Adaptive Coding for Transmission of Position Information Over the Limited-band Communication Channel [1]

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Abstract: The problem is considered of transmission object position estimates using band limited communication channels. Due to advances in technology, it is feasible to have a tracking filter attached to the sensor, and transmit only the condensed information as a tradeoff between computation and communication requirements. This paper proposes a scheme, which minimizes transmission data rate by sending adaptively encrypted innovations. It is assumed that the second derivative of the object position is bounded above by the known constant, and the sample rate is the design parameter. Adaptive tuning procedure for the coder range parameter is proposed and numerically studied.

Keywords: Track Fusion, Band Limited Communication Channels, Observer-Based Synchronization

1. INTRODUCTION

A fast growth of interest to the problem of estimation and control under information constraints has been observed within the control community during the last decade. A number of research groups are studying limit possibilities of control under such constraints (Nair and Evans, 2003; Nair and Evans, 2004; Matveev and Savkin, 2004) and others. The requirement of finite data rate demands the use of coders (quantizers) in the control loop which makes the system *hybrid* and makes its analysis more complex (Liberzon, 2003). The known results are close to complete for the equilibrium stabilization problem. However, they leave open similar questions for many other important problems. If the desired state vector of the system is time-varying (e.g. for tracking problems) no tight bounds for transmission rate ensuring convergence of the error to zero are known.

In a number of engineering applications (e.g. in distributed sensor networks, or remote surveillance systems), there is no possibility to mount advanced measurement/estimation devices on the transmitter (target) side (Liberzon, 2003; Nair and Evans, 2003; Matveev and Savkin, 2004; La Scala and Evans, 2005; Evans *et al.*, 2005; Malyavej *et al.*, 2006). In these cases only measurements of some scalar output variable of the transmitter system are available. Such a problem was studied in (Fradkov *et al.*, 2006), where results on observer-based synchronization of chaotic systems, represented in the Lurie form are given, and optimality of the binary coding for coders with one-step memory is established. In the present paper the results of (Fradkov *et al.*, 2006) are applied to the problem of state estimation via the limited-band communication channel for transmission of position information. The system considered consists of sensor node, which contains a tracking filter and encoder, bandwidth limited communication channel, and receiver node with a decoder. Some simplifying assumptions are made in the paper. It is assumed that the sensor uses linear Kalman filter for tracking, and that the receiver knows all the sensor parameters. Further simplifications are achieved by assuming that there are no clutter measurements, that probability of detection equals one, and thus there are no data association issues. The source (sensor) node performs a simple Kalman filter based tracking. The channel is assumed bandwidth limited, but otherwise noiseless. In the present paper the results of (Mušicki *et al.*, 2006) are expanded to *adaptive* coding.

2. CODING PROCEDURES

The following time-varying coder with memory is used in the present work for observations transmission over the communication channel. This procedure is based on the results of (Nair and Evans, 2003; Brockett and Liberzon, 2000; Liberzon, 2003; Fradkov *et al.*, 2006).

2.1 Primitive coder

Let y be an observation signal to be transmitted over the channel. A uniform scaled coder function $q_{\nu,M}(y)$ is defined as

$$q_{\nu,M}(y) = \begin{cases} \delta \cdot \langle \delta^{-1} y \rangle, & \text{if } |y| \le M, \\ M \operatorname{sign}(y), & \text{otherwise,} \end{cases} \qquad (1)$$

where $\langle \cdot \rangle$ denotes round-up to the nearest integer function, $\operatorname{sign}(\cdot)$ is the signum function; M is the coder *range parameter*; $\delta = 2^{1-\nu} M$ is length of the *discretization*

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interval. The *range interval* $\mathcal{I}$ of length $2M$ is defined as $\mathcal{I} = [-M, M]$. Evidently, $|y - q_{\nu,M}(y)| \leq \delta/2$ for all y such that $y : |y| \leq M + \delta/2$ and all values of $q_{\nu,M}(y)$ belong to the range interval $\mathcal{I}$. Notice that the interval $\mathcal{I}$ is equally split into 2^ν parts. Therefore, the cardinality of the mapping $q_{\nu,M}$ image is equal to $2^\nu + 1$, and each codeword symbol contains $R = \log_2(2^\nu + 1) = \log_2(2M/\delta + 1)$ bits of information. Thus, the discretized output of the considered coder is found as $\bar{y} = q_{\nu,M}(y)$. Measurements of the plant output and codeword transmission of the over the channel are produced with the given constant sampling period T_s. It is assumed that the coder and decoder make decisions based on the same information.

