ROBUST CONTROL OF ONE CLASS OF NONLINEAR SYSTEMS WITH A DISTRIBUTED DELAY

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Abstract

The paper presents a novel robust control of one class of nonlinear single-input single-output (SISO) systems with distributed delay on state used the auxiliary loop method. The problem is solved by introducing an auxiliary loop and using two Khalil observers to compensate for a generalized disturbance signal containing information not only about the parametric uncertainty of the installation model, but also information about distributed delay and disturbances. The designed control law is investigated in MATLAB. To illustrate the obtained results, an example of a nonlinear system with a distributed time delay is given. It is shown that the proposed control law under conditions of parametric uncertainty ensures the achievement of the goal with a given accuracy.

Key words

robust control, dynamic plant, disturbances, dynamic accuracy, distributed delay, nonlinear plant, auxiliary loop, observer, parametric uncertainty

1 Introduction

In control theory, various solutions to time delay problems have been obtained [Hassan et al., 2011; Karimi, 2020; Morarescu, 2016 et al.;Kharitonov, 2013; Fridman, 2014; Tsykunov, 2014]. The importance of solving this problem is due to its practical significance. A number of papers have shown that in some cases the linearization of systems with different types of delay allows relatively adequate description of real processes, but, for example, in the theory of oscillations [Rubanik, 1969], bioinformatics [Riznichenko, 2010; Murray, 2009], in the theory of visco-elasticity [Rabotnov, 1977], such an approach is unacceptable, since it leads to rather crude or erroneous results. Thus, in [Fridman, 2002] it is shown that the exclusion of even a small time delay in the system of differential equations can significantly change the picture of the behavior of solutions.

One of the types of systems with delay is systems of differential equations with distributed delay. Mathematical models including distributed delay are proposed in [Volterra, 1982] and are used in such fields as biology [Murray, 2009], industry [Morarescu et al., 2016; Zitek et al., 2001], cosmonautics [Crocco, 1951; Fiagbedzi et al., 1987]. Taking into account the distributed delay makes it possible to do the models of these systems correspond to reality. An example of a system with this type of delay can be taken from the textile industry, in particular the process of manufacturing metallised fabric [Zitek et al., 2001]. The metallised fabric is composed of a large number of discrete fibres which do not change their length during the process, only their position relative to each other and the number of fibres in the cross section varies according to the speed of rotation of the technological shafts. The lengths of individual fibres are random variables, ranging from a minimum to a certain maximum, and this fibre length distribution includes a distributed delay.

It is relevant to solve control problems in the conditions of disturbances [Andrievsky, Furtat, 2020a, 2020b], a priori uncertainty of the parameters of models of control plants [Annaswamy et al., 2021]. Solutions of various robust control problems are proposed in the class of problems with distributed delay. So, in [Karimi,2020], with the help of theory $H-\infty$, an adaptive synchronization algorithm is synthesized for a plant with a nonstationary discrete and distributed delay by state, and in [Tsykunov, 2016], using an auxiliary loop and an observer of variables, the problem of robust control of plant with distributed delay and unknown order of mathematical modelis solved. A solution to the problem of robust control of the combustion process in the chambers of rocket engines is proposed [Xie et al.,2002]. In paper

[Morarescu et al., 2016], a linear analysis of the stability of synchronized equilibria in networks of diffusely coupled oscillators was carried out, taking into account the distributed delay. In [Alexandrov et al., 2023], a nonlinear system with distributed delay for angular stabilization of a solid is investigated. A distributed delay based controller for simultaneous periodic disturbance rejection and input-delay compensation [Yuksel et al., 2023]. In [Mirkin, 2004], the approximation of control laws with distributed delay is considered. The optimized design of a reliable resonator with a distributed time delay is considered in [Pilbauer et al., 2019].

In this paper, in the class of problems of robust control of plants with distributed delay, a solution to the problem of control of a nonlinear system with distributed delay using the auxiliary loop method, is proposed. The problem is solved by introducing an auxiliary loop [Tsykunov, 2007] and using two Khalil observers [Atassi et al., 1999]. To illustrate the result obtained, a numerical example of a control system for a nonlinear dynamic plant with a distributed delay is given. It is shown that the synthesized control under conditions of parametric uncertainty ensures the achievement of the control goal with a given accuracy.

