MULTIPARAMETRICAL OPTIMAL CORRECTION FOR CHAOS SUPRESSION IN A FAMILY OF DUFFING-VAN DER POL OSCILLATORS

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Abstract

A technique of chaos suppression in chaotic dynamical systems presented in this paper is based on the idea of multiparametrical correction of the system's parameters. The aim of correction is the chaotic system stabilization so that with the natural demand of minimal dynamic change of its parameters the modification of system's chaotic attractor into the stable limit set will be provided. The words "minimal changing" mean that the feature of proposed technique lies in providing the achievement of an optimum regime by the system. This regime can be obtained with Pontryagin maximum principle under special kind of dynamic changing of parameters. The optimal corrective function can be found for each parameter. The value of these functions is the possibility to localize the unique limit set in the phase space of the system and to investigate the peculiarities of the optimal transient process which provides the modification of chaotic attractor into this set. The results of numerical experiments with the family of nonlinear oscillators have confirmed the quality of chaos suppression and efficiency of the offered technique.

Key words

Suppression of chaos, multiparametrical analysis, optimal correction.

1 Introduction

An important feature of chaotic systems is the high sensitivity to the small parametric perturbations. It is especially significant when one needs to control complex dynamics. The practical application of well-known chaos control techniques [Fradkov, Evans and Andrievsky, 2006] arises the investigators' interest to the field of chaos control and to the upgrading the ways of investigation evolving in it.

Recent inclusion of the chaotic systems into the class of controllable objects defined the new concept of system perturbation aims [Shinbrot, Grebogi, Ott and Yorke, 1993]. It lies in the choice of control modifying the system limit set (LS) from unstable into the stable one. For modified LS one can take an unstable system state, unstable cycle or a chaotic attractor of the system. Though unlike the traditional stabilization task the quantitative characteristics of the target set are not given beforehand. Instead of it only the desirable type of the limit set (which must be stable) is postulated.

The developed field of investigation aimed at the qualitative changes in chaotic dynamical systems under certain external periodic parametric perturbations (that is at the chaotic attractor modification) may be called the studies of the problems related to suppression of chaos and non-feedback controlling of chaotic motion (generalize analytic studies of the problems see [Loskutov, 2006]). Note that the basis of the analytical apparatus which allows to estimate the efficiency of the parametric perturbation is constituted by Melnikov criterion. As a rule only one parameter is perturbated.

The presented paper is closely connected with the studies mentioned. But we focus our attention on the investigation of the technique which provides the achievement of the optimal regime by the system during the chaos suppression process. Thus we assume that the most complete understanding of the chaotic dynamics disappearance is possible only in multiparametrical analysis.

In this context, notice several established fields of investigation which exploit the idea of multiparametrical approach to the non-linear phenomena investigation. Important results widely applicable to the problems of mechanics were obtained through the development of the techniques of the classical theory of stability (multiparametrical theory of stability [Seyranian, Mailybaev, 2003]). We study linearized equations depending on several parameters. The application of analytical apparatus of multiparametrical bifurcation analysis of eigenvalues to the oscillator systems allows studying the space of parameters, defining the features of the stability (instability) areas boundaries and discovering new mechanic effects. Thus a productive approach in dynamics, a multiparametrical investigation of chaos transitions [Kuznetsov, Turukina, Savin, 2002], is based on the application of the ideas of the renormalization group (RG) analysis. The analysis of solutions of RG equations makes it possible to study various extreme situations which take place in multiparametrical analysis of chaos appearance in dynamic systems (universal characteristics, self-similarity laws, peculiarities of parameters' space organization at the edge of chaos [Kuznetsov, Kuznetsov and Sataev, 2005]).

In this work the object of multiparametrical optimal correction is nonlinear generalization of the classical self-oscillatory system – periodically forced Duffing-van der Pol (DvdP) oscillator:

$$\ddot{x} - \mu(1-x^2)\dot{x} + \alpha x + \beta x^3 = f\cos(\omega t),\ \mu > 0\,. \quad (1)$$

A large number of physical and engineering problems [Lakshmanan, Murali, 1996] are reduced to the investigation of the equation (1). A particular representative of the family of DvdP oscillators is defined by the restrictions of parameter values α and β of the unperturbed potential: $U(x) = \alpha x^2/2 + \beta x^4/4$. According to the condition $dU(x)/dx = 0$ DvdP oscillator has three equilibria: $x^e_{(1)} = (0,0)$ and $x^e_{(2,3)} = (\pm\sqrt{-\alpha/\beta}, 0)$. We shall study two most outstanding and interesting physical situations: (i) single-well potential ($\alpha > 0$, $\beta > 0$) and (ii) double-well potential ($\alpha < 0$, $\beta > 0$).

