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THE PROPERTY OF INVARIABLE INHERITED GENETIC POSITIONS IN REAL TRAJECTORIES OF LOGISTIC MAP

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Abstract

The numerical simulation of computers is an important approach for studying chaotic systems. The basic structure of real trajectories of the logistic map can be observed by using the iterative method with lower positions, which includes the root gene position, the common gene position and the individual gene position. The invariable inherited genetic position of a real trajectory as the root gene position represents the fundamental characteristic of the trajectory, which will be inherited by each true numerical solution within the real trajectory. In this paper, the two types of real trajectories of the logistic map are introduced, which include the divergence trajectory and the convergence trajectory. The property of the root gene position of divergence trajectories is presented. The maximum number of the root gene position is proved as the length of the root gene position is equal to 1. Experimental results show the map of the root gene position when the precision of initial conditions is specified to 1 and 2. As the precision of initial conditions is set to 2, the root gene position in the length of 3 possesses a great percentage, which is close to 50%. The similar situation occurs at the root gene position in the length of 1 when the precision of initial conditions is specified to 1. It suggests the dense distribution of the root gene position in length. Moreover, the permutation and combination of the common gene positions occur in real trajectories with the same root gene position.

Key words

Numerical simulation, logistic map, real trajectory, root gene position, chaos.

1 Introduction

The numerical simulation of computers is an important approach for studying dynamical equations. However, a fundamental issue is that whether the numerical simulation of computers precisely reflects the characteristic of dynamical equations. On the one hand, dynamical equations are defined in the continuous real number field for theoretical research. A real trajectory consists of true numerical solutions of dynamical equations. True numerical solutions might possess the infinite precision of digit position. On the other hand, the numerical simulation of computers for dynamical equations is always a finite limit on the computational precision. In comparison with the continuous space of real number, the space of the numerical simulation of computers is discrete for dynamical equations. As a result, there exists a widening breach between the continuous space of real number and the finite computational precision of computers.

Some dynamical equations such as the logistic map demonstrate a complex dynamical behavior and finally route to chaos with time in the continuous space of real number. Report [Tan and Chia, 1993] studied numerically the properties of the logistic map with a single sectional discontinuity. Besides, Tan et al. [Tan and Chia, 1995] also showed the bifurcation diagram in the linear-logistic map. The chaotic systems present the properties such as the sensitivity to initial conditions and the ergodicity. Moreover, the long-term prediction is mostly impossible [Ditto and Munakata, 1995]. However, the finite computational precision has a strong effect on the dynamical behavior of chaos. In other words, the property of dynamical equations has the precision-dependent behavior.

Numerical results for the discrete logistic map show that there exists a geometric pattern in the specified interval when convergence is gauged with a finite computational precision, and also show that this pattern cascades into the period-doubling areas [Bresten and Jung, 2009]. Cang et al. [Cang, Qi and Chen, 2010] reported a new four-dimensional smooth quadratic autonomous chaotic system with four-wing hyper-chaotic attractor and coexistence of two double-wing hyper-chaotic attractors under different initial conditions, which is investigated via both numerical simulations and bifurcation analysis. As seen above, period-doubling areas and attractors of dynamical equations have close links with the specified precision.

Apart from the precision-dependent behavior, many researchers focus on the property of numerical orbits and real trajectories. As we know, real chaotic trajectories of dynamical equations are nonperiodic in the continuous space of real number. The numerical simulation of computers is limited to the finite space of real number. Numerical orbits finally fall into the periodicity.

Ref. [Zhou, 2006] reported that one-dimensional unimodal maps may be robustly employed to generate the maximum-length shift-register sequences with the use of the limiter controller. The orbit structure of the discrete maps on a finite set is reported [Lanford, 1998]. It is an important problem how to get long periodic sequence in order to overcome the dynamical degradation of chaotic systems [Li, Chen and Mou, 2005]. In other words, it implies that how to make numerical orbits close to real trajectories.

In order to reveal the property of numerical orbits and real trajectories, a key problem is that how to compute numerical solutions of dynamical equations. It can be divided into two types: the floating-point arithmetic with round-off errors and the computing method of the specified precision without computational errors.

