THE STUDY OF SELF-SYNCHRONIZATION OF VIBROEXCITERS WITH INNER DEGREE OF FREEDOM USING NUMERICAL METHODS

Abstract

The question about possibility of provision of synphase rotation of two unbalanced rotors of vibration plant in selfsynchronizing regime is discussed. In this case under consideration only the antiphase rotating is "natural stable". The possibility of using for this aim the vibroexciters with additional degree of freedom of rotors is studied.

Description Of The Plant

Consider a solid body fixed by means of springs with certain characteristics and realizing plane oscillations. On the bearer two coaxed unbalanced masses with one rotating degree of freedom are fixed. Within each of debalances a definite additional mass which can oscillate along the debalance's axis is fixed.Fig.1

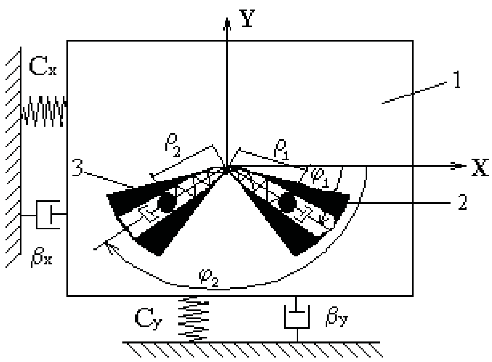

Figure 1: A vibrostand with two debalances having mobile centers of mass

Thereby the vibrostand has two translational degrees of freedom, x and y, each i - debalance having one rotational φ_i and one translational degree of freedom, ρ_i defined by displacement of the additional mass from the axis of rotor.

System of the equation for a plant with two vibroexciters, each having two inner degrees of freedom

Let us consider the equation of motion obtained in [1] for a plant with two vibroexciters, each having two inner degrees of freedom:

$$
\begin{aligned}
&[I+m_0\varepsilon^2+m(r+\rho_s)^2+m\sigma_s^2)]\ddot{\varphi}_s+K_s(\dot{\varphi}_s-\omega)+2m[(r+\rho_s)\dot{\rho}_s+\sigma_s\dot{\sigma}_s]\dot{\varphi}_s-\\
&\quad-[m_0\varepsilon+m(r+\rho_s)](\ddot{x}\sin\varphi_s+\ddot{y}\cos\varphi_s)-m\sigma_s(\ddot{x}\cos\varphi_s-\ddot{y}\sin\varphi_s)+\\
&\quad+m[(r+\rho_s)\ddot{\sigma}_s-\sigma_s\ddot{\rho}_s]=0 \qquad\qquad (\ s=1,2\ )
\end{aligned}
$$

$$
\begin{aligned}
M\ddot{x}=&\sum_{i=1}^{2}[m_0\varepsilon+m(r+\rho_i)](\ddot{\varphi}_i\sin\varphi_i+\dot{\varphi}_i^2\cos\varphi_i)-\\
&-m\sum_{i=1}^{2}[\ddot{\rho}_i-\sigma_i\ddot{\varphi}_i-2\dot{\sigma}_i\dot{\varphi}_i]\cos\varphi_i+\\
&+m\sum_{i=1}^{2}[(\dot{\rho}_i-\sigma_i\dot{\varphi}_i)\dot{\varphi}_i+\ddot{\sigma}_i+\dot{\rho}_i\dot{\varphi}_i]\sin\varphi_i-\beta_x\dot{x}-C_x x \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
M\ddot{y}=&\sum_{i=1}^{2}[m_0\varepsilon+m(r+\rho_i)](\ddot{\varphi}_i\cos\varphi_i-\dot{\varphi}_i^2\sin\varphi_i)+\\
&+m\sum_{i=1}^{2}[\ddot{\rho}_i-\sigma_i\ddot{\varphi}_i-2\dot{\sigma}_i\dot{\varphi}_i]\sin\varphi_i+\\
&+m\sum_{i=1}^{2}[(\dot{\rho}_i-\sigma_i\dot{\varphi}_i)\dot{\varphi}_i+\ddot{\sigma}_i+\dot{\rho}_i\dot{\varphi}_i]\cos\varphi_i-\beta_y\dot{y}-C_y y \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{\rho}_s+\beta_\rho\dot{\rho}_s+\omega_\rho^2\rho_s&=\sigma_s\ddot{\varphi}_s+2\dot{\sigma}_s\dot{\varphi}_s+(r+\rho_s)\dot{\varphi}_s^2-(\ddot{x}\cos\varphi_s-\ddot{y}\sin\varphi_s)\\
\ddot{\sigma}_s+\beta_\sigma\dot{\rho}_s+\omega_\sigma^2\sigma_s&=-(r+\rho_s)\ddot{\varphi}_s-2\dot{\rho}_s\dot{\varphi}_s+\sigma_s\dot{\varphi}_s^2+(\ddot{x}\sin\varphi_s+\ddot{y}\cos\varphi_s) \qquad (3)
\end{aligned}
$$