In case (i) parameters causing chaotic behavior will be chosen for $\alpha >> \beta > 0$ that is for a very small value β. This particular situation is thoroughly studied in [Sanjuán, 1996]. It is shown that two routes to chaos: period-doubling and the intermittency associated with the saddle-node bifurcations are peculiar to the system. The characteristic feature of the case lies in the fact that the coordinates of the points $x^e_{(2,3)}$ are imaginary. The typical chaotic state with the parameter values $\alpha = 1$, $\beta = 0.00025$, $\mu = f = 5$, $\omega = 2.4665$ investigated in the work is shown in Fig.1,(a) where phase variables of the first-order system

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\alpha x_1 - \beta x_1^3 + \mu(1 - x_1^2)x_2 + f\cos(x_3),$$
$$\dot{x}_3 = \omega$$

are plotted on the axes. It is equivalent to the equation (1) and useful from both a theoretical and a numerical point of view.

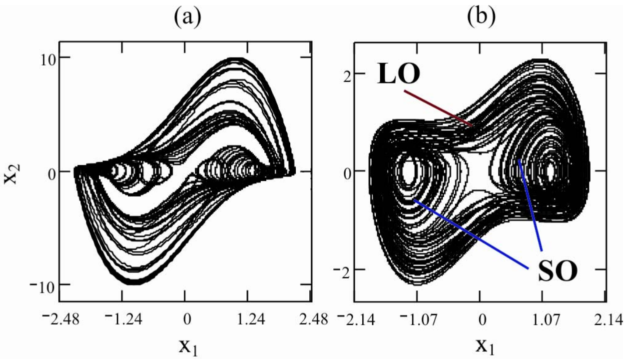

Figure 1. Chaotic regimes of a family of Duffing-van der Pol oscillators: (a) – single-well potential; (b) – double-well potential.

The case (ii) will be studied for $|\alpha| > \beta$, when chaotic attractor includes two types of unstable orbits, closed and separated by a separatrix when unforced. Fig. 1,(b) illustrates the systems chaotic state under investigation at $\alpha = -1.44$, $\beta = \mu = 1$, $f = 0.45$, $\omega = 1.2$. As you can see, the first type of motions surrounds all the three equilibrium states (large orbit (LO)). The second type (small orbit (SO)) occurs in the vicinity of one of the points $x^e_{(2,3)}$ and substantially depends on initial conditions.

Both models demonstrate a wide range of dynamic regimes including chaos and are of much interest for the studies of nonlinear phenomena and chaotic dynamics control [Li, Ji, Hansen and Tan, 2006].

2 General Problem Statement

Let $\dot{x} = f(x,p)$, $x \in R^n$ be a continuous-time dissipative chaotic system with the vector of real parameters $p = (p_1, p_2, ..., p_m)$ accessible for certain external perturbations. We assume that initial values of the parameters lie in the area of chaotic behavior of the trajectories. The trajectories are globally restricted ($\| x(t) \| < D$) and for all the initial conditions $x(0) = x_0 \in B_A$ the chaotic attractor A_p is the limit set of the system ($B_A(A_p) \subseteq R^n$ is the basin of attractor A_p). The structure of the attractor is determined by the set

$$E = \{x^e_{(k)}\}_{k=\overline{1,s}} = \{x \subset R^n \mid f(x,p) = 0\}\,.$$

Here s – the number of unstable equilibrium states in the system's phase space with respect to which the unstable cycles embedded into the attractor are localized.

In the general case the idea of space correction of the chaotic dynamic system parameters lies in the initial system transformation to the form

$$\dot{x} = f(x, p^*)\,, \quad (2)$$

where the accessible parameters are corrected according to the rule $p^*_j(t) = p_j(1 + h_j(t))$, $j = \overline{1,m}$.

The aim of correction is the stabilization of the system (2) so that with regard to the natural small demand $\| p^* - p \| \to \min$ the modification of A_p into the stable limit set M could be provided.

