A PASSIVE VIBRATION ISOLATOR INCORPORATING A COMPOSITE BISTABLE PLATE

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Abstract

Decreasing the natural frequency of a passive vibration isolator would improve its performance by extending the isolation frequency bandwidth. However, there is trade-off between low natural frequency (low stiffness) and large static displacement. This can be mitigated by employing a nonlinear spring with high-static-low-dynamic-stiffness (HSLDS). In this paper a passive isolator with HSLDS is designed by exploiting the negative stiffness of a composite bistable plate.

1 Introduction

The use of passive isolators is ubiquitous in engineering systems (Hartog, 1985; Rivin, 2001). In the simplest case when the isolator is linear, a low natural frequency, which is desirable, can only be achieved by having a large static deflection, which is undesirable. This disadvantage can be overcome by employing isolation mounts with a nonlinear characteristic that provides a High-Static-Low-Dynamic-Stiffness (HSLDS) resulting in a small static deflection, and a small stiffness resulting in a low natural frequency and hence a greater frequency range over which there is vibration isolation (Rivin, 2001). There are a number of ways to obtain this desirable nonlinear characteristic. Platus (Platus, 1999) exploited the buckling of beams under axial load in a specific configuration to achieve a negative stiffness in combination with a positive stiffness, and hence low dynamic stiffness. Others have achieved the same by connecting linear springs with positive stiffness in parallel with mechanical elements of negative stiffness (Alabuzhev *et al.*, 1989; Carrella *et al.*, 2007) or using magnets as source of negative stiffness (Carrella *et al.*, accepted for pubblication). In this paper it is proposed to exploit the negative stiffness exhibited by a square bistable composite plate in order to achieve the desired HSLDS characteristic for implementing an efficient passive vibration isolation system. Most in general, bistable structures are those which exhibit two stable equilibrium positions. Examples of bistable systems can be found in many fields. For example the flight mechanism of the diptera can be considered as a bistable mechanism, (Brennan *et al.*, 2003). Great interest has been recently focussed on the employment of bistable structures for aeronautical applications. Namely, these 'morphing' structures allow, for instance, in-flight change of the geometry of an aircraft with consequent benefit in terms of versatility and fuel consumption, (Mattioni *et al.*, 2006*b*; Mattioni *et al.*, 2005). Further details on the design and manufacture of composite bistable structures can be found in reference (Mattioni *et al.*, 2006*a*).

The passive vibration isolator based on a bistable plate is shown schematically in Fig. 1 in its loaded condition. The mass is placed in the centre (point P) of the bistable plate initially curved, which flattens under the weight. At this point the plate does not exert any restoring force and the weight is resisted only by the vertical mechanical spring. The force displacement characteristic of the plate can be computed with a commercial FE software (in this case ABACUS) or experimentally and then fitted with a cubic polynomial with negative linear coefficient and a positive cubic term. This way it is possible to determine the maximum negative stiffness, which is required to occur at the static equilibrium position. This defines the coefficient of the mechanical

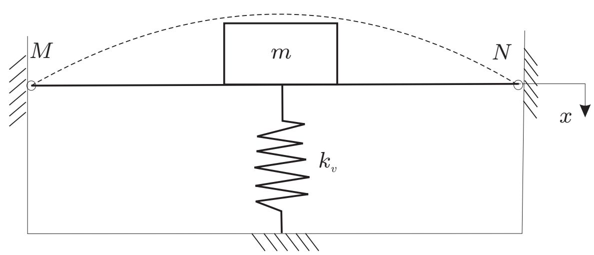

Figure 1. Schematic representation of a passive isolation system with a bistable plate and a mechanical spring. The dashed line indicate the plate in its initial shape. The addition of the mass m causes the plate to become approximately horizontal (solid line)

spring that is required to make the total dynamic stiffness (for oscillations about the static equilibrium position) very small (theoretically zero).

