ADJOINT MULTIVARIABLE REPETITIVE CONTROL ALGORITHM FOR ACTIVE CONTROL OF VIBRATION

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Abstract: In this paper a well-known adjoint SISO Repetitive Control algorithm is extended to the multivariable case and its convergence properties are analysed. The new adjoint algorithm is validated using a simulation of an experimental facility consisting of a passive Naval vibration isolation mount that is combined with six active control channels located in a Stewart platform style arrangement. The algorithm utilises a multivariable FIR system description that is derived from frequency response function measurements. The adjoint repetitive control algorithm is used to eliminate a harmonic disturbance where the first harmonic coincides with the fundamental mount resonance of the passive component. The simulation results show that the repetitive control algorithm ultimately achieves good vibration isolation but due to the wide spread of system eigenvalues at the harmonic frequency, convergence is slow.

Keywords: Learning control; Repetitive Control; Signal processing; Real-time control width (maximum), left justified.

1. INTRODUCTION

Many signals in engineering are periodic, or at least they can be accurately approximated by a periodic signal over a large time interval. This is true, for example, of most signals associated with engines, electrical motors and generators, converters, or machines performing a task over and over again. Hence it is an important control problem to try to track a periodic signal with the output of the plant or try to reject a periodic disturbance acting on a control system. In order to solve this problem, the research area of Repetitive Control has emerged within the control community. The basic and defining philosophy in Repetitive Control is to utilise information obtained during previous periods to improve current performance through a process of learning. Since first proposed in Inouye, Nakano, Kubo, Matsumoto, and Baba (1981), an increasing number of diverse applications of RC have been reported in the literature during recent years. Examples include, robotics Kaneko and Horowitz (1997),motor control Kobayashi, Kimuara, and Yanabe (1999), rolling mills Garimella and Srinivasan (1996) and vibration control Hillerström (1996).

This study concentrates on a well-known adjoint based Repetitive Control algorithm for SISO systems. Initially, as a new result, this paper extends the SISO algorithm first introduced in Chen and Longman (2002) and subsequently analysed in Hätönen, Freeman, Owens, Lewin, and Rogers (2004) to the multivariable (MIMO) case. As a main contribution, the stability properties of the MIMO algorithm are analysed and estimates for convergence speed in the multivariable setting are established.

In the simulation part of the paper the algorithm is applied in the context of active control of vibration. To be more precise, the algorithm is implemented on an experimental Naval vibration isolation mount. This simulates problems within the marine environment where vibration propagation from propulsion and auxiliary machinery can cause both significant passenger and crew discomfort and also leads to the generation of acoustic noise from the hull. Such acoustic noise creates a severe detection hazard in Naval vessels and is also problematic for civil vessels such as those used by fisheries research organisations. It is shown that the adjoint based algorithm is capable of achieving good isolation following convergence.

The rest of the paper is organized as follows: Section 2 defines formally the Repetitive Control problem. This is followed by Section 3, which analyses the stability properties of the algorithm and provides estimates for convergence speed. In particular, it is shown that large singular value spread in the plant at disturbance frequencies can dramatically slow down the convergence speed. After this, Section 4 reports the simulation work on the Naval mount Finally, Section 5 concludes the paper and gives directions for future research.

2. REPETITIVE CONTROL

The starting point in Repetitive Control is a standard state-space representation

$$\begin{cases} x(t+1) = \Phi x(t) + \Gamma u(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \end{cases} \qquad (1)$$

where $x \in R^{n \times n}, u \in R^r$ and $y \in R^s$. Furthermore, Φ, Γ and C are matrices of appropriate dimensions. From now on it is assumed that the $r = s$ (i.e. the system is square) and that the system is both controllable and observable. Furthemore, without loss of generality, it is assumed that the system (1) is stable. Throughout, the notation $G(q) := C(qI - \Phi)^{-1}\Gamma$ is used to represent the transfer function matrix corresponding to (1).

