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ESTIMATION OF VISCOUS FRICTION PARAMETERS IN ACROBOT

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Abstract

This paper deals with estimation of coefficients of viscous friction for a model of an acrobot. The acrobot represents an underactuated nonlinear dynamic system, where typically not all states are measurable. Moreover effect of noise corruption on remaining measured states is often non negligible. However, except for friction coefficient, all remaining parameters of the model can usually be measured directly.

To overcome mentioned difficulties and to take advantage of abundant prior knowledge, we applied hybrid extended Kalman filter to this task. Using Monte Carlo (MC) simulations we approximated probability density functions of friction coefficients estimate and showed that the bias and variance of the estimate can be controlled by properly designed experiment.

Key words

Filtering and Identification, Nonlinear Dynamics, Stochastic systems

1 Introduction

Research concerned with robotic walking design in past few decades became very actual topic, mostly aimed at improvement of robustness and energy consumption of control algorithms. The pursue for energy efficient walking design lead to use of unstable robot configurations with underactuated variables. Model of acrobot serves as a basic concept for such underactuated walking robot design. Although simple in appearance, acrobot represents highly nonlinear underactuated dynamic system and control algorithms designed for acrobot can be seen as an important step towards underactuated walking.

Such an approach is applied for example in [Anderle et. al., 2010] and [Anderle and Čelikovský, 2010b] where it is showed that using a well designed state feedback control algorithm, it is possible to track a cyclic reference trajectory designed for walking like movement of the acrobot.

To deal with the fact that not all state variable are measurable in [Anderle and Čelikovský, 2010a] a nonlinear observer of acrobot was proposed.

Bipedal walking robot acrobot can be extended to a more complicated version including four links joined by two additional joints and thus robot is in addition able to perform movement in knees. Each leg can be modeled by one acrobot.

However mostly the mathematical model considered assumes that the viscous friction in joints can be neglected and thus parameters of mathematical model of bipedal robot can be obtained by direct physical measurement. Assumption that viscous friction can be neglected often can not be satisfied and damping effect on the motion of the robot is often considerable.

To include viscous friction effect in a mathematical model is more or less straight forward procedure, depending on precision of mathematical modeling, but the determination of the values of friction coefficients is not possible by direct measurement and these coefficient have to be estimated.

The situation is complicated by the fact that not all the states of the system are known and so in order to estimate the parameters one has to estimate remaining unknown states as well. The Kalman filter has become the algorithm of choice for state estimation from noise corrupted data in many practical applications, e.g [Iwasaki and Kataoka, 1989]. In this article we use hybrid extended Kalman filter (HEKF) which is an extension of Kalman filter to nonlinear systems with continuous dynamics.

Aim of this article is to investigate whether it is possible to estimate parameters of viscous friction using experimental identification and to determine at least approximately how accurate the resulting estimate of these unknown parameters will be. To exploit the abundant prior knowledge, deal with the fact that not all states are measured and also that the noise corruption of measurement is non negligible we used HEKF.

2 Model of the acrobot

In this article we consider following model of acrobot, depicted on fig. 1.

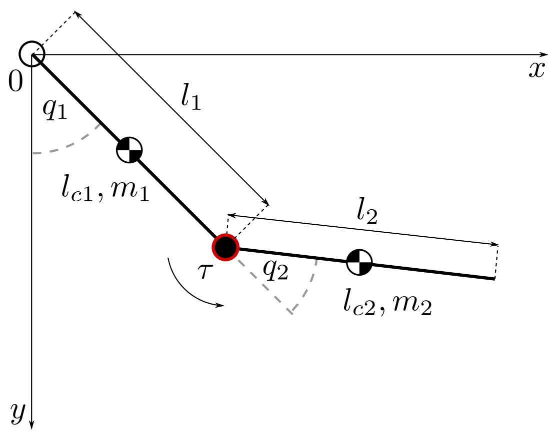

Figure 1. The acrobot

Physical quantities that describe model of acrobot are summed up in tab. 1.