2 Problem statement

Consider a plant model in the form

$$\begin{array}{l} \dot{x}(t) = Ax(t) + D\int\limits_{-h}^{0} y(t+\theta)d\theta \\ +F(y) + Bu(t) + \Gamma f(t), \\ \qquad y(t) = Lx(t), \\ \qquad x(\theta) = \varphi(\theta), \quad \theta \in [-h; 0], \end{array} \tag{1}$$

where $x \in \mathbb{R}^n$, $y(t)$ and $u(t)$ are scalar output and input signals, h is a time delay, $\varphi(\theta)$ is a continuous initial function, $f(t)$ is an external bounded perturbation, A, D, B, Γ and L are constant matrices of appropriate dimensions, $F(y)$ is a smooth vector function.

The required performance is given by the reference model

$$\begin{array}{c} \dot{x}_m(t) = A_m x_m(t) + D_m \int\limits_{-h}^{0} y_m(t+\theta)d\theta + B_m g(t), \\ y_m(t) = L_m x_m(t), \end{array} \tag{2}$$

where $x_m \in \mathbb{R}^{n-m}$, $y_m(t)$ and $g(t)$ are scalar reference output and input, $A_m,\ D_m,\ B_m$ are well-known matrices of appropriate dimensions, the initial conditions of (2) are zero.

It is require to design the control law ensuring the fulfillment of the goal

$$|y(t) - y_m(t)| < \delta, \ \ t \geq T_0, \tag{3}$$

where $\delta > 0$ is a required accuracy, T_0 is the time after which from the beginning the goal holds.

Assumptions

1. The pair $A,\ B$ is controllable and the pair $A,\ L$ is observable.
2. Matrix elements A, D, B, Γ and L are unknown, but it belongs to a known compact set Ξ.
3. The polynomial $\det[L(I_n s - A)^+ B]$ is Hurvitz, where s is a complex variable, $(I_n s - A)^+$ is a transposed matrix of algebraic complements of the matrix $(I_n s - A)$, $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix.
4. The external perturbation $f(t)$ and the reference input $g(t)$ are smooth bounded functions.
5. The function $F(y)$ is bounded.
6. Elements of the vector function $F(y)$ satisfy the global Lipschitz conditions.
7. The derivatives of y and u are not measured.

Transforming the equations (1) and (2) into the input-output form and using the Laplace transform, one gets

$$Q(s)y(s) = G(s)\int\limits_{-h}^{0} e^{s\theta}d\theta y(s) + \sum_{i=1}^{n} Q_i(s)F_i(y)$$

$$+kM(s)u(s) + N(s)f(s) + K(s),$$

$$Q_m(s)y_m(s) = G_m(s)\int\limits_{-h}^{0} e^{s\theta}d\theta y_m(s) + k_m M_m(s)g(s),$$

where $Q(s) = \det(I_n s - A)$, $Q_m(s) = \det(I_n s - A_m)$, $G(s) = \det L(I_n s - A)^+ D$, $G_m(s) = \det L_m(I_n s - A_m)^+ D_m$, $M(s) = \det L(I_n s - A)^+ B$, $M_m(s) = \det L_m(I_n s - A_m)^+ B_m$, $N(s) = \det L(I_n s - A)^+ \Gamma$, $\deg Q(s) = \deg Q_m(s) = n$, $\deg N(s) = n_1 \leq n-1$, $\deg G(s) \leq n-1$, $\deg G_m(s) \leq n-1$, $\deg M(s) = m$, $\deg M_m(s) \leq m$, $Q_i(s)$ is the ith element of the matrix $L(I_n s - A)^+$, $F_i(y)$ is the ith element of the vector function $F(y)$. The polynomials $Q(s)$, $Q_m(s)$, $M(s)$, $M_m(s)$ are monic polynomials, $K(s)$ is a polynomial depending on initial conditions.

Introduce the error $e(s) = y(s) - y_m(s)$:

$$\begin{array}{l} Q_m(s)e(s) = G_m(s)\int\limits_{-h}^{0} e^{s\theta}d\theta e(s) + \\ +\Delta Q(s)y(s) - \Delta G(s)\int\limits_{-h}^{0} e^{s\theta}d\theta y(s) + \\ +\sum\limits_{i=1}^{n} Q_i(s)F_i(y) + kM(s)u(s) + \\ +N(s)f(s) - k_m M_m(s)g(s) + K(s), \end{array}$$

where $\Delta Q(s) = Q_m(s) - Q(s)$, $\Delta G(s) = G_m(s) - G(s)$. By applying the Euclidean division algorithm, we have

$$Q_m(s) = \overline{Q}(s)M(s) + M_1(s),$$

$$G_m(s) = \overline{G}(s)M(s) + M_2(s).$$

Here $\deg \overline{Q}(s) = \gamma$, $\gamma = n - m$, $\deg M_1(s) \le m - 1$, $\deg \overline{G}(s) \le \gamma - 1$, $\deg M_2(s) \le m - 1$.