2.2 Time-varying coder with memory

The *zooming strategy* with a time-varying quantizer (with different values of M for each instant, $M = M_k$) is applied. Zooming is effective for coders with *memory* (Liberzon, 2003). To describe this kind of coders, introduce the sequence of *central numbers* c_k, $k = 0, 1, 2, \ldots$ with the initial condition $c_0 = 0$. At the step k, the coder compares the current measured output y_k with the number c_k, forming the deviation signal $\partial y_k = y_k - vc_k$. Then this signal is discretized with a given ν and $M = M_k$ according to (1). The output signal

$$\bar{\partial} y_k = q_{\nu, M_k}(\partial y_k) \tag{2}$$

is transmitted over the communication channel to the receiver. Then the central number c_{k+1} and the range parameter M_k are renewed based on the available information about the driving system dynamics. The following update algorithm is used in one-step memory coders:

$$c_{k+1} = c_k + \bar{\partial} y_k, \quad c_0 = 0, \quad k = 0, 1, \ldots. \tag{3}$$

Optimization of coders w.r.t. the *bit-per-second* rate $\bar{R} = R/T_s$ is considered in (Fradkov *et al.*, 2006). It is shown that the *binary* coding scheme gives the optimal transmission rate $\bar{R} = 1/T_s$ baud, which yields $M = \Delta/2$ as an optimal value for M, where $\Delta = \sup_t |\delta_y(t)|$ is the upper bound of the transmission error. Therefore, the *signum* function is an optimal one for coder, and (1) turns to

$$q_{0,M_k}(y) = M_k \operatorname{sign} y. \tag{4}$$

It gives the coder output as $\bar{\partial} y_k = M_k \operatorname{sign}(\partial y_k)$.

At the initial stage of the system evolution the error $|\delta_y|$ may exceed the bound Δ, because the initial value $y(0)$ is not known. This initiated the transient mode of the system behavior. The zooming strategy is efficient at this stage. The values of M_k may be precomputed (the *time-based* zooming), or, alternatively, current quantized measurements may be used at each step to update M_k (the *event-based* zooming).

2.3 Adaptive coder

To start, consider the following simple time-based zooming procedure of (Fradkov *et al.*, 2006):

$$M_k = (M_0 - M_{\min})\rho^k + M_{\min}, \quad k = 0, 1, \ldots, \tag{5}$$

where $0 < \rho \leq 1$ is the decay parameter, $M_{\min}$ stands for the limit (minimal) value of M_k. The initial value M_0 should be large enough to capture all the region of possible initial values of y_0.

If parameter $M_{\min}$ is mistakenly chosen too small, the data transmission process by means of the described procedure may failed due to the coder saturation. In the present work, the time-based zooming procedure with the event-based correction is proposed: if the coder is not saturated, the quantizer range M is exponentially decreased; if the coder saturation appears, the quantizer range M is increased. Such an *adaptive* coding makes it possible to maintain the minimal discretization interval δ and, therefore, minimize transmission errors. At the same time, this prevents fail in tracking the signal y_k due to the saturation.

The proposed method for tuning the quantizer range M is described by the following recurrent algorithm:

$$\begin{aligned} \lambda_k &= (\bar{\partial} y_k + \bar{\partial} y_{k-1})/2, \\ M_k &= m + \begin{cases} \rho M_{k-1}, & \text{if } |\lambda_k| \leq 0.5 \\ M_{k-1}/\varrho, & \text{otherwise}, \end{cases} \end{aligned} \tag{6}$$

where $0 < \varrho \leq 1$ is the *decay parameter*; $m = (1-\rho) M_{\min}$, $M_{\min}$ assigns the minimal possible value for M_k. The initial value M_0 should be large enough in comparison with possible y_0. The procedure (6) leads to time-based decreasing of M_k while signs of the successive values of $\bar{\partial} y_k$ alternate. The sort of discrete-time sliding-mode tracking the plant output appears in that case, and M_k recursively tends to the limiting value $M_{\min}$. When the moving average of the transmission error exceeds the threshold, the second alternative of algorithm (6) is realized, and the quantizer range M_k increases.