In repetitive control the design problem is to find a feedback controller that forces the system (1) to track a reference signal $r(t)$ or to reject a load disturbance signal $d(t)$, and it is known that both signals are N-periodic, $r(t) = r(t+N)$ and $d(t) = d(t+N)$. Furthemore, it is easy to see that the load disturbance rejection problem is equivalent to the tracking problem, and therefore from now on only the tracking problem is considered. The inputs into the feedback controller at time point t are previous inputs $u(s)$ for $s < t$ and tracking errors $e(s) := r(s) - y(s)$ for $s \le t$. As is shown in Francis and Wonham (1975), a necessary condition for asymptotic convergence to zero tracking error is that a controller

$$[Mu](t) = [Ne](t) \qquad (2)$$

where M and N are suitable operators, must have an internal model of the reference signal inside the operator M. Because the reference signal is assumed to be N-periodic, the internal model is simply $1 - q^{-N}$, where q^{-1} is the standard delay operator, i.e. $[q^{-1}v](t) = v(t-1)$ for an arbitrary (possibly vector-valued) time-sequence. The simplest algorithm that satisfies this condition is

$$u(t) = u(t-T) + [Ne](t) \qquad (3)$$

and if it is assumed that N is causal LTI-filter, the algorithm can be written using the q^{-1}-operator formalism as

$$u(t) = q^{-N}u(t) + K(q)e(t) \qquad (4)$$

and the control design problem is to select $K(q)$. In the SISO case $(r = s = 1)$ Chen and Longman (2002) proposed the algorithm

$$u(t) = q^{-N}u(t) + bG_o\left(q^{-1}\right)q^{-N}e(t) \qquad (5)$$

where $G_o(q^{-1}) = \sum_{i=1}^{N} g_i q^i$ and g_i are the Markov parameters of the transfer function $G(q) = \sum_{i=1}^{\infty} g_i q^{-i} = C(qI - \Phi)^{-1}\Gamma$. The idea is therefore to use the first N elements of the impulse response to implement causally the control law (5). Note that $G(q^{-1})$ can be understood to be the adjoint system of $G(q)$, see Lewis and Syrmos (1995) for details.

In Hätönen et al. (2004) it has been shown that if the effect of truncation is modelled through multiplicative uncertainty $G(g) = U(g)G_o(g)$, and the phase of the multiplicative uncertainty lies between $\pm 90^o$, and b is selected to be sufficiently small, the algorithm will drive the tracking error to zero asymptotically for any N-periodic reference/disturbance signal. The next section will generalise this algorithm to the multivariable case and analyse theoretically its convergence properties.

3. ADJOINT BASED MULTIVARIABLE ALGORITHM

This section introduces and analyses an adjoint type Repetitive Control algorithm for multivariable systems. The initial results were presented for the first time in the conference publication Hätönen, Daley, Zhang, and Owens (2005), and are extended here to include an analysis of convergence speed.

3.1 Nominal convergence analysis

Assume initially that the Markov parameters G_i of the transfer function matrix $G(g)=\sum_{i=0}^{\infty} G_i q^{-i}$ go to zero after N steps. In this case the algorithm

$$u(t)=q^{-N}u(t)+bG(q^{-1})^T q^{-N}e(t) \qquad (6)$$

is causal, even though the algorithm contains a non-causal element $G(q^{-1})^T=\sum_{i=0}^{\infty} G_i^T q^i$. Using (6) straightforward algebraic manipulations show that the tracking error satisfies the following autonomous system

$$e(t)=q^{-N}\left(I-bG(q)G(q^{-1})\right)e(t) \qquad (7)$$

This equation can used to establish the convergence of the algorithm under the FIR assumption on $G(q)$:

Proposition 1 Assume that the condition

$$\sup_{w=[0,2p]} \bar{s}\left(I-bG(e^{jw})G(e^{jw})^*\right)<1 \qquad (8)$$

where $\bar{s}$ represents the largest singular value of $I-bG(e^{jw})G(e^{jw})^*$ for a given w. This implies that $\lim_{t\to\infty} e(t)=0$.