Decompose the polynomials $\overline{Q}(s)$ and $\overline{G}(s)$ as $\overline{Q}(s) = Q_0(s) + \Delta_1(s)$, $\overline{G}(s) = G_0(s) + \Delta_2(s)$, and divide by the polynomial $M(s)$. Here $Q_0(s)$ is Hurwitz polynomial. As a result of these transformations, the error equation in complex variable takes the form

$$\begin{gathered} Q_0(s)e(s) = G_0(s) \int\limits_{-h}^{0} e^{s\theta} d\theta e(s) + ku(s) \\ + \tfrac{1}{M(s)} \Big(N_1(s)y(s) + N_2(s) \int\limits_{-h}^{0} e^{s\theta} d\theta y(s) \\ + \sum_{i=1}^{n} Q_i(s)F_i(y) + N(s)f(s) \\ -k_m M_m(s)g(s) + K(s) \Big), \end{gathered} \tag{4}$$

where $N_1(s) = \Delta Q(s) - M_1(s) - \Delta_1(s)$, $N_2(s) = \Delta G(s) + M_2(s) + \Delta_2(s)$, $\deg N_1(s) = n - 1$, $\deg N_2(s) \le n - 1$.

Let us highlight the whole components in the expressions as $\frac{N_1(s)}{M(s)} = N_3(s) + \frac{N_4(s)}{M(s)}$, $\frac{N_2(s)}{M(s)} = N_5(s) + \frac{N_6(s)}{M(s)}$, $\deg N_3(s) = \gamma - 1$, $\quad \deg N_5(s) \le \gamma - 1$.

Transform the equation (4) into an operator form

$$Q_0(p)e(t) = G_0(p) \int\limits_{-h}^{0} e(t+\theta)d\theta + ku(t) + \psi_1(t), \tag{5}$$

where

$$\begin{gathered} \psi_1(t) = \left(N_3(s) + \tfrac{N_4(s)}{M(s)} \right) y(t) \\ + \left(N_5(s) + \tfrac{N_6(s)}{M(s)} \right) \int\limits_{-h}^{0} y(t+\theta)d\theta \\ + \sum_{i=1}^{n} Q_i(p)F_i(y) + \sigma_1(t) + \sigma_2(t) + \sigma(t), \\ \sigma_1(t) = \tfrac{k_m M_m(p)}{M(p)} g(t) = k_m g(t) + \tfrac{\Delta M(p)}{M(p)} g(t), \sigma_2(t) \\ = \tfrac{N(p)}{M(p)} f(t) = \Delta N_1(P) f(t) + \tfrac{\Delta N_2(p)}{M(p)} f(t), \\ \sigma(t) = L^{-1} \left\{ \tfrac{K(s)}{M(s)} \right\}, \quad e(t) = L^{-1}\{e(s)\} \end{gathered}$$

are originals from Laplace images, $p = d/dt$ is a differentiation operator, $\deg \Delta N_1(p) = n_1 - m$, $\deg \Delta N_2(p) \le m$.

If the measurement of $n - m - 2$ derivatives of the auxiliary control $v(t)$ is available, then we introduce the control law $u(t)$ in the form

$$u(t) = T(p)v(t). \tag{6}$$

Then the equation (5) will take the following form

$$Q_0(p)e(t) = G_0(p) \int\limits_{-h}^{0} e(t+\theta)d\theta + +kT(p)v(t) + \psi_1(t). \tag{7}$$

If it is impossible to measure the derivatives of the control $v(t)$, we consider the control law in the form

$$u(t) = T(p)\overline{v}(t), \tag{8}$$

where $\overline{v}(t)-$ signal estimation received from an observer [Atassi et al., 1999]

$$\dot{\zeta} = F_0\zeta(t) + B_0(v(t) - \overline{v}(t)), \overline{v}(t) = L\zeta(t). \tag{9}$$