Despite the sufficient lack of the information on the target set M, we can distinguish two possible results of correction:

1) In the simplest case one of the equilibrium states of the system $x^e_* \in E$ will become the stable limit set.
2) Due to the greater variability the stabilization of the cycle $x^\tau_*(t)$, where τ – period of the cycle, is much more interesting. It would appear natural that the localization and characteristics of the periodic solution $x^\tau_*(t)$ depend on the structure and stability character of the equilibrium set E.

We define the set $M_E = \{(x,h) \mid f(x,p^*) = 0\}$ so that at $h \equiv 0$ it will coincide with E. Let

$\rho(M,x)=\inf_{z\in M}\|z-x\|$ be the minimal Euclidian distance between the set M and the point $x\in R^n$.

Then the aim of correction for the both cases mentioned is the provision of the *internal stability* of the target set

$$M(x_*^e,r)=\{x\in R^n \mid \|x-x_*^e\|\le r,\, x_*^e\in M_E\},$$

where r – is the radius of the sphere (determined by the system characteristics, that is by $\|x(t)\|<D$) containing the *unique limit set* (ULS). It means that for all $x_0\in B_A$ at $t\to+\infty$ together with (global) attracted condition

$$\rho(M(x_*^e,r),x(t))\to 0$$

for the solution $x=x(t)$ the conditions of asymptotical orbital (local) stability are also performed in relation to the state $x_*^e\in M_E\subset M(x_*^e,r)$. Observe that at $h\equiv 0$ the target set is unstable.

Hence if the internal stability conditions are performed the set $M(x_*^e,r)$ is invariant and includes either a closed trajectory – stable cycle $x_*^\tau(t)$ ($r=r_0=const$, $0<r_0<D$), or a stable equilibrium set x_*^e ($r\to 0$ at $t\to+\infty$).

High sensitivity of chaotic systems to small parametric changes calls for taking into account the possibility of variations of corrective influence intensity which provides the necessary modification of the system attractor. In this context consider the case of restriction to a corrective influence of the form

$$\|h(\cdot)\|\le a,\ 0<a<<1. \tag{3}$$

As the corrective influence has a dynamic character, let's make the localization and stabilization of the system unique state suit the requirement

$$\frac{1}{2}\int_0^T\|h(t)\|^2\,dt\to\min_{h\in U}, \tag{4}$$

where

$$U=\left\{h(\cdot)\in C[0,T] \mid \|h(t)\|\le a, t\in[0,T], T>>T^*\right\},$$

$C[0,T]$ is a class of continuous bounded functions, defined at the time interval $[0,T]$, T is the final but not fixed moment of time, definable from the transient process duration T^* of the achievement of $M(x_*^e,r)$.

Note that the choice of the function $h(t)$, $t\in[0,T]$, is simultaneously constrained by both the restriction (3) and the demand (4). The first considers obeying the demand of small perturbation on system parameters and presupposes the choice of the minimal quantity $a_{\min}$. The second shows a natural want to carry out the dynamic modification of chaotic attractor with minimal energy costs. Simultaneous compliance with the conditions will allow to identify the unique limit set embedded to $M(x_*^e,r)$.

To sum up we formulate the task of multiparametrical optimal correction the following way: *through the choice of value of the restriction* $a_{\min}$*, it is necessary to find such an admissible process* $(x(t),h(t))$*,* $t\in[0,T]$*, which for all* $x_0\in B_A$*,* $x_0\ne 0$*, meeting the condition* (4) *provides the internal stability of the set* $M(x_*^e,r)$. Note that the solution of the task means the realization of the optimal modification of chaotic attractor A_P into the stable ULS.

It is well-known that by virtue of necessity of provision the system's asymptotic stability the traditional optimal stabilization task is formulated at the infinite half-interval of time $[0,+\infty)$. The achievement of asymptotic state is required at multiparametrical correction as well. Besides, the consideration of the dynamic character of correction draws attention not only to the established regime and its characteristics, but also to the transient process during which the system demonstrates the behavior different from the one typical at $t\to+\infty$. In numerical simulation in time evolution of nonlinear dynamic systems it is always possible to identify the regions corresponding to the transient process and to the established regime as well as to define the transient process duration with the prearranged accuracy δ [Koronovskii, Starodubov and Hramov, 2002].