Report [Blanck, 2005] pointed out that it is possible to effectively compute the forward orbit of iterated maps contrary to often held believes that rounding errors and sensitivity on inputs make this impossible. Curtright et al. [Curtright, Jin and Zachos, 2011] presented that it is probable to construct approximate solutions to functional evolution equations by using a combination of series and conjugation methods, and relative errors could be estimated. The functional conjugation substantially improves the numerical accuracy of formal series approximations for their continuous iterates. Besides, Blanck [Blanck, 2006] also investigated that approximations based on dyadic centred intervals as a means are used to realize exact real arithmetic. It is shown that the field operations can be implemented on these approximations with optimal or near optimal results.

It is obvious that round-off errors have a considerable influence on the numerical orbits. Oteo et al. [Oteo and Ros, 2007] reported that the statistics of the largest double precision error as a function of the map parameter is characterized by jumps whose location is determined by certain boundary crossings in the bifurcation diagram.

Moreover, Gontar et al. [Gontar and Gutman, 1999] examined different ways of obtaining iteration sequences of the simplest 1-D maps in a particular form of quasi-continuous orbits specified by their long laminar segments.

Furthermore, Kim et al. [Kim and Park, 1999] proposed a new method based on the modified genetic algorithm to find a modeling function of nonlinear sequences. Numerical simulations were performed using nonlinear sequences generated by Logistic map function and Henon map function.

As mentioned above, the numerical orbits with round-off errors are based on approximations by using the finite computational precision. The approximations could cunningly conceal the property of true numerical solutions in real trajectories of dynamical equations.

Crutchfield [Crutchfield, 2012] pointed out that for an organism order is the distillation of regularities abstracted from observations, and an organism's very form is a functional manifestation of its ancestor's evolutionary and its own developmental memories. The property of the logistic map with scalable precision is reported [Liu, Zhang, Song, Buza, Yang and Guo, 2012]. It seemed that the real chaotic trajectories of the logistic map are an organism's very form. Each current true numerical solution in a real trajectory inherits its ancestor's gene and develops own characteristic with evolution. The ancestor's gene can be explained as the invariable inherited genetic position of each true numerical solution in the real trajectory of the logistic map, which is referred to the root gene position. Own developmental memories could be described as the difference from each other, which is mentioned as the individual gene position. It is interesting that the round-off error is hiding some important facts.

The iterative method with lower positions in absolute precision is the computing method of the specified precision without computational errors, which allows us to observe the property of real trajectories of dynamical equations. Besides, the basic structure of real trajectories of the logistic map is reported, which includes the root gene position, the common gene position and the individual gene position [Liu, Zhang and Song, 2014].

In this paper, the property of the root gene position of the logistic map is introduced. The maximum number of the root gene position is proved when the length of the root gene position is equal to 1. The map of the root gene position is plotted when the precision of initial conditions of the logistic map is specified to 1 and 2. Moreover, the major length distribution of the root gene position is presented. To the best of my knowledge, the inherited genetic property of real trajectories of the logistic map as the root gene position is reported in detail for the first time.

The rest of this paper is organized as following. Section 2 gives some definitions and the lemma which will be used in the next section. Section 3 introduces the property of numerical trajectories with round-off errors. The property of the root gene position is presented in Section 4. The last section summarizes this work.

2 Preliminary Knowledge

In this section, some definitions and the lemma will be given before the property of the root gene position is described.

Definition 1. *A real number τ can be formulated as $\tau = r_0.r_1r_2\cdots r_i\cdots r_n$, $r_i \in \{0,1,2,3,4,5,6,7,8,9\}$.*

When $r_n \neq 0$, n represents the precision of τ.

Definition 1 is utilized to introduce the lower positions of the true numerical solution and demonstrate the location of the root gene position in the real trajectory.

Definition 2. *Assume that $\mathbf{z}(\gamma)$ represents a normalized form of an integer number γ, i.e.,*

$$\mathbf{z}(\gamma) = z_{\gamma_n} z_{\gamma_{n-1}} \dots z_{\gamma_i} \dots z_{\gamma_0},$$

*where z_{γ_0} is called **the last digit** of γ.*

Definition 2 is used to distinguish the last digit of integer number in arithmetic.

Assume the sets of integer number $\mathbb{Z}^c = \{5\}$ and $\mathbb{Z}^d = \{1,2,3,4,6,7,8,9\}$.