Here $\zeta(t) \in \mathbb{R}^{n-m}$, F_0 is a matrix in Frobenius form with zero lower row, $L = [1, 0, ..., 0]$, $B_0^{\mathrm{T}} = \left[\frac{b_1}{\mu}, ..., \frac{b_{n-m}}{\mu^{n-m}} \right]$. The parameters $b_1, ..., b_{n-m}$ are chosen so that the matrices $F = F_0 + BL$ are Hurwitz, $B^{\mathrm{T}} = [b_1, ..., b_{n-m}]$. Substituting (8) into (5), we obtain the equation

$$Q_0(p)e(t) = \beta T(p)v(t) + \bar{\varphi}(t) + \beta T(p)(\overline{v}(t) - v(t)), \tag{10}$$

where $\overline{\varphi}(t) = \psi_1(t) + G_0(p) \int\limits_{-h}^{0} e(t+\theta)d\theta, \beta = \sup\limits_{k \in \Xi} k.$

Let us choose the polynomial $T(\lambda)$ so that one gets the transfer function $\frac{T(\lambda)}{Q_0(\lambda)} = \frac{1}{\lambda + a_m}$. Then the equation (10) is transformed to the form

$$(p + a_m)e(t) = \beta v(t) + \varphi(t), \tag{11}$$

where $\varphi(t) = \frac{1}{T(p)}\overline{\varphi}(t) + \beta(\overline{v}(t) - v(t))$.

The signal $\varphi(t)$ depends on all the uncertainty of the parameters of the considered system and external disturbances.

Let us introduce an auxiliary loop

$$(p + a_m)\overline{e}(t) = \beta v(t). \tag{12}$$

Taking into account (11) and (12), we write an equation for the mismatch $\zeta(t) = e(t) - \overline{e}(t)$,

$$(p + a_m)\zeta(t) = \varphi(t). \tag{13}$$

Thus, if $n - m - 1$ derivatives of the signal $v(t)$ and the first derivative of $e(t)$ are available for measurement, then by forming $v(t)$in the form of

$$v(t) = -\frac{1}{\beta}(p + a_m)\zeta(t) \tag{14}$$

we obtain that the control law (6), (14) provides the asymptotic stability of the system (4), (7), (14) by the variable $e(t)$. Then the equation of the closed-loop system has the form $(p + a_m)\, e\,(t) = 0$.

If it is impossible to measure the necessary derivatives of the signal $\zeta(t)$, instead of (13), the signal $v(t)$ is introduced as

$$v(t) = -\frac{1}{\beta}(p + a_m)\overline{\zeta}(t), \tag{15}$$

where $\overline{\zeta}(t)$ is the estimate obtained from the observer [Atassi et al., 1999, Furtat et al., (2018)]

$$\begin{aligned} \dot{z}(t) &= \tfrac{b_1}{\mu}(\zeta(t) - z(t)), \\ \overline{\zeta}(t) &= z(t). \end{aligned} \tag{16}$$

Theorem. Let the conditions of assumptions 1-6 hold. Then for any $\delta > 0$ in (1) there are numbers $\mu > 0$, $T > 0$ such that for $\mu \leq \mu_0$ and $t \geq T$ the system (1), (8), (9), (12), (15), (16) ensures the goal (3) and all variables in the closed-loop system are bounded.

Proof. Let us introduce two vectors

$$\sigma^T(t) = \left(v(t), pv(t), ..., p^{n-m}v(t)\right), z_0^T = [\zeta(t), p\zeta(t)]$$

and normalized mismatch vectors
$\overline{\eta}(t) = \Gamma_1^{-1}(\sigma(t) - \varsigma(t)),\ \overline{w}(t) = \Gamma_2^{-1}(z_0(t) - z(t))$, where $\Gamma_1 = \text{diag}\{\mu^{n-m-1}, ..., \mu, 1\}$, $\Gamma_2 = \text{diag}\{\mu, 1\}$. Then from (9) and (16) we have

$$\begin{aligned} &\dot{\overline{\eta}}(t) = \tfrac{1}{\mu}F\overline{\eta} - b_0 p^{n-m}v(t),\ \theta(t) = \mu^{n-m-1}L\overline{\eta}(t), \\ &\dot{\overline{w}}(t) = \tfrac{1}{\mu}\overline{F}\overline{w}(t) - \overline{b}_0 p^2\zeta(t), \tau(t) = \mu L_2\overline{w}(t), \end{aligned} \tag{17}$$