Table 1. Parameters of the acrobot

l_1, l_2	length of 1st, 2nd link	$[m]$
l_{c1}, l_{c2}	center of gravity of 1st, 2nd link	$[m]$
m_1, m_2	mass of 1st, 2nd link	$[kg]$
τ	applied torque	$[N.m]$
g	gravitational acceleration	$[m.s^{-2}]$
μ_1, μ_2	viscous friction parameters	$[N.s/m]$

Two rigid links are joined by a joint. This joint is actuated by a DC motor. Position of the system is uniquely defined by two angles q_1 and q_2, thus the system has two degrees of freedom, yet there is only one control input, torque τ generated by DC motor. Therefore acrobot represents an underactuated system, with degree of underactuation equal to one and underactuated angle q_1. The state vector $\boldsymbol{x}$ of acrobot is composed from generalised co-ordinates - angles q_1, q_2 and generalised velocities $\dot{q_1}$, $\dot{q_2}$. Measured state variables of our laboratory model are q_1, q_2 thus

$$
\begin{aligned}
&\boldsymbol{x} = (q_1,\ \dot{q_1},\ q_2,\ \dot{q_2})^T, && \\
&\boldsymbol{q} = (q_1,\ q_2)^T, && \dot{\boldsymbol{q}} = (\dot{q_1},\ \dot{q_2})^T, \\
&\boldsymbol{u} = (0\ ,\tau)^T, && \boldsymbol{y} = (q_1,\ q_2)^T.
\end{aligned}
$$

Remaining unmeasured states $\dot{q_1}$ and $\dot{q_2}$ have to be estimated. Directly measurable parameters of acrobot are l_1, l_2, m_1, m_2, g and parameters l_{c1}, l_{c2} can be computed, unknown parameters are μ_1, μ_2.

2.1 Equations of motion

To obtain equations of motion (EoM) for acrobot we use classical Lagrangian approach [Landau and Lifshitz, 1976]. If we introduce following substitution

$$
\begin{aligned}
\theta_1 &= m_1 l_{c1}^2 + \tfrac{1}{12} m_1 l_1^2 + m_2 l_1^2, \\
\theta_2 &= m_2 l_{c2}^2 + \tfrac{1}{12} m_2 l_2^2, \\
\theta_3 &= m_2 l_1 l_{c2}, \\
\theta_4 &= m_1 l_{c1} + m_2 l_1, \\
\theta_5 &= m_2 l_{c2}, \\
\theta_6 &= \mu_1, \\
\theta_7 &= \mu_2,
\end{aligned}
\tag{1}
$$

then resulting equations of motion (EoM) of acrobot in Lagrange formalism are

$$
\mathbf{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \mathbf{G}(\boldsymbol{q}) = \boldsymbol{u} \tag{2}
$$

where matrices

$$
\mathbf{D} = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos(q_2); & \theta_2 + \theta_3 \cos(q_2) \\ \theta_2 + \theta_3 \cos(q_2); & \theta_2 \end{bmatrix}, \tag{3}
$$

$$
\mathbf{C} = \begin{bmatrix} -2\theta_3 \sin(q_2)\dot{q_2} + \theta_6; & -\theta_3 \sin(q_2)\dot{q_2} \\ \theta_3 \sin(q_2)\dot{q_1}; & \theta_7 \end{bmatrix}, \tag{4}
$$

$$
\mathbf{G} = \begin{bmatrix} \theta_4 g \sin(q_1) + \theta_5 g \sin(q_1 + q_2) \\ \theta_5 g \sin(q_1 + q_2) \end{bmatrix}. \tag{5}
$$

Matrix $\mathbf{D}$ contains inertia terms, matrix $\mathbf{C}$ contains centripetal, Coriolis force and viscous friction terms and matrix $\mathbf{G}$ contains gravity terms.

3 Hybrid extended Kalman filter

The Kalman filter is a mathematical tool used for estimation of state vector of linear systems in state-space description from noise corrupted measurements. The hybrid extended Kalman filter is extension of classical discrete Kalman filter to nonlinear systems, moreover HEKF considers system with continuous-time dynamics and discrete-time measurements, for detailed information see for example [Simon, 2006].