Lemma 1. *$\forall \alpha, \beta \in \mathbb{Z}^d$, let $\gamma = \alpha \times \beta$, z_{γ_0} is the last digit of γ, the following is true:*

$$z_{\gamma_0} \in \mathbb{Z}^d. \tag{1}$$

Proof: the results of $\gamma = \alpha \times \beta$ are summarized in Table 1. Since the set $\{2,4,6,8\} \subset \mathbb{Z}^d$, we can easily obtain Eq. (1). The proof is thus completed.

3 Numerical Trajectory with Round-off Errors

Floating-point arithmetic in modern computers is by far the most widely used computational method of implementing real-number arithmetic. In general, computers possess the finite computational precision. In fact, it is too difficult for computers to simulate a dynamical equation with absolute precision through floating-point arithmetic. On the one hand, the precision of true numerical solutions of the dynamical equation will be truly very large with time. It is inevitable to truncate extra digit positions of numerical solutions with finite computational precision. On the other hand, the computational precision could make a great impact on long-term behavior of the dynamical equation. In other words, the precision-dependent behavior of the dynamical equation will produce different trajectories even though same initial conditions are specified.

In order to explain the problem we give examples for floating-point arithmetic with the specified finite computational precision. Assume that a computer possesses the finite computational precision which can be specified freely by users.

The typical dynamical equation will refer to the logistic map which can be described as

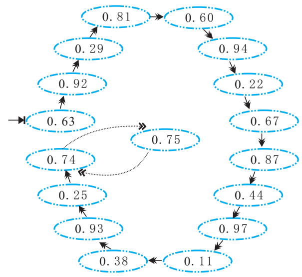

Figure 1. The numerical trajectory of the logistic map is shown for x_0=0.2 and a=3.95 with the computational precision of 2.

$$x_{n+1} = a\, x_n\,(1 - x_n), n = 0, 1, 2, \cdots \tag{2}$$

where a represents the control parameter which is a real number in the interval $(3,\ 4)$. x_n is the numerical solution of the n-th iteration which belongs in the interval $(0,\ 1)$. x_0 stands for an initial value of x_n.

Initial conditions of the logistic map are given as $x_0 = 0.2$ and $a = 3.95$. The computational precision of the computer is set to 2. The computing procedure is shown as following.

$x_1 = a\, x_0\,(1 - x_0) = 3.95 \times 0.2 \times (1 - 0.2) = 0.632$. The precision of x_1 is equal to 3. The last digit position exceeds the computational precision of the computer. Therefore, the lowest digit position will be truncated. Thus, $x_1 = 0.63$.

$x_2 = a\, x_1\,(1 - x_1) = 3.95 \times 0.63 \times (1 - 0.63) = 0.920745$. The precision of x_2 is equal to 6. The lowest four digit positions will be truncated. Thus, $x_2 = 0.92$.

Figure 1 shows the numerical trajectory of the logistic map with $x_0 = 0.2$ and $a = 3.95$ when the computational precision is specified to 2.

From Figure 1, we can see that the numerical trajectory possesses two parts: the delay part and the cycle part. The length of the delay part is equal to 15, which ranges from $x_1 = 0.63$ to $x_{15} = 0.25$. The length of the cycle part is equal to 2, which includes $x_{16} = 0.74$ and $x_{17} = 0.75$. The period of the cycle part is very short.

We increase the computational precision of the computer in order to further observe the precision-dependent behavior.

Assume that initial conditions of the logistic map are given as $x_0 = 0.215$ and $a = 3.95$. The value of the control parameter a does not change. The computational precision of the computer will increase to 3. The computing procedure is listed as follows.

$x_1 = a\, x_0\,(1 - x_0) = 3.95 \times 0.215 \times (1 - 0.215) = 0.66666125$. The precision of x_1 is equal to 8. Lower five digit positions exceed the computational precision. Therefore, 0.00066125 will be truncated as round-off errors. Thus, $x_1 = 0.666$.

$x_2 = a\, x_1\,(1 - x_1) = 3.95 \times 0.666 \times (1 - 0.666) = 0.8786538$. The precision of x_2 is equal to 7. There-

Table 1. The results of $\gamma = \alpha \times \beta$ and the set $\mathbb{S}$ of z_{γ_0}.