where $\overline{F} = \overline{F_0} + \overline{B_0}L_2$, $b_0^T = [0, ..., 1]$, $\overline{b_0}^T = [0, 1]$, $\theta(t) = v(t) - \overline{v}(t)$, $\tau(t) = \zeta(t) - \overline{\zeta}(t)$. Let transform equations (17) into equivalent equations with respect to the outputs $\theta(t)$ and $\tau(t)$

$$\begin{aligned} \dot{\eta}(t) &= \tfrac{1}{\mu}F\eta(t) - bpv(t),\ \theta(t) = \mu^{n-m-1}L\eta(t), \\ \dot{w}(t) &= \tfrac{1}{\mu}\overline{F}w(t) - \overline{b}p\zeta(t),\ \tau(t) = \mu L_2 w(t), \end{aligned} \tag{18}$$

where $b^T = [1, 0, ..., 0]$, $\overline{b}^T = [1, 0]$.

Equations (17) and (18) are equivalent with respect to the outputs $\theta(t)$ and $\tau(t)$, since they are vector-matrix forms of the same equations

$$\begin{aligned} &\left(p^{n-m} + \tfrac{b_1}{\mu}p^{n-m-1} + ... + \tfrac{b_{n-m}}{\mu^{n-m}}\right)\theta(t) = p^{n-m}v(t), \\ &\left(p^2 + \tfrac{d_1}{\mu}p + \tfrac{d_2}{\mu_2}\right)\tau(t) = p^2\zeta(t). \end{aligned}$$

Taking into account (8) and (14), equation (17) takes the form

$$(p + a_m)y(t) = -\mu(p + a_m)L_2 w(t). \tag{19}$$

From where we have $y(t) = -\mu L_2 w(t)$.

Let take Lyapunov function in the form

$$V(t) = \eta^T(t)H\eta(t) + w^T(t)H_1 w(t), \tag{20}$$

where the positive-definite matrices H and H_1 are solutions of the equations

$$HF + F^T H = -2\rho_1 I,\ \ H_1\overline{F} + \overline{F}^T H_1 = -2\rho_2 I.$$

Calculating the derivative of $V(t)$ along the trajectories of the closed-loop system (17), one gets

$$\begin{aligned} \dot{V}(t) &= -2\tfrac{\rho_1}{\mu}|\eta(t)|^2 - 2\tfrac{\rho_2}{\mu}|w(t)|^2 - \\ &-2\eta^T(t)Hpv(t) - 2w^T(t)H_1 p\zeta(t). \end{aligned} \tag{21}$$

Let us rewrite the equations (17) in the form

$$\begin{aligned} &\mu_1\dot{\eta}(t) = F\eta(t) - \mu_2 bpv(t), \\ &\mu_1\dot{w}(t) = \overline{F}w(t) - \mu_2\overline{b}p\zeta \\ &(p + a_m)y = -\mu_2(p + a_m)L_2 w(t). \end{aligned} \tag{22}$$

Let us use the lemma [Brusin, 1995].

Lemma [Brusin, 1995]. If the system is described by the equations $\dot{x} = f(x, \mu_1, \mu_2)$, where $x \in \mathbb{R}^n$, $f(x, \mu_1, \mu_2)$ is a continuous Lipschitz function with respect to x and for $\mu_2 = 0$ has a bounded closed domain of dissipativity $\Omega = \{x/F(x) < C\}$, where $F(x)$ is positive-definite, a continuous, piecewise smooth function, then there exists $\mu_0 > 0$ such that for $\mu_1 < \mu_0$ and $\mu_2 < \mu_0$ the original system has the same Ω dissipativity domain if for some numbers $C,\ \overline{\mu}_1,\ \ \mu_2 = 0$ the condition is met

$$\sup_{|\mu_1| \leq \overline{\mu}_1}\left(\left(\tfrac{\partial F(x)}{\partial x}\right)^T f(x, \mu_1, 0)\right) \leq -C,\ F(x) = C.$$

In this case, if $\mu_2 = 0$ in (22), this is equivalent to the fact that all derivatives are measured and two exponentially stable systems are added $\mu_1\dot{\eta}(t) = F\eta(t), \dot{w}(t) = \overline{F}w(t)$. As already proved, in this case all the variables in the system are limited and the conditions of the lemma are fulfilled. In other words, in the area of Ω $y(t) \to 0,\ \ |v(t)| < k_1,\ \ |\zeta| < k_2$, and from (13) and (6) it follows that $|pv| < k_3$, $|p\zeta| < k_4$, where $k_1,\ k_2,\ k_3,\ k_4$ are some positive constants.