Acrobot's equations of motion with discrete measurement of outputs can be represented in the following form

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}, t) \\
\boldsymbol{y}_k &= \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{v}_k) \\
&= [x_{1,k}, x_{3,k}]^T + [v_{1,k}, v_{2,k}]^T \\
\boldsymbol{w}(t) &\sim (0, \mathbf{Q}) \\
\boldsymbol{v}_k &\sim (0, \boldsymbol{R}_k)
\end{aligned}
$$

where $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}, t)$ are derived EoM, $\boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{v}_k)$ is function which describes how is state vector transformed on output, $\boldsymbol{w}(t)$ is continuous-time white noise[1] with covariance $\mathbf{Q}$ and $\boldsymbol{v}_k$ is discrete-time white noise with covariance $\mathbf{R}_k$ and index k means that quantity is evaluated at discrete time instants $t_k = kT_s$ and T_s stands for sampling period. Between the measurements estimation of the state vector is propagated according to continuous EoM and the covariance of estimation error is propagated according to (7).

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}, \boldsymbol{u}, \boldsymbol{w}_0, t) \tag{6}$$
$$\dot{\mathbf{P}} = \mathbf{AP} + \mathbf{PA}^T + \mathbf{LQL}^T \tag{7}$$

where

$$\mathbf{A} = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}^+_{k-1}}, \quad \mathbf{L} = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{w}}\right|_{\hat{\boldsymbol{x}}^+_{k-1}}, \quad \boldsymbol{w}_0 = 0 \, .$$

At each measurement time t_k, we update the a prior state estimate $\boldsymbol{x}^-_k$ and the a prior covariance matrix estimate $\mathbf{P}^-_k$ as derived in [Simon, 2006].

$$\mathbf{K}_k = \mathbf{P}^-_k \mathbf{H}^T_k (\mathbf{H}_k \mathbf{P}^-_k \mathbf{H}^T_k + \mathbf{M}_k \mathbf{R}_k \mathbf{M}^T_k)^{-1} \tag{8}$$
$$\hat{\boldsymbol{x}}^+_k = \hat{\boldsymbol{x}}^-_k + \mathbf{K}_k(\boldsymbol{y}_k - \boldsymbol{h}_k(\hat{\boldsymbol{x}}^-_k, \boldsymbol{v}_0, t_k)) \tag{9}$$
$$\mathbf{P}^+_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}^-_k (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{M}_k \mathbf{R}_k \mathbf{M}^T_k \mathbf{K}^T_k \tag{10}$$

where $\mathbf{I}$ denotes identity matrix and

$$\mathbf{H}_k = \left.\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}^-_k}, \quad \mathbf{M}_k = \left.\frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{v}}\right|_{\hat{\boldsymbol{x}}^-_k}, \quad \boldsymbol{v}_0 = 0 \, .$$

3.1 Parameter estimation

It is possible to estimate both states and parameters of acrobot using HEKF. We can consider unknown parameters of acrobot as state variables and augment the state vector of acrobot as follows

$$\boldsymbol{x}' = (\boldsymbol{x}^T, \theta_6, \theta_7)^T. \tag{11}$$

Parameters θ_6, θ_7 are not time varying thus

$$\dot{x}'_i = 0, \qquad i = 6,\ 7, \tag{12}$$

and augmented EoM of acrobot are

$$\dot{\boldsymbol{x}}' = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{w}, t) \\ \mathbf{0} + \boldsymbol{w}_\theta \end{bmatrix} \tag{13}$$
$$\boldsymbol{y}_k = \boldsymbol{h}_k(\boldsymbol{x'}_k, \boldsymbol{v}_k) \tag{14}$$

where $\boldsymbol{w}_\theta \sim (0, \mathbf{Q}_\theta)$ is an artificial white noise. Covariance matrix $\mathbf{Q}_\theta$ is used for control of the identification algorithm and value we used is $\mathbf{Q}_\theta = \sigma^2_\theta \mathbf{I}$ where σ_θ denotes variance of unknown parameters and $\mathbf{I}$ denotes identity matrix.

Now we can use HEKF to estimate state vector $\boldsymbol{x}'$ of augmented dynamic system.

4 Simulation results

The results of the proposed identification procedure are strongly dependent on the level of the measurement noise. Although corruption of the measurement by the measurement noise can be suppressed only by the use of more precise sensors, there are ways how to affect the precision of the unknown parameters estimate. Depending on the amount of data available we can either adjust the parameter $\mathbf{Q}_\theta$ or we can change the input signal u.