Hence in choosing a moment of time from the condition $T>>T_\delta^*$ where T_δ^* is the transient process duration of the system meeting the target set $M(x_*^e,r)$ specified with the accuracy δ the general task defined at the infinite half-interval of time $[0,+\infty)$, can be reduced to the similar task re-defined at the interval $[0,T]$. Then with the given number δ for the numerical estimation of transient process duration T_δ^* the confirmation for all the $t>T_\delta^*$ of the condition: $\|x(t)-x_*(t)\|<\delta$, where $x_*\in M(x_*^e,\varepsilon)$ is system attractor, is necessary.

Taking into account the dependence of T_δ^* on the initial condition and restriction $\|h(t)\|\le a$, the transient process duration can be found from:

$T^*=\max\{T_\delta^*(x_0,a) \mid \|x(t,x_0)-x_*(t)\|<\delta,\ \forall\, t>T_\delta^*(x_0,a)\}$.

With the estimation of T^* being done, the choice of the time interval length $[0,T]$, $T>>T^*$, at which the problem solution is being investigated becomes possible.

3 The Offered Solution

The solution of the problem formulated in the work lies in the combination of optimal control theory techniques with numerical tests of chaos suppression quality.

The condition of the maximum principle [Pontryagin, Boltyanski, Gamkrelidze, 1962] forms the basis of the analytical apparatus that permits to obtain locally optimal corrective functions.

Let's introduce the Hamilton-Pontryagin function

$$H(x,h,\psi)=\psi^T f(x,p^*)-\frac{1}{2}\|h\|^2$$

for the system (2). To make the vector-function $h^*(t)\in U$ and the trajectory $x^*(t)$ corresponding to it with bounded conditions $x_0^*\in B_A$, $x^*(T)\in M(x_*^e,r)$ optimal with respect to (4), there should exist such a nonzero vector function $\psi(t)\in R^n$, satisfying the system

$$\dot\psi(t)=-H_x(x^*(t),h^*(t),\psi(t),$$

in which the function $h^*(t)=h^*(x(t),\psi(t))$ satisfies the condition of maximum

$$H(x^*(t),h^*(t),\psi(t)) = \max_{h\in U} H(x^*(t),h,\psi(t)) \equiv 0\,. \quad (5)$$

At the same time in the points x_0^* and $x^*(T)$ the conditions of transversality $\psi_0 \perp \Omega(x_0^*)$ and $\psi(T) \perp \Omega(x^*(T))$, where $\Omega(x_0^*)$ and $\Omega(x^*(T))$ are the tangent manifolds to the sets B_A and $M(x_*^e, r)$ in the points $x_0^* \in B_A$ and $x^*(T) \in M(x_*^e, r)$ correspondingly should be performed.

The condition (5) plays a special role in our task.

The solution of the maximization task $H(x(t),u,\psi(t)) \to \max_{u\in U}$ using the equation $H_h(x^*(t),h,\psi(t)) = 0$ and the restriction $h \in U$ gives the desired corrective function

$$h^*(t) = \begin{cases} \tilde{h}(t), & if \ \ \tilde{h}(t) \in U, \\ a \cdot sign(\tilde{h}(t)), & if \ \ \tilde{h}(t) \notin U, \end{cases} \quad (6)$$

where $\tilde{h}(t) = \psi^T(t) f_h(x(t), p^*(t))$. As a result we have the system

$$\begin{cases} \dot{x} = H_\psi(x, h(x,\psi),\psi), & x(0) = x_0, \\ \dot{\psi} = -H_x(x, h(x,\psi),\psi), & \psi_0 \perp \Omega(x_0), \end{cases} \quad (7).$$

Its integration with regard to (6) gives the desired optimal pair $x^*(t)$, $h^*(t) = h(x^*(t),\psi(t))$, $t \in [0,T]$.

To make the solution optimal we should satisfy the conditions of transversality at both ends of the trajectory. It is easy to perform the condition $\psi_0(0) \perp \Omega(x_0)$ at the left end using the equation $\sum_{i=1}^{n} \psi_{0i} x_{0i} = 0$. The condition at the right end is performed automatically as it is shown below.

