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TESTING FUNCTIONAL OUTPUT-CONTROLLABILITY OF TIME-INVARIANT SINGULAR LINEAR SYSTEMS

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Abstract
After introducing the concept of functional output-controllability for singular systems as a generalization of the concept that is known for standard systems. This paper deals with the description of a new test for calculating the functional output-controllability character of finite-dimensional singular linear continuous-time-invariant systems in the form

$$\left.\begin{array}{l} E\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{array}\right\} \qquad (1)$$

where $E, A \in M = M_n(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$, $C \in M_{p\times n}(\mathbb{C})$.
The functional output-controllability character is computed by means of the rank of a certain constant matrix which can be associated to the system.

Key words
Singular systems, functional output-controllability.

1 Introduction

It is well known that many physical problems as for example electrical networks, multibody systems, chemical engineering, Convolutional codes among others, use state space representation as (1) for its description.

This linear system can be described with a input-output relation called transfer function obtained by applying Laplace transformation to equation (1)

$$\left.\begin{array}{l} sE\dot{X} = AX + BU \\ Y = CX, \end{array}\right\},$$

obtaining the following relation

$$H(s) = C(sE - A)^{-1}B. \qquad (2)$$

The controllability concept of a dynamical standard system is largely studied by several authors and under many different points of view, (see [Cardetti and Gordina, 2008], [Chen, 1970], [Kundur, 1994] for example). Nevertheless, controllability for the output vector of a system has been less treated, (see [Domínguez-García and García-Planas, 2011], [García and Domínguez-García, 2013], [Garcia-Planas, Souidi and Um, 2012], [Germani and Monaco, 1983] for example).

The functional output-controllability generally means, that the system can steer output of dynamical system along the arbitrary given curve over any interval of time, independently of its state vector. A similar but least essentially restrictive condition is the pointwise output-controllability.

In this paper functional output-controllability property for singular systems is analyzed generalizing the study realized for standard systems and a test to study this property is presented. A partial result for regularizable systems can be found in [García and Tarragona, 2013].

2 Singular Systems

In this paper, it is considered the singular state space systems as one that has been introduced in equation (1)

$$\left.\begin{array}{l} E\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{array}\right\},$$

where x is the state vector, y is the output vector, u is the input (or control) vector, $A \in M_n(C)$ is the state matrix, $B \in M_{n\times m}(C)$ is the input matrix and $C \in M_{p\times n}(C)$ is the output matrix.

For simplicity we will write the systems by a quadruple of matrices (E, A, B, C).

One particular class of such systems are those called regular, which are those that satisfy the following relation $\det(\lambda E + \mu A) \neq 0$ for some $(\lambda, \mu) \in \mathbb{C}^2$, or those systems (called regularisable), which through a feedback proportional and/or derivative and/or an output injection proportional and/or derivative become regular. More concretely (E, A, B, C) is regularisable if and only if there exist matrices $F^B_E, F^B_A \in M_{m\times n}(\mathbb{C})$, $F^C_E, F^C_A \in M_{n\times p}(\mathbb{C})$, such that the system $(E + BF^B_E + F^C_E C, A + BF_A + F^C_A, B, C)$ is regular.

Remark 2.1. *If a singular system is regular there exists a unique solution for any consistent initial condition.*
Remember that an initial condition is called consistent with the system, if the associated initial value problem has at least one solution.

A manner to understand the properties of the system is treating it by purely algebraic techniques. The main aspect of this approach is defining an equivalence relation preserving these properties.

The equivalence relation considered is such that derived after to make the following elementary transformations: basis change in the state space, basis change in the input space, basis change in the output space, proportional feedback, derivative feedback, proportional output injection, derivative output injection and a premultiplication by an invertible matrix.

More concretely.

