi matrix is determined as follows:

$$J_j = \left[J_j^{<1>} \; J_j^{<2>} \right], \tag{10}$$

$$J_j^{<1>} = \begin{bmatrix} \dfrac{h_k(\alpha_{2j} + \delta\alpha_{2j}, t_{p_j}) - h_k(\alpha_{2j}, t_{p_j})}{\delta\alpha_{2j}} \\ \dfrac{\theta_k(\alpha_{2j} + \delta\alpha_{2j}, t_{p_j}) - \theta_k(\alpha_{2j}, t_{p_j})}{\delta\alpha_{2j}} \end{bmatrix},$$

$$J_j^{<2>} = \begin{bmatrix} \dfrac{h_k(\alpha_{2j}, t_{p_j} + \delta t_{p_j}) - h_k(\alpha_{2j}, t_{p_j})}{\delta t_{p_j}} \\ \dfrac{\theta_k(\alpha_{2j}, t_{p_j} + \delta t_{p_j}) - \theta_k(\alpha_{2j}, t_{p_j})}{\delta t_{p_j}} \end{bmatrix},$$

where $\delta\alpha_{2j}$, δt_{p_j} are small deviations of parameters α_2, t_p from their values at j-th iteration. Solving the matrix equation (9) in $\mathbf{x}_{j+1}$, we shall get iteration formulas to define parameters α_2, t_p:

$$\alpha_{2j+1} = \alpha_{2j} + \frac{\Delta h_{kj}\left(\frac{\partial \theta_k}{\partial t_p}\right)_j - \Delta \theta_{kj}\left(\frac{\partial h_k}{\partial t_p}\right)_j}{\left(\frac{\partial h_k}{\partial t_p}\right)_j \cdot \left(\frac{\partial \theta_k}{\partial \alpha_2}\right)_j - \left(\frac{\partial \theta_k}{\partial t_p}\right)_j \cdot \left(\frac{\partial h_k}{\partial \alpha_2}\right)_j};$$

$$t_{p_{j+1}} = t_{p_j} + \frac{-\Delta h_{kj}\left(\frac{\partial \theta_k}{\partial \alpha_2}\right)_j + \Delta \theta_{kj}\left(\frac{\partial h_k}{\partial \alpha_2}\right)_j}{\left(\frac{\partial h_k}{\partial t_p}\right)_j \cdot \left(\frac{\partial \theta_k}{\partial \alpha_2}\right)_j - \left(\frac{\partial \theta_k}{\partial t_p}\right)_j \cdot \left(\frac{\partial h_k}{\partial \alpha_2}\right)_j}, \quad (11)$$

where $\left(\frac{\partial h_k}{\partial \alpha_2}\right)_j$, $\left(\frac{\partial h_k}{\partial t_p}\right)_j$, $\left(\frac{\partial \theta_k}{\partial \alpha_2}\right)_j$, $\left(\frac{\partial \theta_k}{\partial t_p}\right)_j$ are partial derivatives on the j-th iteration, which are defined according to (10);
$\Delta h_{kj} = h_k - h_k(\alpha_{2j}, t_{p_j})$ is the height deviation from the preset terminal condition;
$\Delta \theta_{kj} = \theta_k - \theta_k(\alpha_{2j}, t_{p_j})$ is deviation of the trajectory angle inclination from the preset terminal condition.

As a convergence condition, simultaneous fulfillment of the following inequalities is assumed: $\Delta h_k \le 10m$, $\Delta \theta_k \le 0.1°$.

While defining the angle of attack, limitation (2) fulfillment is checked. In case of non-fulfillment, the angle of attack is assumed to be equal to the corresponding boundary value.

The next step is numerical integration of equations (1). The angle of attack program with parameters, determined according to (11), is used.

Time interval $\Delta\tau$ is assumed to be 5 seconds. All computational operations of control correction are performed in time, which is several orders of magnitude less than $\Delta\tau$.

To meet the conditions on height and trajectory inclination angle, in case of insufficient velocity it is necessary to introduce a horizontal acceleration leg to achieve the required velocity.

Terminal condition of velocity is achieved after the conditions on height and trajectory inclination angle are met, by means of introducing a horizontal acceleration leg to achieve the velocity of $6M$.

This control algorithm makes it possible to fulfill terminal motion conditions both for all cases of atmospheric density disturbance used for simulation and for impaired (up to 10%) aerodynamic characteristics of the aircraft.

For cases of the most unfavorable disturbances Figure 1 shows nominal (1) and command (2 - atmospheric density disturbance, 3 - deviation of aerodynamic characteristics) dependence between the angle of attack and time.

In both cases a horizontal acceleration leg should be introduced to achieve preset terminal velocity (leg B, Fig. 1).

Motion is simulated under simultaneous performance of atmospheric disturbance and deviation of aerodynamic characteristics. Simulation revealed that command one-channel control allows for preset terminal conditions of height and trajectory inclination angle only for trajectories of the first type (with deviations of aerodynamic characteristics up to 1.5%). Preset terminal velocity is provided by means of additional leg of horizontal aircraft acceleration.