The role of the condition (5) is not reduced to finding (6) only. An important feature of the system (7) is that it can not be stable at the variables x and ψ simultaneously. The trajectories of chaotic systems are restricted with the area $\| x(t) \| < D$, which can be a sphere such that the trajectories which lie or appear in it remain there at $t \to +\infty$. Global restriction of chaotic systems trajectories in this case is referred to as a kind of stability. It is of value that this particular restriction is preserved at small perturbation of systems parameters. Then the introduction of the conjugate system according to the maximum principle leaves the condition $\| x(t) \| < D$ in force and causes the unlimited increase of the norm $\| \psi(t) \|$ at $[0,T]$.

Having substituted $(x^*(t),h^*(t),\psi(t))$ into the function H we get an equation

$$H(x^*(t),h^*(t),\psi(t)) = \psi^T(t) f(x^*(t), p^*(h^*(t))) - \\ -\frac{1}{2} \| h^*(t) \|^2 = 0.$$

Hence in regard with $\| h^*(t) \| \le a$ we have

$$\psi^T(t) f(x^*(t), p^*(h^*(t))) = \frac{1}{2} \| h^*(t) \|^2 \le \frac{a^2}{2}\,.$$

As $\psi(t)$ is not identically equal to null (otherwise $h^*(t) \equiv 0$) and the norm $\| \psi(t) \|$ increases unlimitedly at $t \to T$, we get the estimation

$$\| f(x^*(t), p^*(h^*(t))) \| \le \frac{a^2}{2 \| \psi(t) \|} \to 0\,. \quad (8)$$

From (8) we have the following conclusion. The corrected trajectory of the system (2) comes nearer to (is forced out) the vicinity of $M(x_*^e, r)$ on a regular basis. It means that the choice of a point from the set B_A automatically leads to the achievement of the target set by the corresponding trajectory $x(t)$ and to make the process $(x^*(t), h^*(t))$ $t \in [0,T]$ optimal, it is enough to satisfy the condition of transversality at the left end of the trajectory. That is in the point x_0^* the condition $\psi_0 \perp \Omega(x_0^*)$ must be fulfilled, where $\Omega(x_0^*)$ is the tangent manifold to the set B_A in the point $x_0^* \in B_A$.

Note that the achievement of the target set, which takes place in the case, doesn't necessarily lead to its internal stability. Virtually the character of the stability is specified by the choice of restriction on the corrective perturbation. That is why in addition to the conditions of the maximum principle the lower value of $a_{\min}$, at which the set $M(x_*^e, r)$ is internally stable, must be found by the numerical tests of the restrictions $\| h \| \le a$.

4 Simulation Studies

In this part we present and discuss the two results of the optimal modification in correcting parameters of a family of DvdP oscillators.

Equation (1) is equivalent to the first-order corrected system

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = -\alpha^* x_1 - \beta^* x_1^3 + \mu^* (1 - x_1^2) x_2 + f \cos(x_3), \quad (9) \\ &\dot{x}_3 = \omega, \end{aligned}$$

where

$$\alpha^* = \alpha(1+h_1),\ \beta^* = \beta(1+h_2),\ \mu^* = \mu(1+h_3).$$

The Hamilton-Pontryagin function corresponding to (9) looks like

$$H(x,h,\psi) = \psi_1 x_2 + \\ + \psi_2(-\alpha^* x_1 - \beta^* x_1^3 + \mu^* (1 - x_1^2) x_2 + f \cos(x_3)) + \\ + \psi_3 \omega - \frac{1}{2} \sum_{j=1}^{3} h_j^2. \quad (10)$$

On the basis of (10) according to (6) we get the general form of corrective functions

$$h_j^*(t) = \begin{cases} \tilde{h}_j(t), & if \ \ \| \tilde{h}(t) \| \le a, \\ a \cdot sign(\tilde{h}_j(t)), & if \ \ \| \tilde{h}(t) \| > a, \end{cases} \quad (11)$$

where

$$\tilde{h}_1(t) = -\alpha x_1(t) \psi_2(t),$$

$$\tilde{h}_2(t) = -\beta x_1^3(t) \psi_2(t),\ \tilde{h}_3(t) = \mu x_2 \psi_2(t)(1 - x_1^2),$$

and a system of conjugate variables

$$\dot{\psi}_i = -H_x(x, h^*, \psi),\ i = \overline{1,3}\,. \quad (12)$$

The dynamics of the corrected system was studied at the time interval $[0,T]$, $T >> T^*$, using the standard procedure of the numerical integration (Runge-Kutta fixed-step-size forth-order technique).