Definition 2.1. *Two systems (E_i, A_i, B_i, C_i), $i = 1, 2$, are equivalent if and only if there exist matrices $P \in Gl(n;\mathbb{C})$, $Q \in Gl(n;\mathbb{C})$, $R \in Gl(m;\mathbb{C})$, $S \in Gl(p;\mathbb{C})$, $F^B_E, F^B_A \in M_{m\times n}(\mathbb{C})$, $F^C_E, F^C_A \in M_{n\times p}(\mathbb{C})$ such that*

$$\begin{array}{l} E_2 = QE_1P + QB_1F^B_E + F^C_E C_1P, \\ A_2 = QA_1P + QB_1F^B_A + F^C_A C_1P, \\ B_2 = QB_1R, \\ C_2 = SC_1P. \end{array} \tag{3}$$

For this equivalence relation a canonical form is provided, that is to say a quadruple of matrices which is equivalent to a given quadruple and which has a simple form from which we can directly read off the invariants and structural properties of the corresponding singular system.

For a better understanding, we will give the following notations: I_ℓ denotes the ℓ-order identity matrix, $N_i = \text{diag}\,(N_{i_1}, \ldots, N_{i_t}) \in M_{n_i}(\mathbb{C})$, $i = 1, 2, 3, 4$, $N_{i_j} = \begin{pmatrix} 0 & I_{n_{i_j}-1} \\ 0 & 0 \end{pmatrix} \in M_{n_{i_j}}(C)$, $J = \text{diag}\,(J_1, \ldots, J_t) \in M_{n_5}(\mathbb{C})$, $J_i = \text{diag}(J_{i_1}, \ldots, J_{i_s})$, $J_{i_j} = \lambda_i I_{i_j} + N$.

Proposition 2.1. *A system (E, A, B, C) is regularisable if and only if it can be reduced to (E_r, A_r, B_r, C_r) where:*
$E_r = (\,\text{diag}\,(I_1, I_2, I_3, I_4, N_1, S_1)\,)$,
$A_r = (\,\text{diag}\,(N_2, N_3, N_4, J, I_5, S_2)\,)$,
$B_r = \begin{pmatrix} B_1^t & 0 & 0 & 0 & 0 \\ 0 & B_2^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}^t$
and
$C_r = \begin{pmatrix} C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

Remark 2.2. *1. The standard part of the system is maximal among all possible reductions of the system.*
2. Not all parts i),..., vi), necessarily appears in the decomposition of the system.
3. The part vi) is the strictly singular part.

3 Functional Output-Controllability. Case Standard

This section recalls the standard case that will help better understand the singular case.

The output-controllability generally means, that the system can steer output of dynamical system independently of its state vector.

Definition 3.1. *A standard system is functional output-controllable if and only if its output can be steered along the arbitrary given curve over any interval of time. It means that if it is given any output $y_d(t)$, $t \geq 0$, there exists t_1 and a control u_t, $t \geq 0$, such that for any $t \geq t_1$, $y(t) = y_d(t)$.*

Proposition 3.1 ([Chen, 1970]). *A system is functional output-controllable if and only*

$$\text{rank}\, C(sI - A)^{-1}B = p$$

in the field of rational functions

A necessary and sufficient condition for functional output-controllability is given as follows.

Proposition 3.2 ([Chen, 1970], [Ferreira, 1976]).

$$\text{rank} \begin{pmatrix} sI - A & B \\ C & 0 \end{pmatrix} = n + p,$$

The functional output-controllability can be computed by means of the rank of a constant matrix in the following manner

Theorem 3.1 ([García and Domínguez-García, 2013]). *The system (A, B, C) is functional output-controllable if and only if*

$$\text{rank}\, oC_{\text{f}}(A, B, C) = \text{rank} \begin{pmatrix} C & & & & \\ CA & CB & & & \\ CA^2 & CAB & CB & & \\ \vdots & & \ddots & & \\ CA^n & CA^{n-1}B & \ldots & CAB & CB \end{pmatrix} = (n + 1)p$$

The null terms are not written in the matrix.