$\alpha \diagdown \beta$	1	2	3	4	6	7	8	9
1	$1\times1=\underline{1}$	$1\times2=\underline{2}$	$1\times3=\underline{3}$	$1\times4=\underline{4}$	$1\times6=\underline{6}$	$1\times7=\underline{7}$	$1\times8=\underline{8}$	$1\times9=\underline{9}$
2	$2\times1=\underline{2}$	$2\times2=\underline{4}$	$2\times3=\underline{6}$	$2\times4=\underline{8}$	$2\times6=1\underline{2}$	$2\times7=1\underline{4}$	$2\times8=1\underline{6}$	$2\times9=1\underline{8}$
3	$3\times1=\underline{3}$	$3\times2=\underline{6}$	$3\times3=\underline{9}$	$3\times4=1\underline{2}$	$3\times6=1\underline{8}$	$3\times7=2\underline{1}$	$3\times8=2\underline{4}$	$3\times9=2\underline{7}$
4	$4\times1=\underline{4}$	$4\times2=\underline{8}$	$4\times3=1\underline{2}$	$4\times4=1\underline{6}$	$4\times6=2\underline{4}$	$4\times7=2\underline{8}$	$4\times8=3\underline{2}$	$4\times9=3\underline{6}$
6	$6\times1=\underline{6}$	$6\times2=1\underline{2}$	$6\times3=1\underline{8}$	$6\times4=2\underline{4}$	$6\times6=3\underline{6}$	$6\times7=4\underline{2}$	$6\times8=4\underline{8}$	$6\times9=5\underline{4}$
7	$7\times1=\underline{7}$	$7\times2=1\underline{4}$	$7\times3=2\underline{1}$	$7\times4=2\underline{8}$	$7\times6=4\underline{2}$	$7\times7=4\underline{9}$	$7\times8=5\underline{6}$	$7\times9=6\underline{3}$
8	$8\times1=\underline{8}$	$8\times2=1\underline{6}$	$8\times3=2\underline{4}$	$8\times4=3\underline{2}$	$8\times6=4\underline{8}$	$8\times7=5\underline{6}$	$8\times8=6\underline{4}$	$8\times9=7\underline{2}$
9	$9\times1=\underline{9}$	$9\times2=1\underline{8}$	$9\times3=2\underline{7}$	$9\times4=3\underline{6}$	$9\times6=5\underline{4}$	$9\times7=6\underline{3}$	$9\times8=7\underline{2}$	$9\times9=8\underline{1}$
$\mathbb{S}$	$\mathbb{Z}^d$	$\{2,4,6,8\}$	$\mathbb{Z}^d$	$\{2,4,6,8\}$	$\{2,4,6,8\}$	$\mathbb{Z}^d$	$\{2,4,6,8\}$	$\mathbb{Z}^d$

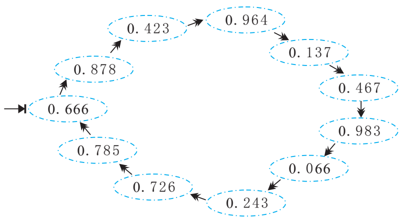

Figure 2. The numerical trajectory of the logistic map is shown for x_0=0.215 and a=3.95 with the computational precision of 3.

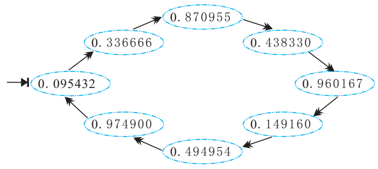

Figure 3. The numerical trajectory of the logistic map is shown for x_0=0.0251 and a=3.9 with the computational precision of 6.

fore, 0.0006538 will be truncated as round-off errors. Thus, $x_2 = 0.878$.

Figure 2 shows the numerical trajectory of the logistic map with $x_0 = 0.215$ and $a = 3.95$ when the computational precision is set to 3.

The numerical trajectory does not possess the delay part for $x_0 = 0.215$ and $a = 3.95$. The length of the cycle part is equal to 11. Apparently, the length of the cycle part becomes larger than before.

We increase the computational precision to 6 for observing the precision-dependent behavior with round-off errors. Assume that initial conditions are given as $x_0 = 0.0251$ and $a = 3.9$. The computing procedure is listed as follows.

$x_1 = a\,x_0\,(1 - x_0) = 3.9 \times 0.0251 \times (1 - 0.0251) =$ 0.095432961. The precision of x_1 is equal to 9. Lower three digit positions exceed the computational precision. Therefore, 0.000000961 will be truncated as round-off errors. Thus, $x_1 = 0.095432$.