At a given value a of the restriction (3) the optimal process $(x^*(t), h^*(t))$, $t \in [0, T]$, may be found by the integration of the $2n$-system of the equations made up of (9) and (12) with the condition (11). In doing so, the initial condition for the conjugate vector in the form

$$\psi_0 = (-x_{02}, x_{01}, 0)^T$$

was used where $(x_{01}, x_{02}, x_{03})^T \in B_A$.

4.1 Single-well Potential

This case is of interest due to the especial sensitivity of the system to very small variations of the parameters. The modification of chaotic attractor into a unique stable set necessary for the correction of all the parameters ($h^*(t) = (h_1^*(t), h_2^*(t), h_3^*(t))$) becomes possible with the restriction $\| h(t) \| \le a_{\min} = 0.0015$.

Fig. 2 shows that ULS represents a stable cycle, preceded by a short transient process. The localization and characteristics of the cycle are determined by the dynamics of the trajectories on the chaotic attractor and differ from the limit cycle of the unforced system ($f = 0$, $h = 0$). As the value of the restriction on the corrective influence below $a_{\min}$ decreases the transition to chaos through the sequence of period-doubling bifurcations is observed.

Qualitative characteristics of ULS found out at multi-parametrical analyses are universal. The cycle localization and bifurcation mechanism of transition of a periodic attractor to chaos are preserved as the number of correctable parameters decreases. For example, the minimal restriction providing the existence of ULS for the corrective function $h^*(t) = (h_1^*(t), 0, 0)$ is small and constitutes $a_{\min} = 0.004$.

4.2 Double-well Potential

The results of correction of a given model are determined by the simultaneous co-existence of several possible established regimes in the phase space. The reasons for the multistability are the peculiarities of the unforced system potential, which is symmetrical and possesses two local minimums. Multistability is preserved under correction and manifests itself in sensitive dependence of ULS localization on the chosen initial condition. As a result the established regime of the corrected chaotic system turns out to be a stable cycle localized in the vicinity either of the state $x^e_{(2)} = 1.2$ or $x^e_{(3)} = -1.2$. The example of the first variant with correction at all parameters is shown in the Fig. 3.

The peculiarity of this case is in longer transient process duration (Fig. 3,4). It is mostly the motion along the large orbit, which bounds the area taken by a chaotic attractor, that is it has the maximal period. The transient process ends with the realization of the transition LO $\to$ SO. It is accompanied by the stability loss of the large orbit and transition to another type of motion (small orbit). The cycle realized in doing so is stable and represents ULS of the system. Thus, the optimal way of the system to ULS may be presented as the following sequence: Chaos $\to$ LO $\to$ Chaos $\to$ SO.

The effect of saturation of corrective functions $h_1^*(t)$, $h_2^*(t)$ at the edge of restriction (Fig.4) which appears on reaching ULS is of great interest. Notice that after chaos is suppressed two variants of dynamics of optimal corrective functions are realized. The first implies the regular switch between the boundaries $+a$ and $-a$. It is observed in the system correction with single-well potential and in the phase of motion along the large orbit in case of double-well potential. The saturation occurs only for the system with double-well potential when the trajectory approaches the small orbit which is ULS of the system. The moment of saturation coincides with the end of transient process and may be taken as a precision criterion of the goal achievement.

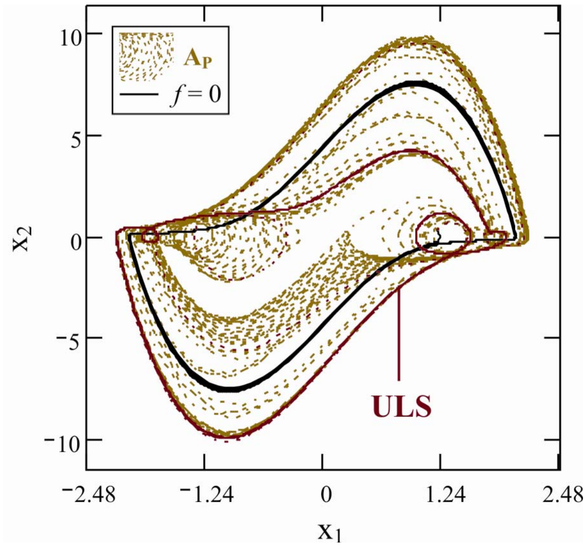

Figure 2. The result of optimal correction of parameters of DvdP oscillator (single-well potential) with restriction $a_{\min} = 0.0015$.

