Comparison between extended Kalman filter and Sigma-Points Kalman filters applied to integrated navigation system INS (inertial Navigation System)/GPS (Global Positioning system) with selective Availability

Ouldali AbdelAziz * .Benzerrouk Hamza **

*Miltary polytechnic school of Algeria-electronic systems Laboratory (e-mail: Aaziz_ouldali@yahoo.fr)
**Saad Dahlab University of Blida- Algeria-Control and systems Laboratory (e-mail: benzerroukhamza82@hotmail.fr)

Abstract: The inertial navigation system means the determination of position, velocity and attitude of vehicles using only inertial sensors, accelerometers and gyrometers for strapdown inertial navigation system. INS/GPS has proven to be a very efficient means of navigation due to the short term accuracy of INS allied to the long term accuracy of GPS. This integration was tested through non linear filters as extended Kalman filter (EKF) and Sigma-points Kalman filters (SPKF), both were compared in this work and were applied to direct mode integration in possible reactivated selective availability (SA) conditions.

Keywords: Inertial navigation system ,Global positioning system, Extended Kalman filters, Sigma points Kalman filters.

1. INTRODUCTION

The integrated systems INS/GPS were widely used in both of military and civilian applications. During these last thirty (30) years , different architectures of integrations were deployed and investigated , using several algorithms in filtering theory, linear and non linear filters were applied to estimate different state navigation compounds as position, velocity and attitude of the vehicles (car, aircraft, underwater,. .etc). the most popular filter used in this kind of integration was and is at this time too the extended kalman filter (EKF). The non linear equation of inertial navigation imply the used of non linear filtering in most of time, EKF is a local filter witch uses a linearization through Taylor development at the first order only, in the case of non derivative non linear function at a given estimate point, creating a singularity point and non efficiency of the EKF, so, by the way, others estimators were introduced to salve this problems , the most interesting filters are the Sigma points Kalman filters (SPKF) (Van der Merwe.R, E. Wan, and S. J. Julier 2004) both Unscented Kalman Filters (UKF) and Central Difference Kalman Filters (CDKF) mean the SPKF, in this case, it is not the non linear function witch is estimate but the RGV , and the density of probability using a deterministic sigma points to estimate at the first and the second order the moment of the RGV, so, the means and the covariance of the state vector can be estimated better than by the EKF, because the accuracy of these kind of estimators is the second and the third order of Taylor development ,in these algorithms, the non linear equations are directly use to propagate the sigma points through the state system equation and the observation equation. The UKF uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points. For the CDKF, it adopts an alternative method in linearization called central difference approximation, like the UKF, CDKF generates several points about the mean based on varying the covariance matrix along each dimension, also it evaluates a non linear function at two different points for each dimension of the state vector that are divided by an appropriate chosen[, the SPKF are powerful estimators and have been to be a superior alternative to the EKF in several applications. We will see in this work the advantages of each estimator according to integrated navigation application.

2. KALMAN FILTERING

In the early 40s, and through great efforts of military research conducted at Massachusetts Institute of Technology , Wiener became interested in the problem of filtering and gave birth to the first filter that bears his name "Wiener filter ." In 1961, Kalman and Bucy introduced a filter that enriches the Wiener filter on two essential points:

- The filter is a recursive filter.

- The filter can be applied to a non stationary process.

The Kalman filter (KF) was developed in a linear model in the presence of additive Gaussian noise. However, the assumption of normality noise is not restrictive to the functioning of the filter. In many situations the KF is still robust vis-à-vis the nature of noise in particular those whose power is low. However, the assumption of linearity remains important.

2.1. Kalman filtering algorithm

The linear Kalman filter is presented under as (Kim.J2004):

- Initialisation : $\hat{X}_0$ et P_0 .
- Prediction :

$$\hat{X}_{k/k-1} = \Phi_k \hat{X}_{k-1}$$
$$P_{k/k-1} = \Phi_k P_{k-1} \Phi_k^T + Q_k$$

- Filtering :

$$K_k = P_{k/k-1} H_k^T \left[H_k P_{k/k-1} H_k^T + R_k \right]^{-1}$$
$$\hat{X}_k = \hat{X}_{k/k-1} + K_k (Z_k - H_k \hat{X}_{k/k-1})$$
$$P_k = P_{k/k-1} - K_k H_k P_{k/k-1}$$

- $k = k + 1$

Important note : the initialization is not more important in the linear case to ensure the convergence of the filter.