Proof. Equation (7) can be written as $\left(I-q^{-N}\left(I-bG(q)G(q^{-1})^T\right)\right)e(t)=0$, and due to the multivariable Nyquist stability theorem, stability is guaranteed if the characteristic loci of $-z^N\left(I-bG(z)G(z^{-1})^T\right)\Big|_{z=e^{jw}}$ encircles the (-1,0) point as many times as there are unstable poles in $-q^N\left(I-bG(q)G(q^{-1})^T\right)$.Because $G(q)$ is assumed to be a FIR system, $-q^N\left(I-bG(q)G(q^{-1})^T\right)$ is a stable system, and therefore for stability it is required that the characteristic loci does not encircle the (−1,0) point. A sufficient condition for this is that $\sup_{w=[0,2p]} \bar{s}\left(I-bG(e^{jw})G(e^{jw})^*\right)<1$, which completes the proof.

Note that condition in Proposition 1 can always be met, if $b<1/\sup_{w\in[0,2p]} \bar{s}\left(G(e^{jw})\right)^2$. In summary, if the algorithm satisfies the FIR assumption and b is sufficiently small, the algorithm will drive the tracking error to zero in the limit.

Remark 1 (Non-square systems) Note that if the plant has more outputs than inputs, a straightforward extension of the results in this section show that the algorithm converges to input $u(t)=\left(I+G(q^{-1})^T G(q)\right)^{-1} G(q^{-1})^T r(t)$, which is the well-known least squares solution. Also the robustness results in the following sections can easily be extended to this non-square case, but they omitted due to space limitations.

3.2 Remarks on convergence speed

Note that resulting error evolution equation (7) can be written in the frequency domain as

$$e(e^{jw})=\left(i-bG(e^{jw})G(e^{jw})^*\right)e^{-jwN}e(e^{jw}) \qquad (9)$$

Assume now that the ratio between the largest and smallest eigenvalue $l_{\max}$ and $l_{\min}$ at the fundamental frequency or one its harmonics is large, i.e. the condition number of the plant is large. In this case the learning gain has to made very small in comparison to the smallest eigenvalue $l_{\min}$. Let v be the (complex) vector corresponding to smallest singular value, and e_r the projection of $e(e^{jnw_o})$ on to v, i.e $e_r(e^{jnw_o})=v^* r(e^{jnw_o})$. Along the vector v the error evolution in the frequency domain at the frequency nw_o can be approximated as

$$\begin{aligned} e_r(e^{jnw_o}) &= \left(I-bG(e^{jnw_o})G(e^{jnw_o})^*\right)e^{-jnw_oN}e_r(e^{jnw_o}) \\ &= e^{-jnw_oN}\left(I-b\,l_{\min}\right)e(e^{jnw_o}) \approx e^{-jnw_oN}e(e^{jnw_o}) \end{aligned} \qquad (10)$$

which shows that $\left|e_r(e^{jnw_o})\right| \approx \left|e^{-jnw_oN}e(e^{jnw_o})\right| = \left|e_r(e^{jnw_o})\right|$, and therefore if the plant is ill-conditioned, hardly any 'learning' takes place along the direction of the eigenvector associated with the smallest eigenvalue. In summary, when the plant is ill-conditioned at the fundamental frequency or the associated harmonics, it can be expected that the resulting convergence rate can be extremely slow .

In order to come up with an estimate for the time constants of the algorithm, in this section a similar approximation technique is used as in Sievers and von Flotow (1992). As a starting point the the loop gain L(q) of the feedback system (7), which in this case is $L(q)=\frac{b}{1-q^{-N}}q^{-N}G(q)G(q^{-1})^T$, is expanded in the following way

$$(q-1)^{-1}R_0+(q+1)^{-1}R_0+ \sum_{n=1}^{N/2-1}\left(q-e^{jnw_o}\right)^{-1}R_n+\left(q-e^{-jnw_o}\right)^{-1}R_n+O(q) \qquad (11)$$

where R_n are the residue matrices obtained from the equation

$$R_n=L(q)\left(q-e^{jnw}\right)\Big|_{q=e^{jnw_o}} \qquad (12)$$

and $O(q)$ contains the poles of $G(q)G(q^{-1})^T$, which are all the in the origin due to the FIR assumption. Note that in this expansion it is assumed that N is an even number, and this will be also assumed for the rest of this section. A similar expansion can be done for an odd N, but these developments are omitted due to space limitations.