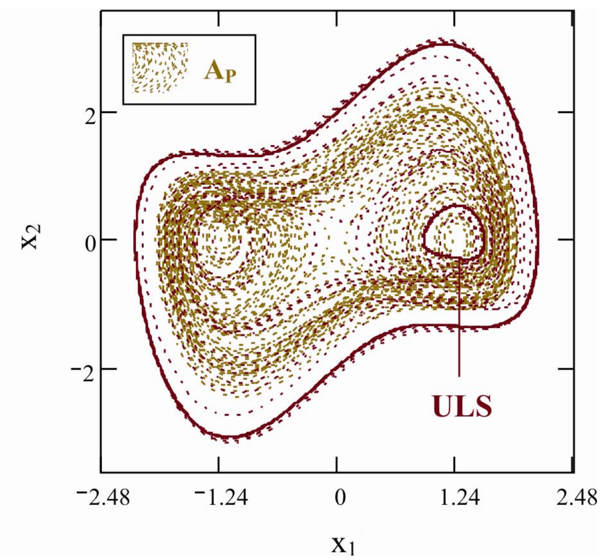

Figure 3. The result of optimal correction of parameters of DvdP oscillator (double-well potential) with restriction $a_{\min} = 0.05$.

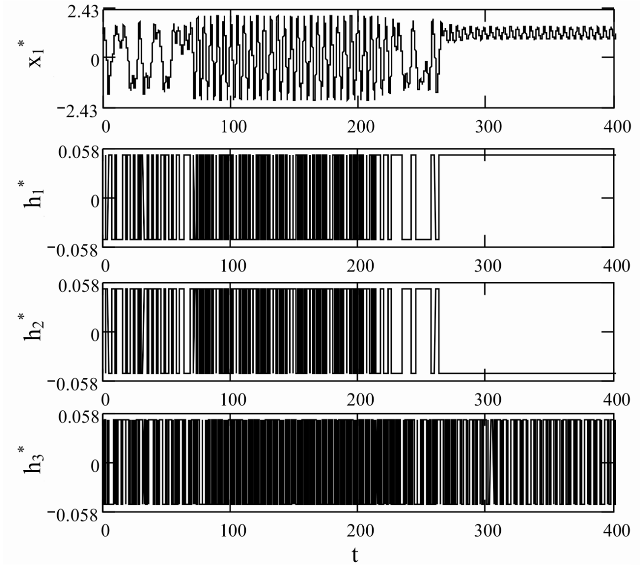

Figure 4. Time realization of corrected regime of DvdP oscillator (double-well potential), $a_{\min} = 0.05$.

The loss of ULS stability is different from chaotization of the oscillator with single-well potential. If the restriction is less than $a_{\min} = 0.05$ there takes place the increase of transient process duration, acquiring more irregular character. In doing so the duration of the phase of movement along the large orbit decreases. In further decrease of restriction the chaotic attractor becomes the system limit set again. Notice that the peculiarities mentioned above are preserved for the corrective function $h^*(t) = (h_1^*(t), 0, 0)$ with $a_{\min} = 0.07$.

5 Conclusions

As shown in this work, multiparametrical optimal correction is an effective technique of investigation of chaos suppression possibility in nonlinear oscillators. Dynamic correction of system parameters space is a way of chaos control techniques perfection, which focuses attention on the evolution of the given chaotic attractor to the stable set. System correction provides us with general information about physical characteristics of the described object in the form of the system reaction on an overall parametric perturbation. Then comparison and evaluation of perturbation efficiency on a concrete parameter become possible. In practical applications when only a limited number of parameters are correctable the single parameter perturbation becomes more justified and its results are predictable. The application of the correction technique to a family of DvdP oscillators showed that in both cases under investigation ULSs localization allows to identify the regime of the system from which the transition to chaotic behavior begins. The effect of corrective functions' saturation observed earlier [Talagaev, Tarakanov, 2007] in the correction of Lorenz system is described.

Hence the value of the presented technique is in the possibility to localize the unique stable set in the phase regime of the system and to study the peculiarity of the optimal transient process which provides the modification of a chaotic attractor in this set. In this approach internal peculiarities of the system are taken into consideration. Evidently, it opens new ways for investigation of the questions of bifurcation features of optimal transient processes in chaotic attractors not understood before.

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