The equations (1), (2), (6) describe the coder algorithm. The same algorithm is realized by the decoder. Namely, the decoder calculates the variables $\tilde{c}_k$, $\tilde{M}_k$ based on received codeword flow similarly to c_k, M_k.

2.4 Time-varying coder of full order

The sequence of the central numbers $\{c_k\}$ defined by (3) is produced by one-step memory coder, corresponding zero-order extrapolation of the measured plant output. This coder ensures asymptotically exact transmission of the sensor observations if the plant model is an unperturbed integrator, no sensor errors present and the channel is ideal (Wong and Brockett, 1997; Nair and Evans, 1997; Matveev and Savkin, 2004; Malyavej and Savkin, 2005). Let us turn to the full-order coder, which may be used for asymptotically exact transmission of the sensor observations for more common class of plant models. Namely, consider the following LTI discrete-time plant model

$$x_{k+1} = Ax_k + Bu_k + E\varphi_k, \quad y_k = Cx_k, \tag{7}$$

where $x_k \in \mathbb{R}^n$ is the plant state vector, $u_k \in \mathbb{R}^m$ is the external input signal, applied to the plant, $y_k \in \mathbb{R}$ is the scalar plant output, measured by the sensor, $k = 0, 1, \ldots$. The matrices A, B, C, E and the signal u_k are assumed to be known both at the transmitter and the receiver nodes, the sensor output y_k is subjected to coding/decoding procedure to be transmitted over the communication channel with limited capacity. The *external disturbance* vector $\varphi_k \in \mathbb{R}^s$ is an irregular process, unmeasured by the sensor. The pair (A, C) is assumed to be observable. As above, we neglect the sensor or channel errors and the plant model (7) imperfection.

Consider the following full-order (Kalman) observer for the plant (7)

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L_k\bar{\partial}y_k, \quad \hat{y}_k = C\hat{x}_k, \tag{8}$$

where the error signal $\bar{\partial}y_k$ is defined by (2), where $\partial y_k = y_k - \hat{y}_k$, $L_k \in \mathbb{R}^{n\times 1}$ is the observer matrix gain (column vector), $k = 0, 1, \ldots$. It should be noticed that the coding procedure (2), (3) may be considered as a particular case of the observer-based coding procedure (8), where $A = B = L = 0$, $n = 1$. Also notice that for the considered data transmission scheme, the observer output $\hat{y}_k$ plays a role of the central number c_k. Matrix L_k is to be found at the stage of the observer design. The least-square optimal solution for L_k may be found as precomputed Kalman gain sequence. In the present work, the constant matrix $L_k \equiv L$ is taken, and the pole-placement technique is used to find the matrix gain L. To calculate the range parameter M_k, the time-based zooming procedure (5) or adaptive algorithm (6) may be applied. The observation algorithm (8), (6) is reproduced at the receiver node based on the values of $\bar{\partial}y_k$, transmitted over the channel.

Let us study accuracy of the observer (8). To this end, write down the state error equation for the plant model (7) and observer (8). Subtracting (8) from (7), one obtains:

$$\begin{aligned} e_{k+1} &= (A - L_kC)e_k + Bu_k + L_k\zeta_k - E\varphi_k, \\ \partial y_k &= Ce_k, \end{aligned} \tag{9}$$