2 A SDOF model for a bistable plate

In this section, a single-degree-of-freedom (SDOF) model for a bistable plate is proposed. The main focus is on the snap-through mechanism that marks the passage from one stable state to the other. Mattioni *et al.* in reference (Mattioni *et al.*, 2006*a*) have performed the static analysis of a composite bistable plate. A static load was applied in the centre of a rectangular plate hinged at the corners and the reaction force was computed. This reaction force is equivalent to the restoring force of the plate and allows the plate stiffness to be estimated. Arrieta *et al.* (Arrieta *et al.*, 2007) applied an harmonic force at the centre of a square plate and measured its response. The test was an initial attempt to study the dynamic of the structure but has shown a rich and complex dynamic behaviour. In particular, there seems to be a frequency, for a given amplitude of the excitation force, at which the plate 'snaps' from one stable position to the other. These two observations imply that, to a first approximation, the dynamics of the snap-through mechanism may be modelled by a single degree of freedom, namely the vertical displacement (out of plane motion) at the excitation point.

2.1 Static analysis

A qualitative load-deflection curve of a bistable plate obtained with a Finite Element Analysis (FEA) is shown as the dashed line in Fig. 2. The curve corresponds to a square plate with the corners pinned and a quasi-static load applied in the centre. The force on the y-axis is the reaction force of the plate measured at the corners and the displacement on the x-axis is the relative displacement between the supports and the application point of the force. The numerical load-deflection characteristic can be approximated to a cubic polynomial with negative linear coefficient and nonlinear cubic term

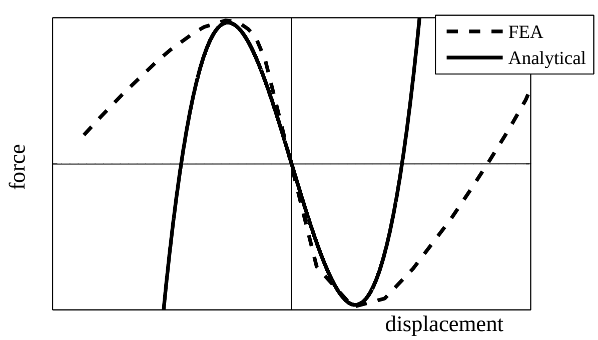

Figure 2. Load-deflection characteristic. The dashed line is obtained with finite element analysis (FEA), the solid line is the plot of the function $f_p = -k_1\,x + k_3\,x^3$

$$f_p(y) = -k_1\,y + k_3\,y^3 \tag{1}$$

where the coefficients k_1 and k_3 can be determined by ensuring the function (1) is symmetric and passes through the peaks of the numerical curve. The approximate analytical restoring force of the plate is also shown in Fig. 2 as the solid line. This function is a reasonable approximation between the peaks in the force-displacement curve, where the vibration isolation will be operating. Other approaches to obtain the coefficients may be applied, such as a least squares fit over a desired range of displacements.

As shown in (Carrella *et al.*, 2007; Alabuzhev *et al.*, 1989), a mechanical model of a system with a cubic restoring force with a negative linear coefficient and positive cubic term is equivalent to a system with two oblique springs of equal coefficients, k_o. This is shown by the system in Fig. 3 with the vertical spring, k_v, omitted. The stiffness of the plate can be now derived

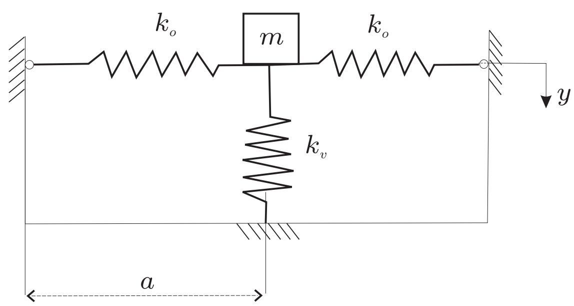

Figure 3. Mechanical SDOF model of the isolation system with a bistable plate, represented by two oblique springs of identical coefficient k_o, and a vertical mechanical spring k_v

from the restoring force as

$$k_p(y) = -k_1 + 3\,k_3\,y^2 \tag{2}$$

A sketch of the non-linear plate stiffness is shown with the dash-dot line in Fig. 4. There is a displacement range with negative stiffness which has a minimum at the zero displacement, which is the desired static equilibrium position of the vibration isolator. The instability due to the negative stiffness makes the bistable-plate alone unsuitable for vibration isolation. The dashed line in Fig.4 is the stiffness of a mechanical spring, which is constant and positive, k_v. This spring stiffness can be chosen to be slightly greater than the minimum stiffness of the plate, k_1, so that the total stiffness, given by their sum, is small and positive.