To investigate properties of proposed identification procedure, we carried out 100 Monte Carlo simulations of HEKF with different realizations of noise vector $\boldsymbol{v}_k$, the covariance matrix of the measurement noise was $\mathbf{R}_k = \sigma^2_v \mathbf{I}$ and we set $\sigma^2_v = 0.0025$. The covariance matrix of process noise was $\mathbf{Q} = \sigma^2_w \mathbf{I}$ and we set $\sigma^2_v = 10^{-8}$.

Then we determined sample means $\bar{\hat{\theta}}_6$, $\bar{\hat{\theta}}_7$ and sample variances ${s_6}^2$, ${s_7}^2$, we computed these values as follows

$$\begin{aligned} \bar{\hat{\theta}}_i &= \frac{1}{n} \sum_{l=1}^{n} \hat{\theta}_l \\ {s_i}^2 &= \frac{1}{n-1} \sum_{l=1}^{n} (\hat{\theta}_l - \theta_i)^2 \\ n &= 100 \qquad i = 1,2 \qquad j = 6,7. \end{aligned}$$

We repeated this procedure for different amplitude of the input signal u and different values of the covariance matrix $\mathbf{Q}_\theta = \sigma^2_\theta \mathbf{I}$ where σ_θ denotes variance of unknown parameters. Length of one simulation was $120\ [s]$.

4.1 Effect of the input signal on the parameters estimation

To evaluate the effect of the input signal on the quality of the unknown parameters estimate we used pseudo random binary signal, e.g. on fig. 2, to excite the system from it's natural stable equilibrium. We used different values of amplitude to discover how the estimate depends on the excitation signal. We set the covariance matrix $\mathbf{Q}'$ of the process noise $\boldsymbol{w}'$ of augmented system as follows

$$\mathbf{Q}' = \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_\theta \end{bmatrix} = \begin{bmatrix} \sigma_w \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_\theta \mathbf{I} \end{bmatrix} = \begin{bmatrix} 10^{-8}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & 10^{-8}\mathbf{I} \end{bmatrix}$$

[1]This notation is only an analogy of notation used for discrete Kalman filter because continuous white noise doesn't exist.

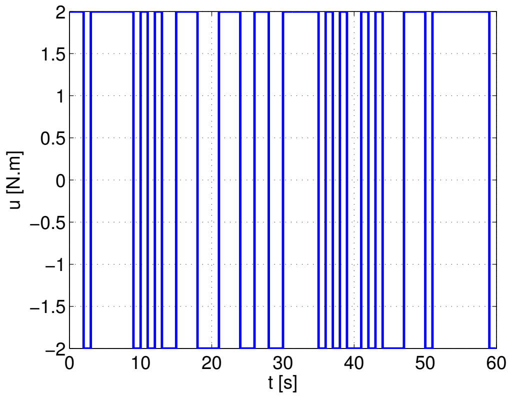

Figure 2. Pseudo random binary signal

We found out that choice of input signal is a very important part of the experiment. All state variables of the system should be sufficiently excited and the higher level of measurement noise is present the better excitation of the system is required. Example of identification process for different values of amplitude of the input signal are depicted on fig. 3.

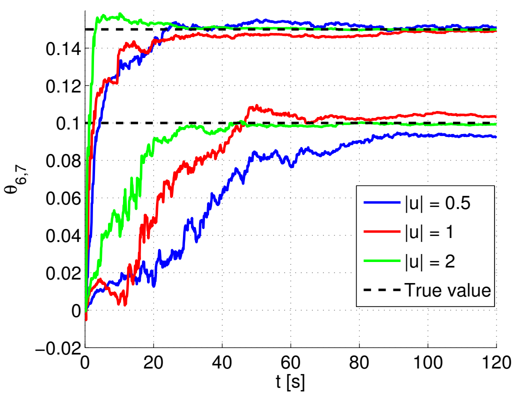

Figure 3. Time behavior of $\hat{\theta}_6$, $\hat{\theta}_7$ under various amplitude of u

From fig. 3 we can see that convergence of the parameters estimate is faster when the amplitude of the input system is larger and slower when the amplitude is smaller.