where $e_k = x_k - \hat{x}_k$ is the *observer state error*. To examine the state error behavior, let us assume that the deviation signal $\partial y_k = y_k - \hat{y}_k$ is bounded, $|\partial y_k| \le M_k + \delta_k/2$ for all $k = 0, 1, 2, \ldots$, where $\delta_k = 2^{1-\nu}M_k$ (see Sec. 2.1). Under this assumption we may represent the signal $\bar{\partial}y_k$ in (8) as $\bar{\partial}y_k = \partial y_k + \zeta_k = y_k - \hat{y}_k + \zeta_k = y_k - C\hat{x}_k + \zeta_k$, where ζ_k is the bounded "*coding error*",

$$|\zeta_k| \le \delta_k/2 = M_k/2^{\nu}. \tag{10}$$

Particularly, for the binary coder (4) it is valid that $|\zeta_k| \le M_k$. Inequality (10) makes it possible to find the upper bound of the state estimation error $\max_k |e_k|$ and, consequently, to find the upper bound of the output transmission error $\max_k |\partial y_k|$, applying the H_∞-norm technique. Under the assumption that $\|\varphi_k\| \le \bar{\varphi}$, we may use the same approach for evaluation the upper bound of the errors effected by the external disturbance φ_k.

It should be noticed that the adaptive quantizer with a memoryless (static) coder was considered in (Goodman and Gersho, 1974). A more general nth order observer-based coding procedure was proposed in (Andrievsky *et al.*, 2007), where application to pitch motion control of the Helicopter laboratory set-up was presented. Note also that the adaptive coding for a special case of first-order system is analyzed in (Gomez-Estern *et al.*, 2007).

3. REMOTE SURVEILLANCE SYSTEM WITH BIT-RATE CONSTRAINED COMMUNICATION CHANNEL

3.1 Problem of target position transmission with data-rate limitations

Based on the papers (Sciacca and Evans, 2002; La Scala and Evans, 2005; Evans *et al.*, 2005; Malyavej *et al.*, 2006; Mušicki *et al.*, 2006), consider the remote surveillance system having several sensors connected to a multiplexer, which in turn is connected via a limited and/or time-varying data rate communication channel to a fusion centre, where target tracking based on observations provided by the sensors are performed (Sciacca and Evans, 2002; Evans *et al.*, 2005). In the case of limited channel data rate, the problem arises of transmission the position information from all sensors without loss to the fusion centre. To simplify the exposition, we assume at the sequel that the channel capacity is divided between the different sensors, and consider a single sensor supplying information within the data rate limit.[1] More definitely, we consider the object tracking problem, assuming that the observation signal $y(t)$ (the object coordinates in the given reference frame) is coded with symbols from a finite alphabet at discrete time instants $t_k = kT_s$, where T_s is the sampling period, $k = 0, 1, \ldots$. Coded symbol is transmitted over a digital communication channel with a finite capacity. Observation noise, transmissions delay, and transmission channel distortions are neglected. Assume, that coded symbols are available at the receiver side at the same sampling instant t_k, as they are generated by the coder.

The problem is to find coder/decoder pair algorithm, minimizing the data flow over the channel without loss of the target position information.

3.2 Target Model

Consider the case of tracking a target moving with near constant ground speed V in one dimension of the horizontal plane with measurements of position by means of the remote sensor with the sampling period T_s. Target position should be transmitted over the communication channel to the fusion centre. The sensor errors, communication time delay and channel drop outs are neglected. The target lateral acceleration $a(t)$ is assumed to be bounded irregular process, $|a(t)| \le \bar{a}$ for all t. No additional information concerning the target acceleration is available. A state-space model for such a scenario is given by (7), where $B = \mathbf{0}$, $u_k \equiv 0$, $\bar{\varphi} = \bar{a}$,

$$A = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad C = [1, 0], \quad E = T_s \begin{bmatrix} T_s/2 \\ 1 \end{bmatrix}. \tag{11}$$

3.3 Coder design

The coder (2), (5) (or (6)), (8) design parameters are: digit capacity ν; sampling period T_s; range parameters M_0, $M_{\min}$, ρ; observer matrix gain L_k. At the sequel the *binary* coder (4) and the constant matrix $L_k \equiv L$ are taken.