$x_2 = a\,x_1\,(1 - x_1) = 3.9 \times 0.095432 \times (1 - 0.095432) = 0.3366664601664$. The precision of x_2 is equal to 13. Therefore, 0.0000004601664 will be truncated as round-off errors. Thus, $x_2 = 0.336666$.

Figure 3 shows the numerical trajectory of the logistic map with $x_0 = 0.0251$ and $a = 3.9$ when the computational precision is set to 6.

From Figure 3 we can see that the length of the cycle part is equal to 8 for the computational precision of 6. The initial conditions of $\{x_0 = 0.215,\ a = 3.95\}$ and $\{x_0 = 0.0251,\ a = 3.9\}$ do not include the delay part. The length of the period is not remarkably increasing with the enhancing computational precision.

As mentioned above, the enhancing computational precision uncertainly leads to increasing the length of the cycle part. Besides, round-off errors are completely out of control. In fact, the finite computational precision of computers and the round-off error have a malign effect on dynamical equations. Some characteristics of dynamical equations could be conceal by round-off errors.

The simulated numerical trajectories might be different from the real trajectories. We take an example to explain the problem. Assume that initial conditions of the logistic map are given as $x_0 = 0.2$ and $a = 3.95$. We use the computational precision of 2 and 3 for observing the deviation of trajectories.

Figure 4 shows the simulated numerical trajectories for $x_0 = 0.2$ and $a = 3.95$ with the computational precision of 2 and 3 in comparison with the real trajectory.

In Figure 4, $Iter_{Precision}$ represents the computational precision of the computer with the floating-point arithmetic. The true numerical solution is computed with absolute precision. In other words, a real trajectory consists of true numerical solutions which do not include any computational error.

From Figure 4 we can see that the simulated numerical trajectories are very close to the real trajectory at the first few iterations. Starting from the seventh iteration, the simulated numerical trajectories gradually deviate from the real trajectory. The accumulated round-off er-

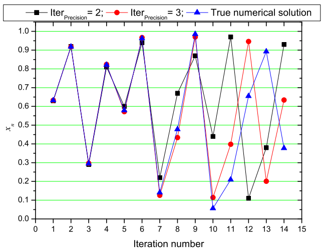

Figure 4. Trajectories include the real trajectory, the simulated numerical trajectories with the computational precision of 2 and 3 when the initial conditions are given as x_0=0.2 and a=3.95.

rors step by step expand with the increasing number of iterations. Moreover, the simulated numerical trajectories are different from each other even though same initial conditions are given.

The complex dynamical behavior is dependent on the round-off error or otherwise. The question is too difficult for the numerical simulation of computers. As so far, there has not any computer with infinite computational precision for simulating continuous trajectories. Many research works are presented to devote more approaches to this issues. In the next section the root gene position is utilized to explore the property of real trajectories.

4 Property of Root Gene Positions

Real trajectories of the logistic map naturally possess the basic structure which includes the root gene position, the common gene position and the individual gene position. The individual gene position represents the ergodicity and randomness of the dynamical equation. The common gene position can divide the trajectory into several groups. The root gene position is able to identify a real trajectory. In other words, every true numerical solution in the real trajectory possesses the same root gene position which locates in the lower digit positions.

4.1 Two Types of Real Trajectories

In order to observe basic structure of the logistic map, the iterative method with lower positions will be utilized to get lower positions of true numerical solutions in the real trajectory.

The logistic map can be rewritten as

$$x_n = f(a, x_{n-1}), n = 1, 2, \cdots \qquad (3)$$

Assume that p_n is a positive integer. The lower p_n positions of x_n are described as

$$0.r_1 r_2 \cdots \underbrace{r_{i+1} r_{i+2} \cdots r_{i+p_n}}_{lower\ p_n\ positions}$$

The iterative method with lower p_n positions is described as following.

Step 1: computing the dynamical equation $x_n^l = f(a, x_{n-1})$ in absolute accuracy;

Step 2: if the total number of positions of x_n^l is less than or equal to p_n, x_n^l will be returned for the next iteration; otherwise, the lower p_n positions will be converted into $0.r_{i+1} r_{i+2} \cdots r_{i+p_n}$ for the next iteration.

Assume that initial conditions are given as $x_0 = 0.3$, $a = 3.8$ and $p_n = 10$. We use the iterative method with lower p_n positions to get the real trajectory of the logistic map. The computing procedure is shown as follows.