Another noticeable fact is that convergence of $\hat{\theta}_7$ is much faster than convergence of $\hat{\theta}_6$ explanation to that fact might be that angle q_2 is much better excited, i.e. it's value varies much more than value of angle q_1.

Probability density function (PDF)

$$f(\hat{\theta}_i) = \frac{1}{\sqrt{2\pi s_i^2}} e^{-\frac{(\hat{\theta}_i - \overline{\hat{\theta}}_i)^2}{2s_i^2}}, \ i = 1, 2$$

of estimate $\hat{\theta}_6$, $\hat{\theta}_7$ for 100 Monte Carlo simulations are depicted on fig. 4, 5.

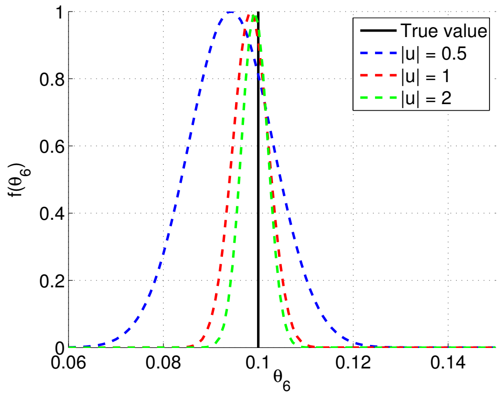

Figure 4. Pdf of $\hat{\theta}_6$

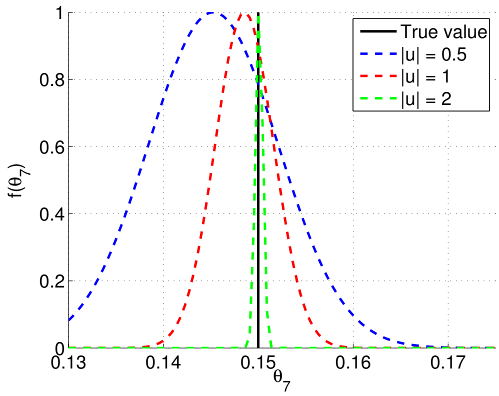

Figure 5. Pdf of $\hat{\theta}_7$

From fig. 4, 5 we can deduce that bias and the variance of the estimates $\hat{\theta}_6$, $\hat{\theta}_7$ are improving due to better excitation of the system.

4.2 Effect of $\mathbf{Q}_\theta$ on the parameters estimation

To evaluate the effect of covariance matrix $\mathbf{Q}_\theta$ on the parameter estimation we again carried out the set of 100 Monte Carlo simulations for different values of matrix $\mathbf{Q}_\theta$ and observed the results. Example of single simulation is depicted on fig. 6.

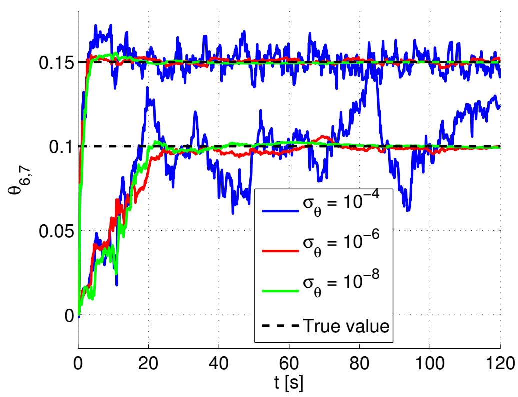

Figure 6. Time behavior of $\hat{\theta}_1$, $\hat{\theta}_2$

From fig. 6 we can see that parameter $\mathbf{Q}_\theta$ works as parameter that controls the sensitivity of the identification algorithm to the measurements. Larger values of $\mathbf{Q}_\theta$ results in faster convergence as the algorithm is more sensitive to the information from the measurement, however the noise from the measurement is amplified too and setting $\mathbf{Q}_\theta$ too large results in oscillatory time behaviour of the parameters estimate.

Probability density function

$$f(\hat{\theta}_i) = \frac{1}{\sqrt{2\pi s_i^2}} e^{-\frac{(\hat{\theta}_i - \overline{\hat{\theta}}_i)^2}{2s_i^2}}, \; i = 1, 2$$

of estimate $\hat{\theta}_6$, $\hat{\theta}_7$ for 100 Monte Carlo simulations are depicted on fig. 4, 5.