Let put $\mu_1 = \mu_2 = \mu$ in (22) and, substituting them in (21), use the estimates

$$\begin{aligned} &-2\eta^T(t)Hpv(t) \leq \tfrac{1}{\mu}|\eta(t)|^2 + \\ &+\mu\|H\||pv(t)| \leq \tfrac{1}{\mu}|\eta(t)|^2 + \mu\|H\|k_3, \\ &-2w^T(t)H_1 p\zeta(t) \leq \tfrac{1}{\mu}|w(t)|^2 + \mu\|H_1\|k_4. \end{aligned} \tag{23}$$

Substituting these estimates in (21), we obtain the inequality
$\dot{V}(t) \leq -\frac{\rho_1}{\mu}|\eta(t)|^2 - \frac{\rho_2}{\mu}|w(t)|^2 - \frac{1}{\mu}(\rho_1 - 1)|\eta(t)|^2 - - \frac{1}{\mu}(\rho_2 - 1)|w(t)|^2 + \mu(\|H\|k_3 + \|H\|k_4)$.

By selecting $\rho_1 > 1$ and $\rho_2 > 1$ one gets

$$\dot{V}(t) \leq -\tfrac{\rho_1}{\mu}|\eta(t)|^2 - \tfrac{\rho_2}{\mu}|w(t)|^2 + \mu\beta, \tag{24}$$

where $\beta = \|H\|k_3 + \|H_1\|k_4$. From where it follows

$$\dot{V}(t) \leq -\beta_1 V(t) + \mu\beta, \tag{25}$$

where $\beta_1 = \min\left\{\frac{\rho_1}{\mu\overline{\lambda}(H)}; \frac{\rho_2}{\mu\overline{\lambda}(H_1)}\right\}$, $\overline{\lambda}(\cdot)$ is the maximum eigenvalue of the corresponding matrix. From (25) we have $V(t) \leq \frac{\mu\beta}{\beta_1}$.

Taking into account the inequality $|w(t)|^2 \leq \frac{1}{\underline{\lambda}(H_1)}V(t) \leq \frac{\mu\beta}{\beta_1}$ from the third equation (22) we have

$|y(t)| = \mu|w(t)| \leq \mu\sqrt{\frac{\mu\beta}{\beta_1}}$. From where it can be seen that for any $\delta > 0$ in (3) there exists μ_0 such that the target condition (3) will be fulfilled.

Further results of the paper may be of interest when applied to the control of the movement of aircraft [Furtat and Putov (2013)], where a delay occurs in relation to the wind action along the hull of the vessel. Also, the application of this method may be interesting for studying the behavior of many physical systems using control theory methods [Furtat and Gushchin (2020), Furtat (2023)]. As well as a delay can occur when control of electromechanical systems, when the coupling of the magnetic field with the rotor poles occurs with an increasing and decreasing delay [Furtat et al. (2022)].

3 Example

Consider a control plant whose mathematical model has the form

$$\dot{x}(t) = \begin{bmatrix} a_1 & 1 & 0 \\ a_2 & 0 & 1 \\ a_3 & 0 & 0 \end{bmatrix} x(t) + \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \int_{-h}^{0} y(t+\theta)d\theta + \\ + \begin{pmatrix} F_1(y) \\ F_2(y) \\ F_3(y) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b_2 \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ \gamma_1 \\ \gamma_2 \end{pmatrix} f(t),$$

$$y(t) = [1\ 0\ 0]\, x(t),\ x_i(\theta) = 1,\ \ \theta \in [-h; 0], i = \overline{1,3}.$$