$$k_{tot} = (k_v - k_1) + 3\,k_3\,y^2 \tag{3}$$

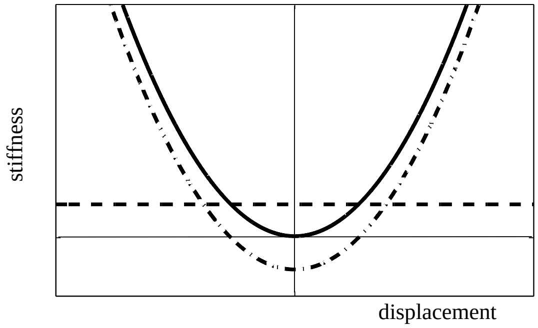

Figure 4. Qualitative plot of the stiffness as function of the displacement for the bistable plate (dash-dot), the mechanical spring (dashed) and the total stiffness of the isolator (solid)

In so doing the stiffness of the isolation mount is very small in the vicinity of the static equilibrium position of the isolator and increases as the displacement increases. This is shown by the solid line in Fig. 4 The restoring force of the isolation system is obtained by integrating Eqn. (3) to give

$$f_{tot} = (k_v - k_1)\, y + k_3\, y^3 = k\, y + k_3\, y^3 \qquad (4)$$

and is plotted in Fig. 5 The major benefit of having

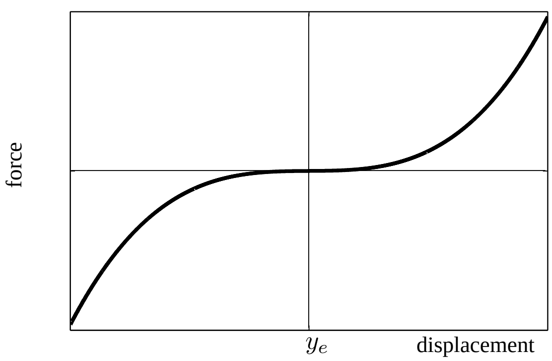

Figure 5. Load-deflection characteristic of the vibration isolator with a bistable plate and a mechanical spring. For oscillations about the static equilibrium position the stiffness is small and this results in a low natural frequency

an isolation mount with the restoring force depicted in Fig. 5 is that the for oscillations about the static equilibrium position the *local* or *dynamic* stiffness is small, and this ensures a very low natural frequency (theoretically zero, if $k_v = k_1$). It is known that a vibration isolator with a low natural frequency has a wider frequency isolation region, (Rivin, 2001; Carrella *et al.*, 2007). It should be noted that, unlike a standard linear isolator, the low natural frequency is not associated with a high static displacement. In fact, at the static equilibrium position the weight of the mass is resisted only by the restoring force of the vertical spring. Thus by designing the bistable plate and dimensioning the vertical spring, it is possible to design a vibration isolator with a High-Static-Low-Dynamic-Stiffness (HSLDS), (Carrella *et al.*, accepted for publication). With the spring force expressed by Eqn. (4) and assuming a viscous dissipative mechanism, the equation of motion for the isolation system with a harmonic force acting on the mass can be written as

$$m\, \ddot{y} + c\, \dot{y} + k\, y + k_3\, y^3 = A\, \cos(\omega\, t) \qquad (5)$$

Eqn. (5) can be recognised as the equation of a hardening Duffing oscillator which has been extensively studied in the literature (for example (Hayashi, 1964; Jordan and Smith, 1999; Carrella, 2008))

3 Conclusions

Nonlinear vibration isolators with high-static-low-dynamic-stiffness (HSLDS) characteristics offer a solution to problem of having to choose between a low natural frequency, desired for a wider frequency isolation bandwidth, and the consequent high static displacement that would result from using a linear softer mount. In this paper a novel approach to obtain the necessary negative stiffness has been proposed. A bistable plate exhibits a snap-through mechanism which can be exploited to achieve the required negative stiffness. A mechanical spring with positive, constant coefficient is then connected in parallel so that the system stiffness is positive and small. This results in a hardening system with a low natural frequency for oscillation about the static equilibrium position. Finally, the dynamic of the system can be studied by solving the nonlinear Duffing equation. Future work will focus on the dynamic model of the isolators and the measurements of the system transmissibility in order to investigate the isolation performance.

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