A rather lengthy calculation then shows that a R_n for $n \geq 1$ is given by

$$R_n = -bg_n e^{jnw_o} G\left(e^{jnw_o}\right) G\left(e^{jnw_o}\right)^* \qquad (13)$$

where it can be proved that

$$g_n = \frac{1}{2^{N/2}\sin(nw_o)^2 e^{jnw_o N/2}\left(\prod_{s=1,s\neq n}^{N/2-1}(\cos(nw_o)-\cos(sw_o))\right)} < 0 \quad (14)$$

for $nw_o \neq 0, p, p/2, 3p/2, 2p$. In the frequency band around $q = e^{jnw_o}$ the loop gain $L(q)$ is dominated by the corresponding compensator pole, and therefore

$$L(q) \approx \left(q - e^{jnw_o}\right)^{-1} R_n + \left(q - e^{-jnw_o}\right)^{-1} R_n \qquad (15)$$

if the plant model does not have strong resonances close to the frequencies nw_o. Furthermore, the locations of the closed loop system poles can be identified using a MIMO root locus argument - a starting point note that the closed-loop system is given by

$$\left(I - q^{-N}\left(I - bG(q)G\left(q^{-1}\right)^T\right)\right)e(t) = 0 \quad (16)$$

and therefore the closed-poles z_i that lie on the root-locus generated by b satisfy the equation

$$l_i\left(-z_i^{-N}\left(I - bG(z_i)G\left(z_i^{-1}\right)^T\right)\right) = -1 \quad (17)$$

where $l_i(M(z_i))$ denotes the ith eigenvalue of an arbitrary square complex matrix $M(z_i)$. Simple algebraic manipulations show that this is equivalent to the condition

$$l_i(-L(z_i)) = 1 \qquad (18)$$

Using the approximation (15) the ith closed loop pole near e^{jnw_o} can be determined by solving the following equation in terms of z_i:

$$l_i\left(\left(z_i - e^{jnw_o}\right)^{-1}\left(-bg_n e^{jnw_o} G\left(e^{jnw_o}\right) G\left(e^{jnw_o}\right)^*\right)\right) = -1 \quad (19)$$

which results in

$$z_i = e^{jnw_o}\left[1 + bg_n l_i\left(G\left(e^{jnw_o}\right)G\left(e^{jnw_o}\right)^*\right)\right] \qquad (20)$$

This equation is very important, since it can be used *a'priori* to estimate the convergence speed of the algorithm. Note that (20) implies that

1. A large eigenvalue spread of $l_i\left(G\left(e^{jnw_o}\right)G\left(e^{jnw_o}\right)^*\right)$ at a frequency nw_o implies low converge rate, since in order to 'stabilize' the largest eigenvalue, a small b is required (in relative terms), but at the same time small b leaves the pole associated with the smallest eigenvalue almost on the unit circle. This finding supports the approximate analysis in (10).
2. g_n term shows the rather complicated effect of the fundamental frequency w_o, sampling time and cycle length N on convergence rate for a fixed $b < 1/\sup_{w \in [0,2p]} \bar{s}\left(G\left(e^{jw}\right)\right)^2$.
3. If a nominal plant $G_o(q)$ is used in algorithm implementation, and the true plant model is given by $y(t) = G(q)U(q)u(t)$, where $U(q)$ represents multiplicative uncertainty and is assumed to be a stable system, it is clear from (20) that a necessary condition for stability is that $l_i\left(G\left(e^{jnw_o}\right)^*\left(U\left(e^{jnw_o}\right)^* + U\left(e^{jnw_o}\right)\right)G\left(e^{jnw_o}\right)\right) > 0$, which is equivalent for $U\left(e^{jnw_o}\right)^* + U\left(e^{jnw_o}\right)$ to be a positive definite matrix. It can in fact be rigorously shown that a sufficient condition for 'robust convergence' is that $U\left(e^{jw_o}\right)^* + U\left(e^{jw_o}\right)$ is a positive-definite matrix for all $w \in [0, 2p]$ and that b is sufficiently small.