2.2 Extended Kalman filter (EKF)

It is the most used technique in non linear filtering . for each time of calculation of the algorithm, the non linear dynamic and the measurement functions are approximated to the first order of Taylor development around the current estimates. The algorithm of EKF is done as this(Kim.J2004):

- Initialisation : $\hat{X}_0$ et P_0 .
- Prediction :

$$\hat{X}_{k+1/k} = f_k(\hat{X}_k)$$
$$P_{k/k-1} = F_k(\hat{X}_k) P_{k-1} F_k(\hat{X}_k)^T + Q_k$$

- Filtering :

$$K_k = P_{k/k-1} H_k^T(\hat{X}_{k/k-1}) \left[H_k(\hat{X}_{k/k-1}) P_{k/k-1} H_k^T(\hat{X}_{k/k-1}) + R_k \right]^{-1}$$
$$\hat{X}_k = \hat{X}_{k/k-1} + K_k \left[Z_k - h_k(\hat{X}_{k/k-1}) \right]$$
$$P_k = P_{k/k-1} - K_k H_k(\hat{X}_{k/k-1}) P_{k/k-1}$$

- $k = k + 1$

2.3 Sigma Point Kalman filters

The sigma-points Kalman filters are used a deterministic sampling points to capture the mean and the covariance of the estimate state vector, according to those definitions (Crassidis J.L 2006), we can present the algorithms of the UKF and the CDKF (resp.) like in the following points(Cho.S.Y2006):

- Initialisation

$$\hat{x}_0 = E[x_0],\ P_{x_0} = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right]$$
$$\hat{x}_0^a = E\left[x_0^a\right] = \left[\hat{x}_0^T \bar{v}_0^T \bar{n}_0^T\right]^T$$
$$P_0^a = E\left[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^T\right]$$
$$= \begin{bmatrix} P_{x_0} & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & R_n \end{bmatrix}$$

- For k=1……. ∞
  1. t=k-1
  2. sigma points
     $$\chi_t^a = \left[\hat{x}_t^a \quad \hat{x}_t^a + \gamma\sqrt{P_t^a} \quad \hat{x}_t^a - \gamma\sqrt{P_t^a}\right]$$
  3. propagation of the sigma points through the system equation

$$\chi_{k/t}^x = f(\chi_t^x, \chi_t^v, u_t)$$
$$\hat{x}_k^- = \sum_{i=0}^{2L} \omega_i^m \chi_{i,k/t}^x$$
$$P_{x_k}^- = \sum_{i=0}^{2L} \omega_i^c (\chi_{i,k/t}^x - \hat{x}_k^-)(\chi_{i,k/t}^x - \hat{x}_k^-)^T$$

  4. Filtering

$$Y_{k/t} = h(\chi_{k/t}^x, \chi_t^n)$$
$$\hat{y}_k^- = \sum_{i=0}^{2L} \omega_i^m Y_{i,k/t}$$
$$P_{\tilde{y}k} = \sum_{i=0}^{2L} \omega_i^c (Y_{i,k/t} - \hat{y}_k^-)(Y_{i,k/t} - \hat{y}_k^-)^T$$
$$P_{x_k y_k} = \sum_{i=0}^{2L} \omega_i^c (\chi_{i,k/t}^x - \hat{x}_k^-)(Y_{i,k/t} - \hat{y}_k^-)^T$$
$$K_k = P_{x_k y_k} P_{\tilde{y}_k}^{-1}$$
$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-)$$
$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k} K_k^T$$

$$\gamma = \sqrt{L + \lambda}\ ,\ \omega_0^m = \frac{\lambda}{L + \lambda},$$
$$\omega_0^c = \omega_0^m + (1 - \alpha^2 + \beta),\ \omega_i^c = \omega_i^m = \frac{1}{2(L + \lambda)}$$
$$1e - 3 < \alpha < 1\ ,\ \beta = 2\ ,\ k = 0$$

2.3.1. Unscented Kalman filter algorithm

For the UKF , the propagation will do in one step and propagated through the non linear function of the dynamic and measurement equations of the system(Haykin.S 2001). We can compare it with the CDKF algorithm :

2.3.2. Central difference Kalamn filter

For the CDKF , it is the approximately the same idea as in UKF algorithm, at the difference of the steps of propagation of the sigma points through the non linear functions of dynamic of the system and measurement equation, behind this the non linear approximation of these functions using the divided differences (Van der Merwe 2004) .