where $\beta_\rho=\frac{h_\rho}{m};\beta_\sigma=\frac{h_\sigma}{m};\ \ \omega_\rho^2=\frac{C_\rho}{m};\ \ M=M^0+2m_0+2m,\ \omega_\sigma^2=\frac{C_\sigma}{m}.$

Here m_o is additional movable mass in the unbalanced mass;
m is the mass of the debalance;

M is the mass of the whole plant;
ε is the eccentricity or the value of displacement of the center of masses of the unbalanced mass from its axis of rotation;
r is length of an unstressed spring inside the rotor;
ρ is the displacement of the movable mass from the rest or the movable mass radius in the stabilized regime;
I is the central moment of inertia of the whole system;
x is the abscissa of the bearer or the horizontal displacement;
y is the ordinate of the bearer or the vertical displacement;
ω_ρ is angular velocity (frequency of the mass's intrinsic oscillations) of mobile mass ;
ω is synchronizable (steady) angular velocity of the rotor;
φ is the angle of the rotation of rotor from the rest;
β_x, β_y are coefficients of the damping of oscillations of the bearer along corresponding axes;
K is the coefficient of electric damping of the motor;
ω_i is the intrinsic (angular) velocity of the unbalanced mass;
σ_s are numbers that equal 1 or -1; the first case corresponds to the debalance's rotation in positive direction and the second case corresponds to the clockwise direction.

The Stationary Regime of Motion of the System

Now, investigate the case when the rotor has only one inner degree of freedom, related with the possibility of displacement of an additional mass radially and elastic and demping forces of all springs of the system are taken into account.

For the stationary synpfase regime of motion of equal vibroexciters, i.e. $\varphi_1 = \varphi_2 = \omega t$; $\rho_1 = \rho_2 = \rho = const$, the system of equations assumed the form

$$\begin{aligned} &[m_0\varepsilon + m(r+\rho_s)](\ddot{x}\sin\omega t + \ddot{y}\cos\omega t) = K(\omega - \omega_s) \\ &M\ddot{x} = 2[m_0\varepsilon + m(r+\rho)]\omega^2\cos\omega t - \beta_x\dot{x} - C_x x \\ &M\ddot{y} = 2[m_0\varepsilon + m(r+\rho)](-\omega^2\sin\omega t) - \beta_y\dot{y} - C_y y \\ &\omega_\rho^2\rho = (r+\rho)\omega^2 - (\ddot{x}\cos\omega t + \ddot{y}\sin\omega t) \end{aligned} \tag{4}$$

where $\omega_\rho^2 = \frac{C_\rho}{m}$; $M = M^0 + 2m_0 + 2m$

For the stationary regime we receive from the second and third equations:

$$x = \frac{2(m_0\varepsilon + m(r+\rho))\omega^2(C - M\omega^2)}{(M\omega^2 - C)^2 + \beta^2\omega^2}\cos\omega t +$$

$$+\frac{2(m_0\varepsilon + m(r+\rho))\omega^3\beta}{(M\omega^2 - C)^2 + \beta^2\omega^2}\sin\omega t \qquad (3)$$

$$y = \frac{2(m_0\varepsilon + m(r+\rho))\omega^3\beta}{(M\omega^2 - C)^2 + \beta^2\omega^2}\cos\omega t + \frac{2(m_0\varepsilon + m(r+\rho))\omega^2(M\omega^2 - C)}{(M\omega^2 - C)^2 + \beta^2\omega^2}\sin\omega t$$

Then

$$\begin{aligned} \ddot{x}\sin\omega t + \ddot{y}\cos\omega t &= -\frac{2\omega^5\beta(m_0\varepsilon + m(r+\rho))}{(M\omega^2 - C)^2 + \beta^2\omega^2} \\ \ddot{x}\cos\omega t - \ddot{y}\sin\omega t &= \frac{2\omega^4(m_0\varepsilon + m(r+\rho))(M\omega^2 - C)}{(M\omega^2 - C)^2 + \beta^2\omega^2} \end{aligned} \qquad (5)$$