$x_1^l = ax_0(1 - x_0) = 3.8 \times 0.3 \times (1 - 0.3)$ = 0.798.

The precision of x_1^l is equal to 3. It is less than $p_n = 10$. Thus, $x_1 = 0.798$.

$x_2^l = ax_1(1 - x_1) = 3.8 \times 0.798 \times (1 - 0.798)$ = 0.6125448.

The precision of x_2^l is equal to 7. It is less than $p_n = 10$. Thus, $x_2 = 0.6125448$.

$x_3^l = ax_2(1 - x_2) = 3.8 \times 0.6125448 \times (1 - 0.6125448)$ = 0.901867938373248.

The precision of x_3^l is equal to 15. It is greater than $p_n = 10$. The lower ten positions starting from the last position will be converted into $0.r_{i+1} r_{i+2} \cdots r_{i+p_n}$ as

$$0.90186\ \underbrace{7938373248}_{lower\ 10\ positions}$$

Thus, $x_3 = 0.7938373248$.

$x_4^l = ax_3(1 - x_3) = 3.8 \times 0.7938373248 \times (1 - 0.7938373248)$ = 0.621906580906641358848.

The precision of x_4^l is equal to 21. It is greater than $p_n = 10$. The lower ten positions will be converted into $0.r_{i+1} r_{i+2} \cdots r_{i+p_n}$ as

$$0.62190658090\ \underbrace{6641358848}_{lower\ 10\ positions}$$

Thus, $x_4 = 0.6641358848$.

Lower ten positions of true numerical solutions in the real trajectory are observed by using the iterative method with lower positions.

Figure 5 shows the real trajectory of the logistic map for $x_0 = 0.3$ and $a = 3.8$ with lower ten positions of true numerical solutions.

$x_1 = 0.798$ is the delay part of the real trajectory. Apart from x_1, the real trajectory exhibits the natural structure with starting from x_2. Each true numerical solution has the same root gene positions of “48”, which are located in the lowest two positions of true numerical solutions. The common gene positions are “448”, “248”, “848” and “048”.

In Figure 5, apart from the root gene position and the common gene position, it is obvious that every true numerical solution is diverging from each other, which is called as the divergence trajectory.

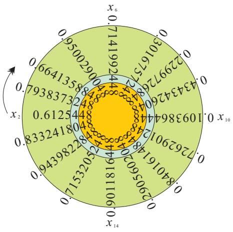

Figure 5. The real trajectory of the logistic map is shown for x_0=0.3 and a=3.8 with lower ten positions of true numerical solutions.

Assume that initial conditions are given as $x_0 = 0.5$, $a = 3.9$. The real trajectory of the logistic map is shown as following.

$x_1 = 0.975$.
$x_2 = 0.0950625$.
$x_3 = 0.335499922265625$.
$x_4 = 0.8694649252589998712310791015625$.

In the next true numerical solution we give the upper three positions and the lower ten positions, because the precision is too large.

$x_5 = 0.442 \cdots 9228515625$.
$x_6 = 0.962 \cdots 6103515625$.
$x_7 = 0.141 \cdots 9853515625$.
$x_8 = 0.475 \cdots 7353515625$.
$x_9 = 0.972 \cdots 2353515625$.
$x_{10} = 0.104 \cdots 2353515625$.

The lower ten positions are rewritten as follows.

$$\begin{aligned}
x_2 &= \qquad 0.0950\underline{625} \\
x_3 &= 0.\cdots 99222\underline{65625} \\
x_4 &= 0.\cdots 07910\underline{15625} \\
x_5 &= 0.\cdots 9228\underline{515625} \\
x_6 &= 0.\cdots 610\underline{3515625} \\
x_7 &= 0.\cdots 98\underline{53515625} \\
x_8 &= 0.\cdots 7\underline{353515625} \\
x_9 &= 0.\cdots \underline{2353515625} \\
x_{10} &= 0.\cdots \underline{2353515625}
\end{aligned}$$

The lower ten positions of the true numerical solution are converging on "2353515625", which is called as the convergence trajectory.

4.2 Property in Precision 1

The root gene positions can identify a real trajectory, since each true numerical solution in the real trajectory inherits the same characteristic from the previous iteration. It is an interesting problem how many trajectories are there in the specified space?