- Initialisation

$$\hat{x}_0 = E[x_0],\ P_{x_0} = E\left[(x_0-\hat{x}_0)(x_0-\hat{x}_0)^T\right]$$

- For k=1…….∞

1. t=k-1
2. Sigma points

$$\hat{x}_t^{av} = [\hat{x}_t \quad \bar{v}]$$

$$P_t^{av} = \begin{bmatrix} P_{x_{k-1}} & 0 \\ 0 & R_v \end{bmatrix}$$

$$\chi_t^{av} = \left[\hat{x}_t^{av} \quad \hat{x}_t^{av} + h\sqrt{P_t^{av}} \quad \hat{x}_t^{av} - h\sqrt{P_t^{av}}\right]$$

3. propagation of Sigma points through the dynamic function

$$\chi_{k/t}^{x} = f\left(\chi_t^x, \chi_t^v, u_t\right)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} \omega_i^m \chi_{i,k/t}^x$$

$$P_{xk}^- = \sum_{i=1}^{L}\left[\omega_i^{c1}\left(\chi_{i,k/t}^x - \chi_{i+L,k/t}^x\right)^2 + \omega_i^{c2}\left(\chi_{i,k/t}^x + \chi_{i+L,k/t}^x - 2\chi_{0,k/t}^x\right)^2\right]$$

4. Sigma points of measurements

$$\hat{x}_{k/t}^{an} = \left[\hat{x}_k^- \quad \bar{n}\right]$$

$$P_{k/t}^{an} = \begin{bmatrix} P_{x_k}^- & 0 \\ 0 & R_n \end{bmatrix}$$

$$\chi_{k/t}^{an} = \left[\hat{x}_{k/t}^{an} \quad \hat{x}_{k/t}^{an} + h\sqrt{P_{k/t}^{an}} \quad \hat{x}_{k/t}^{an} - h\sqrt{P_{k/t}^{an}}\right]$$

5. Update and filtering

$$Y_{k/t} = h\left(\chi_{k/t}^x, \chi_{k/t}^n\right)$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} \omega_i^m Y_{i,k/t}$$

$$P_{\tilde{y}_k} = \sum_{i=1}^{L}\left[\omega_i^{c1}\left(Y_{i,k/t} - Y_{i+L,k/t}^-\right)^2 + \omega_i^{c2}\left(Y_{i,k/t} + Y_{i+L,k/t}^- - 2Y_{0,k/t}\right)^2\right]$$

$$P_{y_k x_k} = \sqrt{\left(\omega_1^{c1} P_x^-\right)}\left[Y_{1:L,k/t} - Y_{L+1:2L,k/t}\right]^T$$

$$K_k = P_{x_k y_k} P_{\tilde{y}k}^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(y_k - \hat{y}_k^-\right)$$

$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k} K_k^T$$

$$h=1,\ \omega_0^m = \left(h^2 - L\right)/h^2,\ \omega_i^m = \frac{1}{2h^2}$$

,

$$\omega_i^{c1} = \frac{1}{4h^2},$$

$$\omega_i^{c2} = \left(h^2 - 1\right)/4h^4$$

Generally we use the optimal value of h =1 (Norgard.M 2000).

3. NAVIGATION SYSTEMS

We have to present briefly the strapdown inertial navigation system and the global positioning system witch are integrated using variants Kalman filters:

3.1 Inertial Navigation System - INS

At first , we have to define the different frames witch are used in inertial equations:

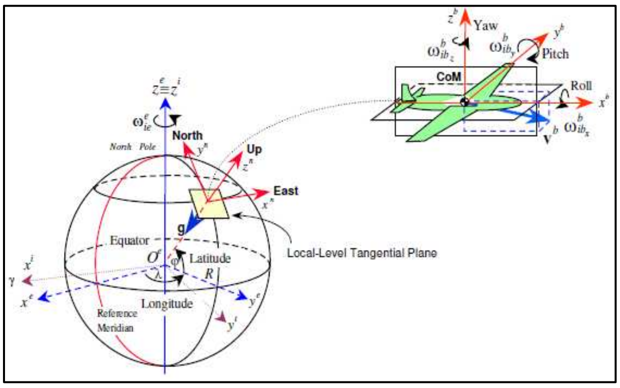

Fig.1. inertial navigation frames

There are four (04) different frames : inertial frame (i), earth frame (e) Navigation frame (n) and body frame (b)

-The mechanisation of strapdown inertial navigation is done as this (Savage 1998):

The attitude of the vehicle is obtained using the following integration:

$$\dot{\rho} = \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = C_{\dot{\phi}\dot{\theta}\dot{\psi}/pqr} \begin{pmatrix} P \\ q \\ r \end{pmatrix} = \frac{1}{\cos\theta} \begin{pmatrix} \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (1)$$

According to the director cosines matrix and attitude integration's matrix:

$$R_{bn} = \begin{pmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{pmatrix} \quad (2)$$