Remark 3.1. *We call*

$$oC_i = \begin{pmatrix} C \\ CA & CB \\ CA^2 & CAB & CB \\ \vdots & & \ddots \\ CA^i & CA^{i-1}B & \dots & CAB & CB \end{pmatrix}, \ \forall i \geq 1.$$

i) If the system (A, B, C) is functional output-controllable, then the matrices oC_i have full row rank for all $0 \leq i \leq n$.

ii) If the matrix oC_{n-1} has full row rank, it does not necessarily the matrix oC_n has full row rank.

4 Functional Output-Controllability. Case Singular

The output-controllability character can be generalized to the singular systems in the following manner.

Definition 4.1. *A regular singular system is functional output-controllable if and only if its output can be steered along the arbitrary given curve over any interval of time. It means that if it is given any output $y_d(t)$, $t \geq 0$, there exists t_1 and a control u_t, $t \geq 0$, such that for any $t \geq t_1$, $y(t) = y_d(t)$.*

Proposition 4.1. *A regular singular system is functional output-controllable if and only*

$$\operatorname{rank} H(s) = p$$

in the field of rational functions

Proof. According to equation (2), $H(s) = C(sE - A)^{-1}B$.

If $\operatorname{rank} H(s) = q$, then $H(s)H(s)^*$ is invertible, then it suffices to consider

$$U(s) = H(s)^*(H(s)H(s)^*)^{-1}Y(s)$$

If $\operatorname{rank} H(s) < q$, we can obtain a $Y(s)$ with $Y(s) \notin \operatorname{Im} H(s)$.

A necessary and sufficient condition for functional output-controllability of regular singular systems is

Proposition 4.2.

$$\operatorname{rank} \begin{pmatrix} sE - A & B \\ C & 0 \end{pmatrix} = n + p,$$

Remark 4.1. *Notice that for $E = I$ the proposition coincides with proposition 3.2*

Remark 4.2. *If* $\operatorname{rank} C < p$ *or* $m < p$ *the system is not functional output-controllable. Then, henceforth and without lost of generality, we will suppose that* $\operatorname{rank} C = p \leq m$.

Proposition 4.2 permit generalize the definition of functional output-controllability to any singular system.

The functional output-controllability can be computed by means of the rank of a certain constant matrix defined in the following manner.

Definition 4.2. *For each system (E, A, B, C) we consider the following matrices.*

$$\begin{aligned} M_0 &= C \\ M_1 &= \begin{pmatrix} A & B & -E \\ C & 0 & 0 \\ 0 & 0 & C \end{pmatrix} \in M_{(n+2p)\times(2n+m)}(\mathbb{C}) \\ M_2 &= \begin{pmatrix} A & B & -E \\ C & 0 & 0 \\ 0 & 0 & A & B & -E \\ 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & C \end{pmatrix} \in M_{(2n+3p)\times(3n+2m)}(\mathbb{C}) \\ &\vdots \\ M_i &= \begin{pmatrix} A & B & -E & 0 & 0 & 0 & \dots & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A & B & -E & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ \vdots & & & & \ddots \\ & & \dots & & & A & B & -E \\ & & \dots & & & C & 0 & 0 \\ & & \dots & & & 0 & 0 & C \end{pmatrix} \\ &\in M_{in+(i+1)p)\times((i+1)n+im)}(\mathbb{C}) \end{aligned}$$

Calling now $M_n = oC_{\mathrm{f}}(E, A, B, C)$, we have the following result.

Theorem 4.1. *The system (E, A, B, C) is functional output-controllable if and only if*

$$\operatorname{rank} oC_{\mathrm{f}}(E, A, B, C) = \operatorname{rank} \begin{pmatrix} A & B & -E & 0 & 0 & 0 & \dots & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A & B & -E & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ \vdots & & & & \ddots \\ & & \dots & & & A & B & -E \\ & & \dots & & & C & 0 & 0 \\ & & \dots & & & 0 & 0 & C \end{pmatrix} = (n+1)p + n^2$$

The null terms are not written in the matrix.