Then, substituting to the system (2) we obtain

$$\begin{aligned} [m_0\varepsilon + m(r+\rho)]\left(-\frac{2\omega^5\beta(m_0\varepsilon + m(r+\rho))}{(M\omega^2 - C)^2 + \beta^2\omega^2}\right) &= K(\omega - \omega_s) \\ \omega_\rho^2\rho = (r+\rho)\omega^2 &+ \frac{2\omega^4(m_0\varepsilon + m(r+\rho))(C - M\omega^2)}{(M\omega^2 - C)^2 + \beta^2\omega^2} \end{aligned}$$

And we rewrite after identical transformations in the following form

$$\begin{aligned} K_s(\omega - \omega_s)[(M\omega^2 - C)^2 + \beta^2\omega^2)] + 2\omega^5\beta(m_0\varepsilon + m(r+\rho))^2 = 0 \\ [(r+\rho)\omega_s^2 - \omega_\rho^2\rho][(M\omega^2 - C)^2 + \beta^2\omega^2)] - 2\omega^4(m_0\varepsilon + m(r+\rho))(M\omega^2 - C) = 0 \end{aligned} \qquad (6)$$

From the second equation for stationary regime we can express ρ :

$$\rho = \frac{r\omega^2[(M\omega^2 - C)^2 + \beta^2\omega^2] - 2\omega^4(m_0\varepsilon + mr))(M\omega^2 - C)}{(\omega_\rho^2 - \omega^2)[(M\omega^2 - C)^2 + \beta^2\omega^2] + 2\omega^4 m(M\omega^2 - C)}$$

This equation is an analytical expression for the coordinate ρ of the additional mass in the steady-state regime.

The Expression for The Displacement of The Movable Mass

Changing the nominator and the denominator of the fraction and taking into account that $\frac{m_0}{M}$ and $\frac{m}{M}$ are small, where as $\frac{C}{\omega^2}$ and $\frac{\beta}{\omega^2}$ tend to zero, we receive

$$\rho \approx r \frac{1}{\frac{\omega_\rho^2}{\omega^2} - 1}\ , \qquad \omega_\rho \neq \omega \tag{7}$$

From here one can see, that for $\omega_\rho < \omega$ value of ρ is positive, i.e. the movable mass move from the axe of rotation, but for $\omega_\rho > \omega$ approach to it.

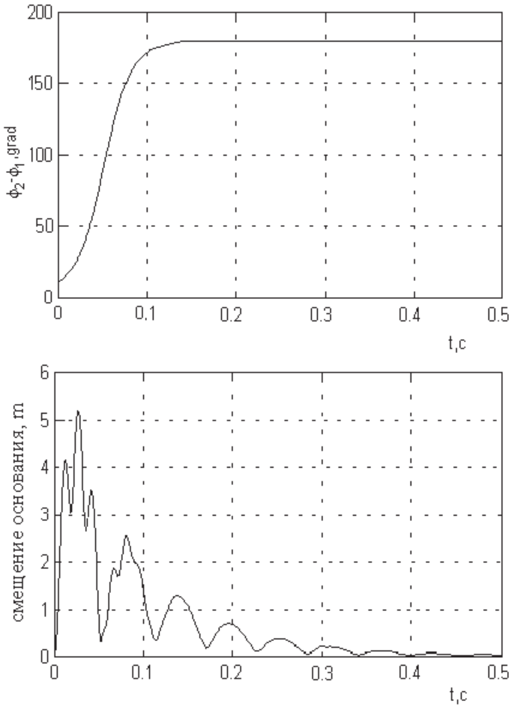

Figure 2: The behaviour of the plant without the movable masses

On the Fig. 2 the changing during the time difference of the phases of rotating and displacement of center of the mass of bearer without additional masses in debalances is shown. The curves correspond to these parameters: $M_0 = 2$ kg, $m_0 = 1$ kg, $r = 0.01$m, $\varepsilon = 0.035$m, $\beta_x = 84$, $C_x = C_y = 12.6\,\frac{H}{m}$, $\omega_s = 300\,\frac{rad}{c}$.