And

$$C_{\dot{\phi}\dot{\theta}\dot{\psi}/pqr} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \quad (3)$$

For position and velocity integration we have to use the following equations in North, East and Down direction of navigation frame (n):

$$v^n = \begin{pmatrix} v_N \\ v_E \\ v_B \end{pmatrix} = \begin{pmatrix} (r_M + h) & 0 & 0 \\ 0 & (r_T + h)\cos\varphi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{\varphi} \\ \dot{\lambda} \\ \dot{h} \end{pmatrix} \quad (4)$$

Where $r^{LLa} = [\varphi \quad \lambda \quad \mathrm{h}]^T$ is the vector of the three positions: latitude, longitude and altitude of the vehicle.

r_M : Meridian radius of the earth in WGS 84 system.

r_T : Tangential radius of the earth in WGS 84 system.

By this, we can integrate the last equation to obtain the position in the navigation frame using the following equation:

$$\dot{r}^{LLa} = \begin{pmatrix} \dot{\varphi} \\ \dot{\lambda} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} (r_M + h) & 0 & 0 \\ 0 & (r_T + h)\cos\varphi & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_N \\ v_E \\ v_B \end{pmatrix} = Dv^n \qquad (5)$$

3.1.1 Errors in inertial sensors

Bias :

$$a_{output} = a_{input} + \delta a_{bias}$$
$$\omega_{output} = \omega_{input} + \delta\omega_{bias}$$

Scale factors:

$$a_{output} = K_a a_{input}$$
$$\omega_{output} = K_\omega \omega_{input}$$

Non linearity:

$$a_{output} = K_0 + K_1 a_{input} + K_2 a_{input}^2 + \dots$$
$$\omega_{output} = L_0 + L_1 \omega_{input} + L_2 \omega_{input}^2 + \dots$$

The inertial navigation system presents some advantages and disadvantages as follow :

-Advantages: complete output solution, good accuracy during short time, high data rate and small size.

-Disadvantages: accuracy decrease after a long time, gravity sensitivity and obligatory external aid for initialization.

So, after have seen generally the inertial navigation system , we will try to define briefly the GPS working as in the follow paragraph.

3.2 Global Positioning System -GPS

The Global Positioning System (GPS) is a spaceborne, radio navigation system. Its world-wide coverage, availability, and high accuracy makes it one of history's most revolutionary developments. In airborne navigation, its complementary characteristics to the inertial navigation system make it an excellent aiding source.

3.2.1 GPS measurements

The principal mode of GPS measurement is the code measurement mode ( ).

Code measure:

$$P_j^i = \rho_j^i + c\,\delta t = \sqrt{(x_j - x^i)^2 + (y_j - y^i)^2 + (z_j - z^i)^2} + c\,\delta t \qquad (6)$$

c: light velocity, ρ_j^i : pseudorange between gps receiver and gps satellite .

In our work the date used to integrate INS were position and velocity from final GPS output. The satellite data are provided in WGS84 coordinates system (Bo.T 2005).

Where GPS has several advantages and disadvantages as following:

Advantages: precision during long term, absolute position and operational conditions.

Disadvantages: multipath problems, dependency to the United state's Department of Defense and atmospheric delays.

4. INTEGRATED SYSTEM

The goal of hybridization is to combine two systems with the advantages of one are the disadvantages of the other and vice versa. The hybridization of such systems should lead to the creation of a new optimal improving the effectiveness of the two merged, and optimizing their respective characteristics in the area in which they live.

We can give the principal modes of INS/GPS integration :

-Non coupled mode.

-Indirect mode loosely coupled.

-Direct mode.

4.1 Mode of INS/GPS integration

In our work, the direct mode was used to implement the INS non linear model of dynamic state equation and a linear model of GPS measurement equation. The following figure shows the direct integration mode (Kim.J 2004):

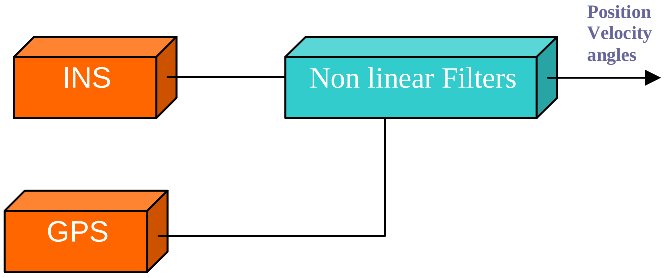

Fig.2. Direct mode integration

This mode imply to use a non linear estimators because there are nonlinear equations to integrate as it is done in the following paragraph.