The matrix L was found at the steady-state mode ($M_k \equiv M$) as a solution to the following constrained optimization problem:

(1) the eigenvalues $z_{1,2}$ of the matrix $A_L = A - LC$ should be bounded, $|z_{1,2}| \le \exp(-\eta T_s)$, where $\eta > 0$ is the given stability margin setting the lower bound of the observer convergence rate;
(2) total output transmission error $Q_y = \max_k |\partial y_k|$, caused by the bounded coding error ζ_k, $|\zeta_k| \le M$, and

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[1] A problem of switching the channel between the sensors (*a sensor selection problem*) is studied thoughtfully in (Evans *et al.*, 2005).

bounded external disturbance φ_k, $|\varphi_k| \le \bar{a}$, should be minimized;

(3) condition $Q_y \le M$ should be fulfilled.

To find the upper bound Q_y of the output transmission error (see *cond. 2*), the H_∞-norm of the systems $\mathcal{S}_1 = \{A_L, L, C\}$ and $\mathcal{S}_1 = \{A_L, E, C\}$ were calculated. Fulfillment of the condition *3* was checked after optimization and only feasible solutions were selected.

3.4 Numerical example

Let $\bar{a} = 10$ m/s^2, $\eta = -1$ (see *cond. 1*). Consider the steady-state mode, $M_k \equiv M$, and pick up the pairs (T_s, M) $\in$ $[0.1, 0.2, \ldots, 1.0] \times [0.5, 2{,}5, 10, 20]$. Resulted output transmission error Q_y as a function of the sampling period T_s for different M is depicted in Fig. 1. It is seen that the problem is infeasible for $M = 0.5$ m if the sampling period exceeds 0.1 s. This means that the data transmission rate $R = 1/T_s$ should not be less than 10 baud to ensure the upper bound of position transmission error $Q_y = 1$ m. If $T_s = 1$ s ($R = 1$ baud), $M = 17$ m may be taken, which gives the guaranteed upper bound $Q_y \approx 34$ m. The surface plot for tracking error versus R and M is shown in Fig. 2.

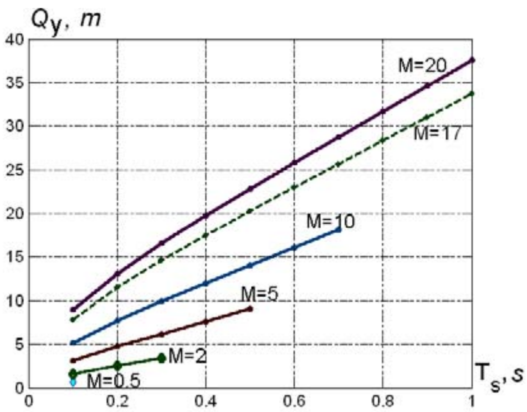

Fig. 1. Tracking error vs sampling period for different values of the range parameter M.

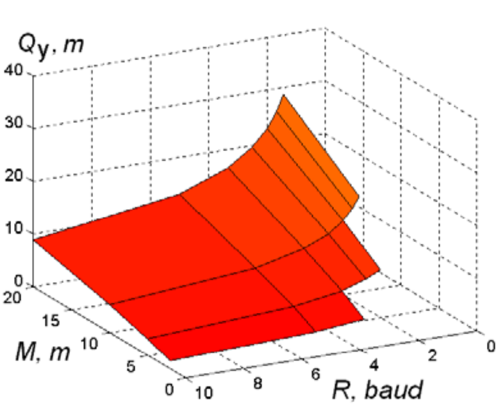

Fig. 2. Tracking error vs sampling rate R and range parameter M.

For simulation, consider the following target motion scenarios. Let the target moves at the horizontal plane, its position data are described in some Earth reference frame (O_g, X_g, Y_g). Target groundspeed is $V = 270$ m/s. The sensor is located at the origin O_g. The first scenario assumes that the target has the constant course angle $\Psi = -2.5$ rad. According to the second scenario, the target turns to $\Psi = -3.45$ rad starting from the point of time $t_{\rm turn} = 60$ s. Course angular velocity $\dot{\Psi}(t)$ is bounded, $|\dot{\Psi}| \le 0.037$ rad/s, which limits the target lateral acceleration a_t: $|a_t| \le 10$ m/s^2.