In Section 4, these points will be explored further in terms of simulation work and results.

4. SIMULATION RESULTS

The plant model used here is based on the facility (see Fig. 1) which was originally developed in association with BAE Systems Marine during the late 1980's. The main purpose of this mount is for testing active isolation schemes for large marine machinery rafts. The active mount consists of a central standard passive elastomeric Naval mount around which are located 6 Ling 30N electro-dynamic shakers. These apply forces in parallel to the passive mount and the 'stinger' attachments are arranged in a hexapod or Stewart platform style such that control can be applied to all six degrees of freedom (three orthogonal translational forces and three orthogonal torques).

Based on the algorithm in section 3, the simulation results use a model of the mount and convergence is shown in figure 2 with learning gain $b = 8 \times 10^{-5}$ and the disturbance signal is $d(t) = \sin(2p \times 60 \times t)$. From the plots it can be seen that the convergence speed of channel 2 is much faster than channel 5 and 6, even though the convergence rate of channel 2 does not satisfy the practical requirements.

By checking the eigenvalues $l_i(G(e^{jnw_0})G(e^{jnw_0})^*)$ at the fundamental frequency $f = 60$, which are given in table 1, the expected convergence rate can be determined from (20).

Fig. 1. Active mount

l_1	607.455
l_2	56.669
l_3	3.351
l_4	0.600
l_5	0.004
l_6	0.098

Table 1. Eigenvalues at 60Hz

From the table of eigenvalues it is clear that the 5th and 6th eigenvalues of the system at the specified frequency are much smaller than the 1st and 2nd ones. Therefore, if the first pole was located at the origin, i.e., $z_1 = 0$, then the value of b would be required to be $b = 0.2963$. However, for this values the position of the fifth pole will be $z_5 = 0.5878 + 0.8090i$ and $|z_5| = 1$. This means when the value of b is chosen to get the fast convergence for mode 1, it will lead to an impractically slow convergence in mode 5. It can readily be seen that in fact, for all values of stabilising b, there will be some modes with very slow convergence for this system. This explains the slow convergence rate demonstrated in all channels in Fig. 2. These convergence properties cannot be significantly improved without further conditioning of the plant.

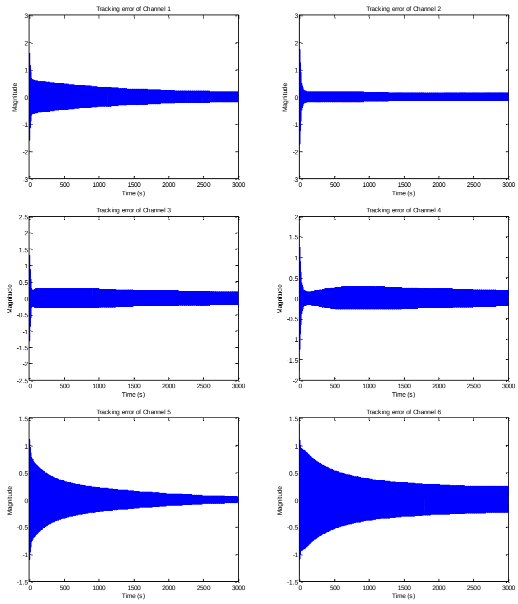

Fig. 2. Simulation results

5. CONCLUSIONS

In this paper, a new adjoint based repetitive control law for multivariable systems has been proposed. The convergence and stability properties of the algorithm have been analysed together with some preliminary robustness results. The algorithm and convergence properties have been verified using an experimentally derived simulation model of a Naval vibration isolation mount. The simulation results show slow convergence properties because of the wide spread of the system eigenvalues at the fundamental frequency. Because of this problem, future work will concentrate on modifications to the algorithm to produce a fast convergence rate for each channel.

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