The Conditions of Temporary Stability by Blekchman-Sperling

Let us return to the first equation (1) and transform it identically using $\frac{1}{1+x} \approx 1 - x$. As a result we receive

$$\omega \approx \omega_s \{1 - \frac{2\beta[m_0\varepsilon + m(r+\rho)]^2}{M^2 K}\} \tag{8}$$

Combining (3) and (4) express ω

$$\omega^2 \approx \frac{\rho}{r+\rho}\omega_\rho^2 \{1 - \frac{2\beta[m_0\varepsilon + m(r+\rho)]^2}{M^2 K}\}^2 \tag{9}$$

Check if the 3rd condition of temporary stability [1]

$$\omega^2 \; [1 + \frac{2(m_0\varepsilon + m(r+\rho))^2 - 4mM(r+\rho)^2}{MI^O}] < \omega_\rho^2 \tag{10}$$

Then, after substituting

$$1 + \frac{2(m_0\varepsilon + m(r+\rho))^2 - 4mM(r+\rho)^2}{MI^O} < \frac{r+\rho}{\rho} \frac{M^4 K^2}{(M^2 K - 2\beta[m_0\varepsilon + m(r+\rho)]^2)^2} \tag{11}$$

Dividing the nominator and the denominator of each fraction, except the last one, in the left part of the inequality by M and taking into account that $\frac{m}{M}$ and $\frac{m_0}{M}$ are infinitely small we receive

$$1 - \frac{4m(r+\rho)^2}{I^O} < \frac{r+\rho}{\rho} \frac{M^4 K^2}{(M^2 K - 2\beta[m_0\varepsilon + m(r+\rho)]^2)^2} \tag{12}$$

Denote

$$a = \frac{M^4 K^2}{(M^2 K - 2\beta[m_0\varepsilon + m(r+\rho)]^2)^2}$$

and suppose, that $a < 1$

Then the last inequality can be written as:

$$1 - \frac{4m(r+\rho)^2}{I^O} < \frac{r+\rho}{\rho} \tag{13}$$

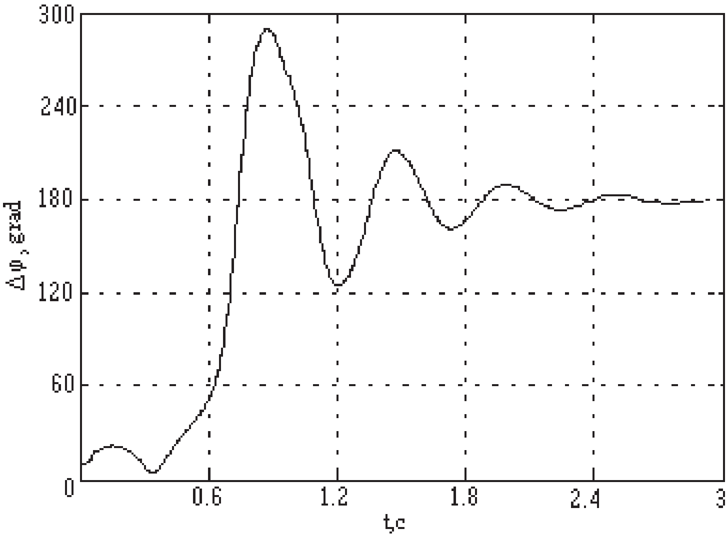

Figure 3: The behaviour of the plant with inner degrees of freedom in debalances

which always fulfils, i.e. m, $I^0 > 0$.

To satisfy the 1st and the 2nd conditions that

$$\omega < \frac{\omega_\rho}{\sqrt{e'^*}}$$

$$\omega < \frac{\omega_\rho \sqrt{M}}{\sqrt{M - 2m}}$$

choose an appropriate ω_ρ and other initial data.

It is always possible to meet 1st condition: $\omega > \omega_\rho$ and 2nd condition of temporary stability by choosing an appropriate ω_ρ and other initial data.

On the Fig 3 the results on the modeling with same initial data as Fig.2 with additional movable masses $m = 0.1; \varepsilon = 0.035$ in the debalances.

Thus it is shown, that first two known conditions of temporary stability can be satisfied by choosing initial data where as third condition is always satisfied. In other words, it is proved that the solution of the steady-state regime satisfies the conditions of the temporary stability from [1].

Conclusion

The system of six non linear differential equation second order, describing the behaviuor of two coaxed undebalanced vibroexciters with inner degree of freedom, fixed on softlyisolated rigid body is researched by numerical methods. It is shown that the unstable synphase regime of selfsynchronizing with two debalances transforms into temporary (hyroscope) stability by means of the introduce the inner degree of freedom in each of the debalances.

Literature

1. Blekchman I.I. Selected Topics in Vibrational Mechanics. World Scientific 2004, 414p.