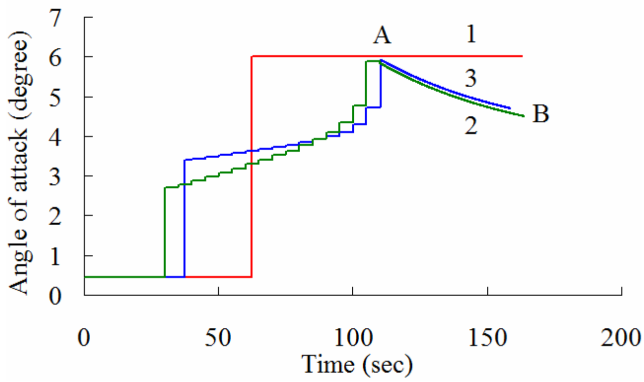

Fig. 1.

To fulfill terminal motion conditions for trajectories of the second type under simultaneous disturbance another control channel should be introduced - that of fuel consumption per second.

5. TWO-CHANNEL COMMAND CONTROL

Angle of attack command is controlled from the moment of climbing to the time t_p according to the algorithm, described in 4. Fuel consumption per second is assumed to be constant and less than maximal value β_{max} ($\beta = 70kg/sec$).

After the moment t_p control is corrected by calculating new values of the parameter α_2 in the and angle of attack and fuel consumption per second program β using formulas (9)-(11), where β is used instead of t_p.

Motion is simulated for trajectories of two types under simultaneous disturbance.

Figures 2, 3 show dependence of the angle of attack (1) and fuel consumption per second (2) of time for trajectories of the first and second type, respectively. Terminal conditions of height and trajectory inclination angle are fulfilled. Terminal velocity is obtained by extra aircraft acceleration (leg AB, Fig. 2, 3).

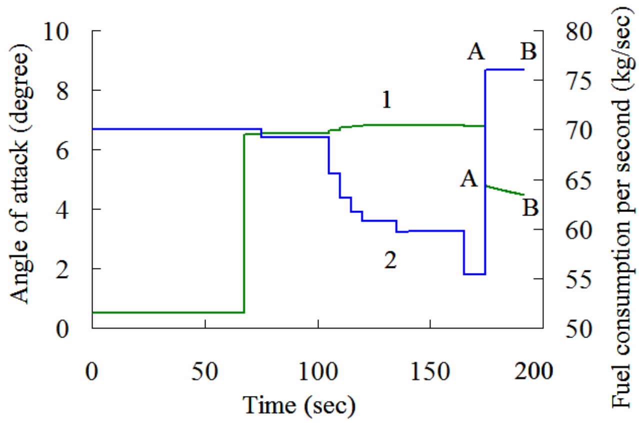

Fig. 2.

6. CONCLUSIONS

1. For the atmospheric density disturbance and deviation of aircraft aerodynamic characteristics up to 1.5%, the

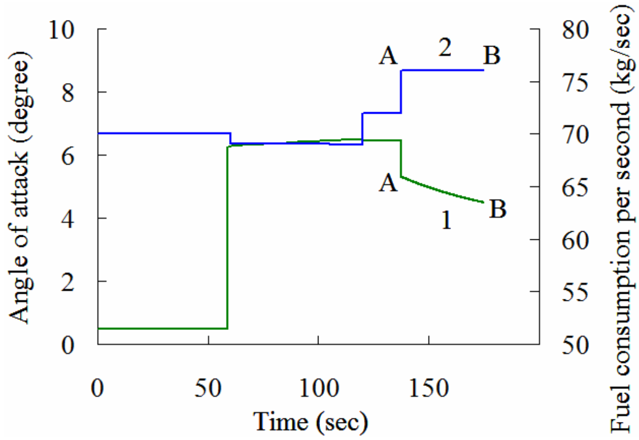

Fig. 3.

algorithm of one-channel control (the angle of attack) suggested in this paper makes it possible to fulfill preset terminal motion conditions when an extra horizontal climbing leg is introduced. This is valid only for the trajectories of the first type.

2. For the atmospheric density disturbance and deviation of aircraft aerodynamic characteristics up to 5% the algorithm of two-channel control (the angle of attack and fuel consumption per second) suggested in this paper makes it possible to fulfill preset terminal motion conditions when an extra horizontal climbing leg is introduced. This is valid for both the first and the second type of trajectories.

REFERENCES

Balakin, V., Bebyakov, A., and Kochyan, A. (2008). Motion optimization of hypersonic launcher aircraft in two-stage aircraft-space system. *Journal of Samara State Aerospace University*, 1, 23–32.

Letov, A. (1969). *Dinamika polyota i upravleniye (Flight dynamics and control)*. Nauka, Moscow.

Nechayev, Y. (1996). *Siloviye ustanovki giperzvukovich i vozdushno-kosmicheskich letatelnich apparatov (Power units in hypersonic and air-and-space aircraft)*. Rossiiskaya Academia Kosmonavtiki im. K.E. Tsiolkovskogo, Moscow.

Sedunov, Y. (1991). *Atmoshera. Spravochnik (Atmosphere. Reference book)*. Gidrometeoizdat, Leningrad.

Shkolny, Y. and Maiboroda, L. (1973). *Atmosphera i upravleniye dvizheniyem letatelnich apparatov (Atmosphere and motion control of aircraft)*. Gidrometeoizdat, Leningrad.