Two identical channels of target tracking, along O_gX_g and O_gY_g axes is considered. The sampling period is taken as $T_s = 1$ s.

Examination of data transmission with time-based zooming. Examine the data transmission system with time-based zooming (5). Pick up $\rho = 0.9$, $M_0 = 25 \cdot 10^3$. Let no *a priory* information on the target position and course angle is available, therefore at $t = 0$ the estimates $\hat{x}_t$, $\widehat{V}_{t_x}$, $\hat{y}_t$, $\widehat{V}_{t_y}$ are zeroized.

Applying for $T_s = 1$ s, $\bar{a} = 10$ m/s^2 procedure of Sec. 3.3 we obtain: $M_{\min} = 20$ (see Fig. 1), $L = [1.26, 0.40]^{\rm T}$, $z_{1,2} = 0.368 \pm 0.0046i$, $|z_{1,2}| = 0.368$. The simulation results for the first scenario are depicted in Figs. 3–5. To feel the estimation error, a distance between the actual and estimated target positions $D = \sqrt{(x_t - \hat{x}_t)^2 + (y_t - \hat{y}_t)^2}$ (in meters) is calculated and its logarithmic measure vs t is plotted in Fig. 5 (curve *1*).

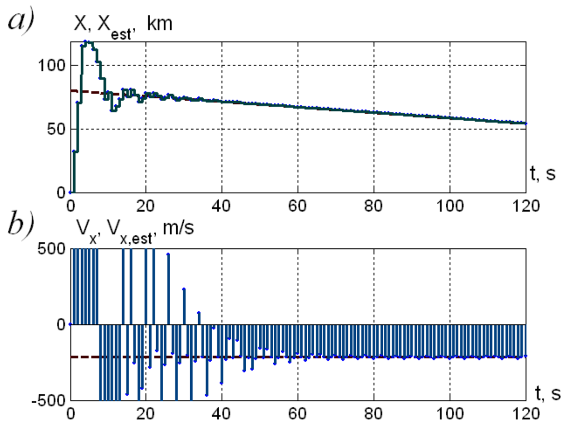

Fig. 3. Target tracking. O_gX_g axis projection. Time-based zooming (5), straight-line motion.

The simulation results for the first scenario are depicted in Figs. 6–8. It is seen that in the course of the turn the error $D(t)$ increases up to 124 m, and then falls to $\overline{\lim_{t\to\infty}} D(t) = \bar{D}$, $\bar{D} \approx 30$ m. Logarithmically scaled time history of $D(t)$ is plotted in Fig. 5 (curve *2*). Time-sequences of binary symbols, transmitted over the each channel are plotted in Fig. 8. Total quantity of information, transmitted during the considered time interval is 240 bit.

Examination of data transmission with adaptive zooming. Examine now the target position transfer algorithm with the adaptive coder (6). Take $M_{\min} = 0.5$, $\rho = 0.5$; the other parameters remain the same as in Sec. 3.4.1.

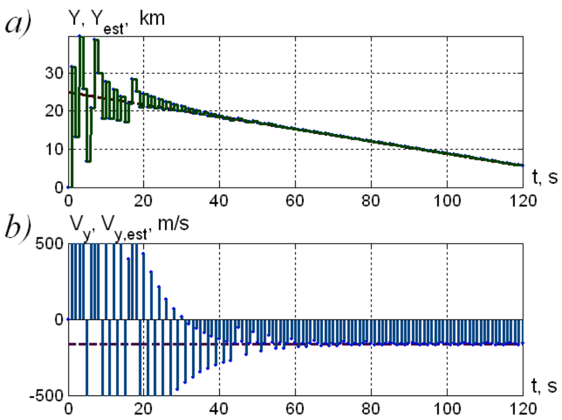

Fig. 4. Target tracking. $O_g Y_g$ axis projection. Time-based zooming (5), straight-line motion.