Remark 4.3. *For $E = I$, the test coincides with the test for standard systems. It suffices to make block elementary row and columns transformations to the matrix $oC_{\mathrm{f}}(I, A, B, C)$:*

$$\operatorname{rank} \begin{pmatrix} A & B & -I & 0 & 0 & 0 & 0 & \dots & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A & B & -I & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ \vdots \\ & & \dots & & & A & B & -I \\ & & \dots & & & C & 0 & 0 \\ & & \dots & & & 0 & 0 & C \end{pmatrix} =$$

$$\operatorname{rank} \begin{pmatrix} I \\ & \ddots \\ & & I \\ & & & C \\ & & & CA & CB \\ & & & CA^2 & CAB & CB \\ & & & \vdots & & \ddots \\ & & & CA^n & CA^{n-1}B & & CB \end{pmatrix}$$

In order to proof this theorem we make use of the equivalence relation defined in 2.1 that permit us to consider an equivalent simple reduced form for the system

Proposition 4.3. *The functional output-controllability character is invariant under equivalence relation.*

Proof.

$$\operatorname{rank}\begin{pmatrix} Q & sF_E^C - F_A^C \\ 0 & S \end{pmatrix}\begin{pmatrix} sE-A & B \\ C & 0 \end{pmatrix}\begin{pmatrix} P & 0 \\ sF_E^B - F_A^B & R \end{pmatrix} = \operatorname{rank}\begin{pmatrix} sE-A & B \\ C & 0 \end{pmatrix}$$

Corollary 4.1. *If a system (E, A, B, C) is functional output-controllable then it is regularizable.*

Proof. The matrix $sS_1 - S_2$ has never full row rank.

So, from now on the systems under consideration are regularizable.

Proof of the Theorem.

Proposition 4.3 permit us to consider the system in its reduced form

$$\begin{aligned}
&\operatorname{rank}\begin{pmatrix} sE-A & B \\ C & 0 \end{pmatrix} = \\
&\operatorname{rank}\begin{pmatrix} sI_1-N_1 & B_1 \\ C_1 & 0 \end{pmatrix} + \operatorname{rank}\begin{pmatrix} sI_2-N_2 & B_2 \end{pmatrix} + \\
&\operatorname{rank}\begin{pmatrix} sI_3-N_3 \\ C_2 \end{pmatrix} + \operatorname{rank}\begin{pmatrix} sI_4-J \end{pmatrix} + \operatorname{rank}\begin{pmatrix} sN_1-I_{n_1} \end{pmatrix} = \\
&n_2 + p_2 + n_3 + n_4 + n_5 + n_1 = n + p_2,
\end{aligned}$$

and the rank is $n + p$ if and only if $p = p_2$. In order to obtain p_2, it suffices to compute the $r_i^{\mathcal{O}}$ numbers associated to the system [Diaz, 2006].

Example 4.1. *We consider a simple electric circuit, with a resistor R, an inductor L and a capcitor C, and where the control input is the voltage source $V_s(t)$ and the voltages of R, L and C are V_R, V_L and V_C respectively. With a mesure equation $y(t) = V_C(t)$ and following the Kirchoff's laws, this circuit can be described as the following singular linear system:*

$$\begin{aligned}
\begin{pmatrix} L & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{I}(t) \\ \dot{V}_L(t) \\ \dot{V}_C(t) \\ \dot{V}_R(t) \end{pmatrix} &= \\
\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1/C & 0 & 0 & 0 \\ -R & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} I(t) \\ V_L(t) \\ V_C(t) \\ V_R(t) \end{pmatrix} &+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} V_s(t) \\
y(t) = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}&\begin{pmatrix} I(t) \\ V_L(t) \\ V_C(t) \\ V_R(t) \end{pmatrix}
\end{aligned} \tag{4}$$

For $L = R = C = 1$ and using Matlab we have that

$$\operatorname{rank} oC_f(E, A, B, C) = 21$$

So the system is functional output controllable.

(see [Dai, 1989] for more information about this kind of systems.