The precision of initial conditions of the logistic map are set to 1. The lowest position of the initial value x_0 and the control parameter a belongs to the set of $\{1, 2, 3, 4, 6, 7, 8, 9\}$, i.e., $\mathbb{Z}^d$. In other words, values of the control parameter of a are appointed in the set of $\{3.1, 3.2, 3.3, 3.4, 3.6, 3.7, 3.8, 3.9\}$. Values of x_0 are specified in the set of $\{0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, 0.9\}$.

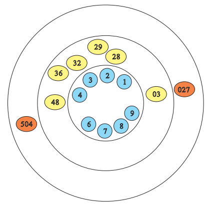

Figure 6. The map of the root gene position is shown for the precision of 1 with initial conditions.

Note that each initial condition can go along with the real trajectory which possesses the root gene position. The initial conditions can be described as the form of (a, x_0). The total number of real trajectories is equal to 64 (8 $\times$ 8) when the precision of initial conditions is specified to 1, which include ($a = 3.1, x_0 = 0.1$), ($a = 3.1, x_0 = 0.2$), $\cdots$, ($a = 3.9, x_0 = 0.1$) and so on. Figure 6 shows the map of the root gene positions for the precision of 1 with initial conditions.

The length of root gene positions ranges from 1 to 3. The total number of the root gene position is equal to 6 when the length of the root gene position is equal to 2, i.e., "03", "28", "29", "32", "36" and "48". The number of the root gene position is the minimum with the length of 3, i.e., "027" and "504".

Note that the total number of the root gene position is equal to 8 when the length of the root gene position is equal to 1, i.e., "1", "2", "3", "4", "6", "7", "8" and "9". From Lemma 1 we can deduce that the maximum number of root gene positions is equal to 8 when the length of the root gene position is equal to 1. In other words, the maximum number of the root gene position in the length of 1 is equal to the element number of the set $\mathbb{Z}^d$.

4.3 Property in Precision 2

In order to further explore the property of the root gene position, the precision of initial conditions of the logistic map is set to 2. The lowest position of the initial value x_0 and the control parameter a belongs to the set of $\{1, 2, 3, 4, 6, 7, 8, 9\}$. Values of the control parameter of a will vary in the set of $\{3.01, 3.02, 3.03, 3.04, \cdots, 3.99\}$. Values of x_0 are specified in the set of $\{0.01, 0.02, 0.03, 0.04, 0.06, \cdots, 0.99\}$.

The total number of real trajectories is equal to 7744 (88 $\times$ 88) when the precision of initial conditions is specified to 2. The total number of root gene positions is equal to 181. The length of root gene positions varies from 1 to 6. Figure 7 shows the map of the root gene positions for the precision of 2 with initial conditions.

The maximum length of the root gene position expands to 6. In comparison with the precision of 1, the number of root gene positions in precision 2 is remarkably increasing.

The total number of root gene positions is equal to 60 when the length of root gene positions is

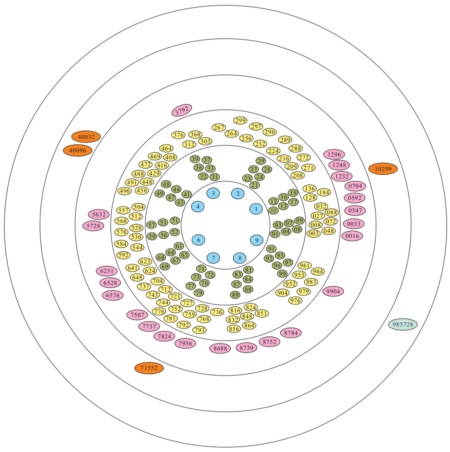

Figure 7. The map of the root gene position is shown for the precision of 2 with initial conditions.

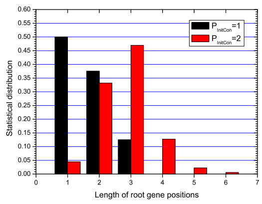

Figure 8. Statistical distribution of the root gene position is plotted by the length.

equal to 2. Note that each element in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ includes six root gene positions. For example, the first position of the root gene position is equal to 0 which includes “01”, “03”, “04”, “07”, “08” and “09”. The first position of the root gene position is equal to 5 which includes “51”, “52”, “53”, “56”, “57” and “59”.