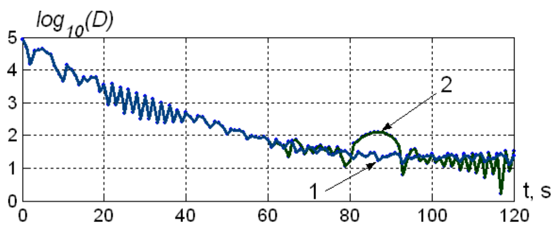

Fig. 5. Distance error (logarithmic scale). Time-based zooming (5). *1* – straight-line motion, *2* – motion with turn.

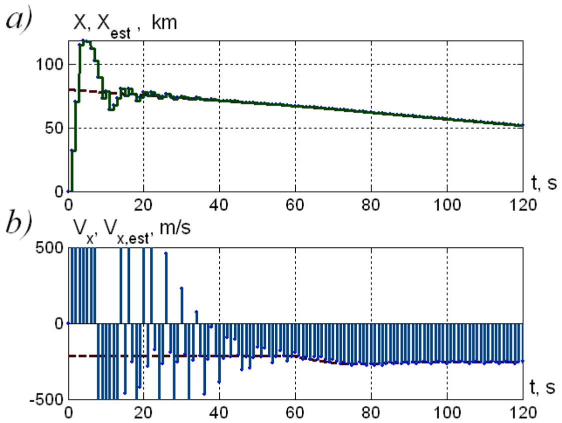

Fig. 6. Target tracking. $O_g X_g$ axis projection. Time-based zooming (5), motion with turn.

The simulations show that the upper bound of the steady-state error $\overline{\lim_{t\to\infty}} D(t) \approx 0.4$ m, the 5%-zone transient time $t_{0.05}$ is $t_{0.05} = 20$ s (i.e. 20 samples), the 1%-zone transient time $t_{0.01} = 33$ s (33 samples). These results are significantly better that those for coder with time-based zooming (5). Some simulation results are plotted in Figs. 9–13.

4. CONCLUSIONS

In the paper a communication scheme minimizing transmission data rate over bandwidth limited noiseless communication channel by sending encrypted innovations is

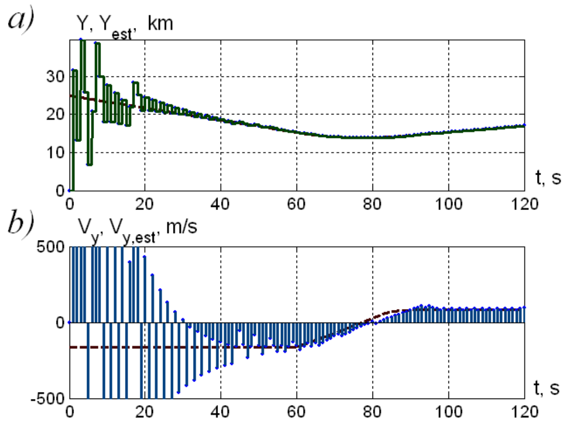

Fig. 7. Target tracking. $O_g Y_g$ axis projection. Time-based zooming (5), motion with turn.

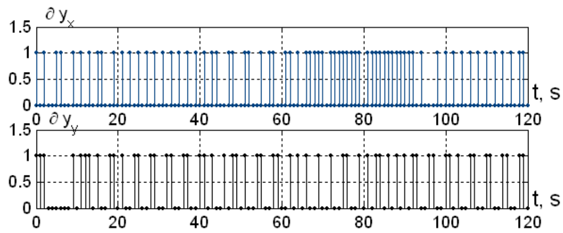

Fig. 8. Sequences of transmitted symbols. Time-based zooming (5), motion with turn.