Example 4.2. *Let (E, A, B, C) be a system with $E = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$*

$$oC_{\mathbf{f}}(E, A, B, C) =$$
$$\begin{pmatrix}
0&1&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0
\end{pmatrix}$$

Using Matlab, it is easy to computing the rank of this matrix, we have

$$\operatorname{rank} oC_{\mathbf{f}}(E, A, B, C) = 13.$$

Then the system it is functional output-controllable.

But if we consider the system (E_1, A_1, B_1, C_1) with $E_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $C_1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

$$oC_{\mathbf{f}}(E_1, A_1, B_1, C_1) =$$
$$\begin{pmatrix}
0&1&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0
\end{pmatrix}$$

As before, using Matlab, it is easy to computing the rank of this matrix,

$$\operatorname{rank} oC_{\mathbf{f}}(E_1, A_1, B_1, C_1) = 11.$$

Then the system it is not functional output-controllable.

Remark 4.4. *i) If the singular system (E, A, B, C) is functional output-controllable, then the matrices M_i has full row rank for all $0 \leq i \leq n$.*

ii) If the matrix M_{n-1} has full row rank, the matrix M_n does not necessarily has full row rank, as it can be seen in the following example.

Example 4.3. *Let (E, A, B, C) with $E = I$, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$.*

$$\operatorname{rank}\begin{pmatrix} A & B & -I \\ C & 0 & 0 \\ 0 & 0 & C \end{pmatrix} = \operatorname{rank}\begin{pmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} = 4 = n + 2p,$$

but

$$\operatorname{rank}\begin{pmatrix} A & B & -I & & \\ C & 0 & 0 & & \\ 0 & 0 & A & B & -I \\ 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & C \end{pmatrix} =$$

$$\operatorname{rank}\begin{pmatrix} 0\,1\,0 & -1 & 0 & 0 & 0 & 0 \\ 0\,0\,0 & 0 & -1 & 0 & 0 & 0 \\ 1\,0\,0 & 0 & 0 & 0 & 0 & 0 \\ 0\,0\,0 & 0 & 1 & 0 & -1 & 0 \\ 0\,0\,0 & 0 & 0 & 0 & 0 & -1 \\ 0\,0\,0 & 1 & 0 & 0 & 0 & 0 \\ 0\,0\,0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} = 6 < 7.$$

Corollary 4.2. *i) The system (E, A, B, C) is functional output-controllable if and only if the matices M_i for all i has full row rank.*

ii) For all $\ell \geq n$ we have that

$$\operatorname{rank} M_{\ell+1} - \operatorname{rank} M_\ell = \operatorname{rank} M_{\ell+2} - \operatorname{rank} M_{\ell+1}.$$

This corollary provides an iterative method to compute functional output-controllability in the following manner.

Step 1: Compute rank M_0. If rank $< p$ the system is not functional output-controllable,

If rank $= p$, then

Step 2: Compute rank M_ℓ. If rank $< (\ell + 1)p + \ell n$ the system is not output controllable.

If rank $= (\ell + 1)p + \ell n$ and $\ell = n$ the system is functional output.controllable, and if $\ell < n$ go to step 2.

Example 4.4. *Let (E, A, B, C) be a system with $E = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$, $A = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, $B = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, $C = \left(\begin{smallmatrix} 1 & 0 \end{smallmatrix}\right)$.*

Then,

$$\operatorname{rank} M_0 = \operatorname{rank} \begin{pmatrix} 1 & 0 \end{pmatrix} = 1 = p$$

$$\operatorname{rank} M_1 = \operatorname{rank} \begin{pmatrix} 0\,0\,1 & -1 & 0 \\ 0\,1\,0 & 0 & 0 \\ 0\,1\,0 & 0 & 0 \\ 0\,0\,0 & 0 & 1 \end{pmatrix} = 3 < 4$$

then the system is not functional output-controllable.

5 Conclusion

In this paper the concept of functional output-controllability for singular systems has been introduced. It has been proven that functional output-controllable systems are necessarily regularizables. A easy test for calculating the functional output-controllability character of finite-dimensional singular linear continuous-time-invariant systems is presented. The test is based in the computation of ranks of certain constant matrices associated to the system.

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