Author does not know the maximum number of root gene positions with the length of 2, because it is difficult for author to deduce the result by using the number theory. Apart from this, it is unclear whether the precision 3 or more of initial conditions have some effect on the maximum number of the root gene position or not. In other words, it is unknown how to prove it by using the theoretical methodology. Moreover, the total number of real trajectories will arrive to 788544 (888 $\times$ 888) as the precision of initial conditions is specified to 3.

The major length distribution of the root gene position represents the maximum percent of the length distribution with the specified precision of initial conditions. Figure 8 shows the statistical distribution of the root gene position by the length.

In Figure 8, the number of the root gene position with the length of 1 and 2 is an overwhelming percentage when the precision of initial conditions is specified to 1. $p_{InitCon}$ represents the given precision of initial conditions. It is obvious that the major length distribution of the root gene position locates in length of 1 for $p_{InitCon} = 1$, which arrives to 50%.

Moreover, the number with the length of 2 and 3 possesses a great percentage for $p_{InitCon} = 2$. The major length distribution of the root gene position locates in length of 3, which is close to 50%.

As seen above, the major length distribution suggests that the number of the root gene position is remarkable dense in the length.

Table 2. The common gene position with the same root gene position of “12” in the real trajectories is shown.

Common gene position	(a, x_0)
$\{112, 512, 312, 912\}$	$\{(3.52, 0.16)\}$
$\{312, 512, 912, 712\}$	$\{(3.27, 0.38), (3.27, 0.34)\}$
$\{112, 712, 912, 312\}$	$\{(3.02, 0.09), (3.02, 0.41)\}$
$\{112, 912, 512, 712\}$	$\{(3.77, 0.62), (3.77, 0.88)\}$

Table 3. The common gene position with the same root gene position of “4” in the real trajectories is shown.

Common gene position	(a, x_0)
$\{24, 44, 84, 64\}$	$\{(3.1, 0.4)\}$
$\{24, 04, 64, 84\}$	$\{(3.46, 0.07)\}$
$\{04, 24, 64, 44\}$	$\{(3.11, 0.48)\}$

4.4 Common Gene Position in Trajectories

In the numerical simulation of the logistic map, the interesting phenomenon is that some real trajectories possess different common gene positions even though they possess the same root gene position. Table 2 shows the common gene position with the same root gene position of “12”.

The initial conditions of $(a = 3.27, x_0 = 0.38)$ and $(a = 3.27, x_0 = 0.34)$ have the same root gene position and common gene position, i.e., the common gene positions “312”, “512”, “912” and “712” with the same root gene position “12”. The initial condition of $(a = 3.52, x_0 = 0.16)$ has the root gene position of “12”. However, the common gene position is “112”, “512”, “312” and “912”. They have the same common gene positions of “512”, “312” and “912”, and have the different ones of “112” and “712”.

The similar situations happen for $(a = 3.02, x_0 = 0.09)$ and so on. Table 3 shows the common gene position with the same root gene position of “4”.

In comparison with Table 2, the digit of the common gene position belongs to the set $\{2, 4, 6, 8, 0\}$ in Table 3. For example, the initial condition of $(a = 3.46, x_0 = 0.07)$ has the root gene position of “4”, the common gene positions “24”, “04”, “64” and “84”. In comparison with $(a = 3.46, x_0 = 0.07)$, the common gene position with the initial condition of ($a = 3.11, x_0 = 0.48$) has the same of “04”, “24” and “64”, and has the different of “44”.

As seen above, the permutation and combination of common gene positions occur in real trajectories with the same root gene position.

5 Discussion and Conclusions

This paper tries to explore the property of real trajectories of the logistic map. The property of the root gene position of true numerical solutions is presented. The major length distribution of the root gene position will change with the specified precision of initial conditions. The length of root gene positions is increasing when the precision of initial conditions enhances. The permutation and combination of the common gene positions could happen with the same root gene position.

Moreover, the root gene position can precisely identify a real trajectory against the numerical trajectories which could be computed by using the floating-point arithmetic with round-off errors. The map of the root gene position might reveal the property of real trajectories of the logistic map and the precision-dependent behavior of dynamical equations.

The topic about the root gene position still remains open. There are many problems to be resolved. For example, it is important how does the major length distribution of the root gene position change with the specified precision of initial conditions? The major length distribution implies the dense distribution of real trajectories. It is the future work that the maximum number of root gene positions in length will be revealed by using number theory.

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