The uncertainty class is given by inequalities: $-3 \leq a_i \leq 3,\ i = \overline{1,3};\ -5 \leq d_j \leq 5,\ j = \overline{1,3};$ $1 \leq b_2 \leq 5;\ -2 \leq \gamma_l \leq 2, l = \overline{1,2}$. Equation of the reference model

$$\dot{x}_m(t) = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix} x_m(t) + \\ + \begin{pmatrix} -2 \\ -1 \end{pmatrix} \int_{-h}^{0} y_m(t+\theta)d\theta + \\ + \begin{pmatrix} 0 \\ 1 \end{pmatrix} g(t), \quad y_m(t) = [1\ 0]x_m(t).$$

$$F(y) = \begin{pmatrix} F_1(y) \\ F_2(y) \\ F_3(y) \end{pmatrix} = \begin{pmatrix} \ln(1+y^2) \\ \ln(1+y^2) + \mathrm{arctg(y)} \\ \mathrm{arctg(y)} \end{pmatrix}.$$

Let us choose a polynomial $T(\lambda) = \lambda^2 + 4\lambda + 4$, $\beta = 10$, $\mu = 0.01$; $a_m = 2$. The auxiliary loop is introduced as $(p+2)\overline{e}(t) = 10v(t)$, and the observer equations (9), (14) have the form

$$\begin{aligned} &\dot{\varsigma}_1(t) = \varsigma_2(t) + \tfrac{6}{\mu}(v(t) - \varsigma_1(t)), \\ &\dot{z}(t) = \tfrac{a_m}{\mu}(\zeta(t) - \varsigma_1(t)), \\ &\dot{\varsigma}_2(t) = \tfrac{8}{\mu^2}(v(t) - \varsigma_1(t)), \\ &\overline{\zeta}(t) = z(t), \\ &\overline{v}(t) = \varsigma_1(t). \end{aligned}$$

The control law is introduced in the form

$$\begin{aligned} &u(t) = \varsigma_1(t) + 4\varsigma_2(t) + 4\dot{\varsigma}_2(t), \\ &v(t) = -\tfrac{1}{10}(2\zeta(t) + \dot{z}(t)). \end{aligned}$$

Figures 1–3 show transients of the tracking error, output of the reference model, and control signal.

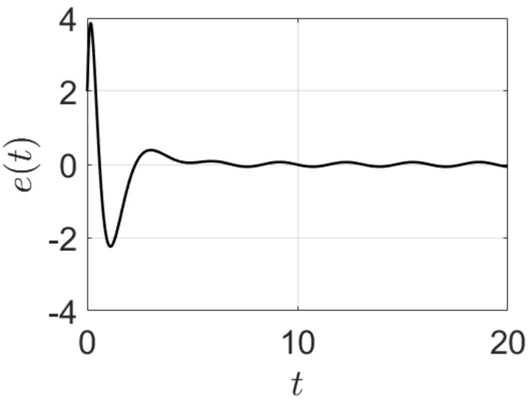

Figure 1. The transients of the tracking error.

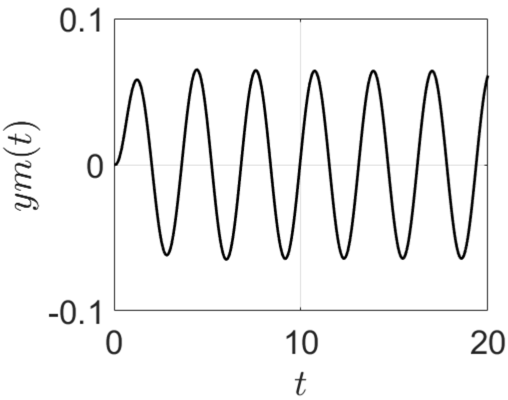

Figure 2. The transients of the output of the reference model.

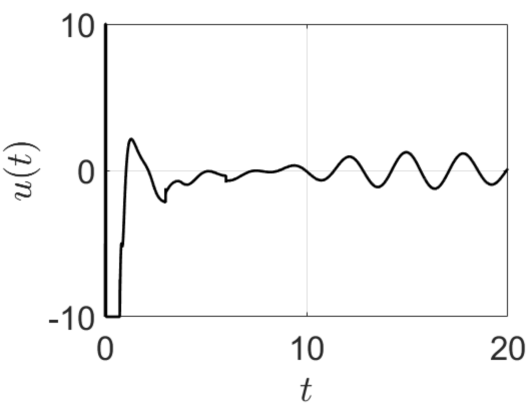

Figure 3. The transients of the control signal.

4 Conclusion

In the proposed paper, a robust plant control system is designed, the dynamic processes in which are described by a nonlinear equation with a distributed time delay in state. To solve the problem, a generalized disturbance signal is generated, and then its compensation is carried out using an auxiliary loop and two observers of variables.

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