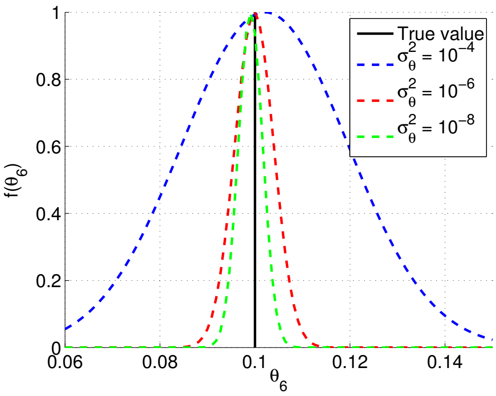

Figure 7. Pdf of $\hat{\theta}_6$

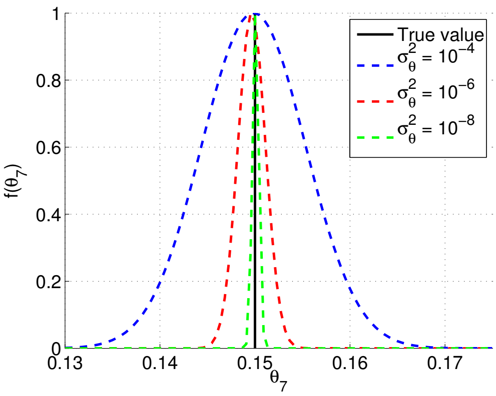

Figure 8. Pdf of $\hat{\theta}_7$

5 Conclusion

In this paper we investigated the possibility to estimate parameters of viscous friction of one leg of bipedal robot, modeled by acrobot. The identification procedure was implemented using HEKF to deal with the noise corruption of the measurement and the fact that not all the state variables were directly measurable. Moreover hybrid EKF considers dynamics of the system in the natural continuous form.

We found this method applicable provided that some a prior estimate of the parameters and enough data are available. We verified that the excitation signal has a direct influence on the bias, variance and convergence speed of the unknown parameters estimate. Also we verified that parameter $\mathbf{Q}_\theta$ works as parameter controlling the sensitivity of the identification procedure to the measurements and also pointed out the possible pitfall in amplification of the noise when one would choose $\mathbf{Q}_\theta$ too large.

All in all HEKF represents an identification procedure that is able to identify the unknown parameters of viscous friction despite aforementioned complications like noise corruption and estimation of unmeasured states.

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References

Anderle, M., Čelikovský, S. (2010a). Position Feedback tracking of the Acrobot walking-like trajectory based on the reduced velocity observer. To appear in *Preprints of the 8th IFAC Symposium on Nonlinear Control Systems*, Bologna, Italy.

Anderle, M., Čelikovský, S. (2010b). Sustainable acrobot walking based on the swing phase exponentially stable tracking. To appear in *The 3rd Annual Dynamic Systems and Control conference*, Boston, USA.

Anderle, M., Čelikovský, S., Henrion, D., Zikmund J. (2010). Advanced LMI based analysis and design for Acrobot walking. In *Internation Journal of Control*, 83:8, pp. 1641 - 1652.

Dolinský, K., Jadlovská, A. (2010). Implementácia výsledkov experimentálnej identifikácie v riadení výukového modelu helikoptéry. In *Electrical Engineering and Informatics: Proceedings of the Faculty of Electrical Engineering and Informatics of the Technical University of Koice*, Košice, Slovakia, pp. 546-550, ISBN 978-80-553-0460-1

Iwasaki, T. and Kataoka, T. (1989). Application of an extended Kalman filter to parameter identification of an induction motor. In *Industry Applications Society Annual Meeting, 1989., Conference Record of the 1989 IEEE*, San Diego, CA, pp. 248-253.

Landau, L. D. and Lifshitz, E. M. (1976). *Mechanics, Third Edition: Volume 1 of Course of Theoretical physics*. Elsevier Science. ISBN 0 7506 2896 0.

Simon, D. (2006). *Optimal State Estimation*. Wiley-Interscience. New Jersey. ISBN 13 978-0-471-70858-2.

Spong, M. (1998). *Underactuated Mechanical Systems, Control Problems in Robotics and Automation*. Springer Verlag. London.