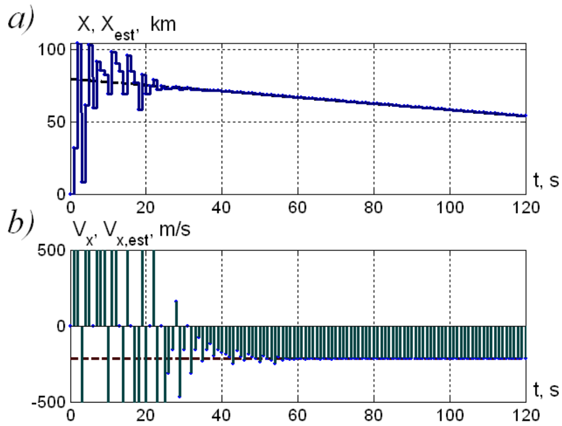

Fig. 9. Target tracking. $O_g X_g$ axis projection. Adaptive zooming (6), straight-line motion.

proposed. The observer-based full-order adaptive coding procedure of (Andrievsky *et al.*, 2007) is applied for target position transmission with data-rate limitations. The simulation results demonstrate efficiency of the proposed method.

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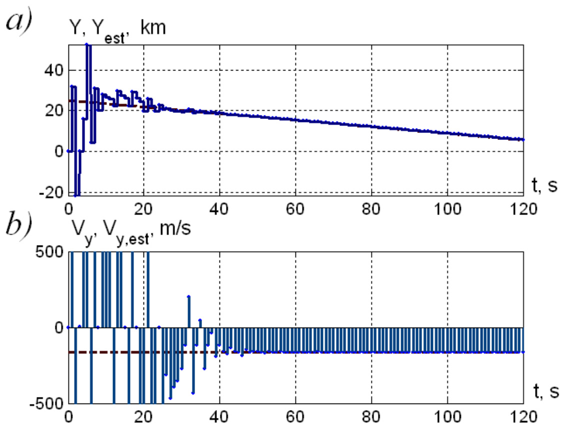

Fig. 10. Target tracking. O_gY_g axis projection. Adaptive zooming (6), straight-line motion.

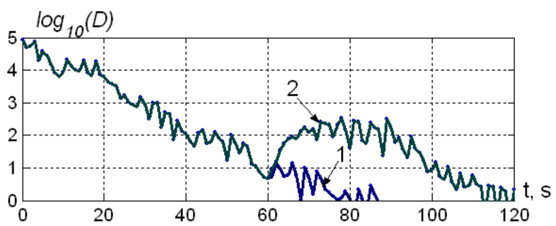

Fig. 11. Distance error (logarithmic scale). Adaptive zooming (6). *1* – straight-line motion, *2* – motion with turn.

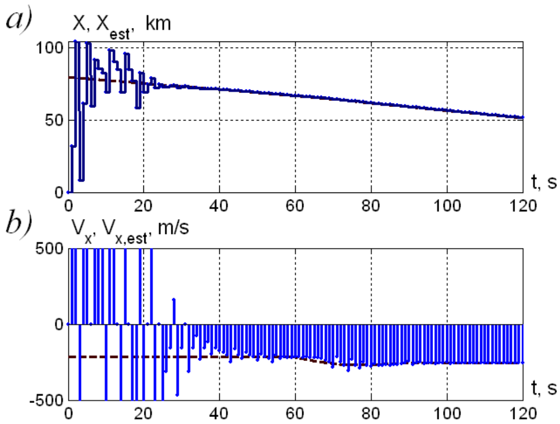

Fig. 12. Target tracking. O_gX_g axis projection. Adaptive zooming (6), motion with turn.

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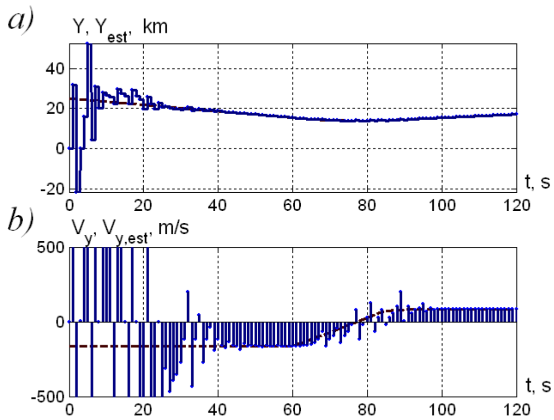

Fig. 13. Target tracking. O_gY_g axis projection. Adaptive zooming (6), motion with turn.

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