4.2 State equations of discrete direct mode

In this case, we have choose to integrate the positions using the distances north, east and down without use latitude ,longitude and altitude as presented in previous paragraph, the velocities are integrated in north ,east and down directions, and the angles integration provide yaw, pitch and roll angles of the vehicle the state equations in discrete time could be written as the following forms (Sukkarieh.S 1999):

$$\begin{bmatrix} p_n(k) \\ v_n(k) \\ \psi_n(k) \end{bmatrix} = \begin{bmatrix} p_n(k-1) + v_n(k-1)\Delta t \\ v_n(k-1) + \left\{ C_n^b(k-1)\left[f_b(k) + \delta f_b(k) + g^n \right] \right\}\Delta t \\ \psi_n(k-1) + E_b^n(k-1)\left[\omega^b(k) + \delta\omega^b(k) \right]\Delta t \end{bmatrix} + \begin{bmatrix} w_{p^n}(k) \\ w_{v^n}(k) \\ w_{\psi^n}(k) \end{bmatrix} \qquad (7)$$

Where $x(k)=f(x(k-1),u(k),w(k))$ is the state vector to estimate and contains three positions, three velocities and three angles of vehicles attitude (Sukkarieh.S 1999).

$$E[w_v(k)]=0$$
$$E[w_v(k)w_v(k)^T]=Q(k)=\begin{bmatrix}\sigma^2_{f^b} & 0\\ 0 & \sigma^2_{\omega^b}\end{bmatrix} \qquad (8)$$

Where

$$\nabla f_x(k)=\begin{bmatrix}\frac{\partial p^n(k)}{\partial p^n(k-1)} & \frac{\partial p^n(k)}{\partial v^n(k-1)} & \frac{\partial p^n(k)}{\partial \psi^n(k-1)}\\ \frac{\partial v^n(k)}{\partial p^n(k-1)} & \frac{\partial v^n(k)}{\partial v^n(k-1)} & \frac{\partial v^n(k)}{\partial \psi^n(k-1)}\\ \frac{\partial \psi^n(k)}{\partial p^n(k-1)} & \frac{\partial \psi^n(k)}{\partial v^n(k-1)} & \frac{\partial \psi^n(k)}{\partial \psi^n(k-1)}\end{bmatrix} \qquad (9)$$

The observation equation from GPS is linear as:

$$Z_{k+1}=H(X_k)+V_k \qquad (10)$$

Where observation matrix is as follow:

$$H_k=\left(\begin{array}{ccc|c|c}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{array}\right) \qquad (11)$$

Where :
$$E[V_k(k)]=0$$
$$E[V_k(k)V_k(k)^T]=Q(k) \qquad (12)$$

$Q(k)$ is the noise covariance matrix of GPS measurement. The noise is assumed white Gaussian additive noise.

5. SIMULATIONS

In our work the duration of simulation was Time=100s, and all EKF, UKF and CDKF were implemented using MATLAB software, in S.A (selective Availability) conditions, an we assumed that all noises are white Gaussian noises ,the simulations data are as in following:

Sample time Δt=0.005s, receiver noise=10m, accelerometer bias=0.05-1g, gyrometer bias=0.02-2°/s ,velocity=150-220m/s, Uncertainty initial in North distance : 1000m, Uncertainty initial in East distance : 1000m, Uncertainty initial in Down distance : 100m , Uncertainty initial in VN : 5m/s, Uncertainty initial in VE : 5m/s, Uncertainty initial in VD : 10m/s, Uncertainty initial in φ(yaw): 1°, Uncertainty initial in θ (pitch) : 1° , Uncertainty initial in ψ (roll) : 1°, and GPS data in SA conditions are augmented from 10m to 1000 m during 40 seconds for positions ,from 5 m/s to 50m/s for velocities and from1° to 10° for attitudes angles . concerning the initialization step of the three filters , it was the same and the following value: 80% from the true values of the state vector. GPS data were used at the frequency of 10Hz and the inertial integration process was made at the frequency of 200 Hz. We can observe in the following figures the simulation results and comment them easily below. All the results are showed on the nine (09) following figures.

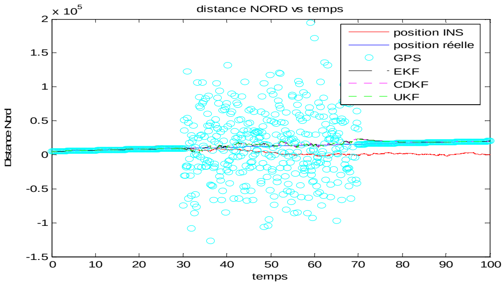

Fig.3. North distance in meter (m) during time (s)

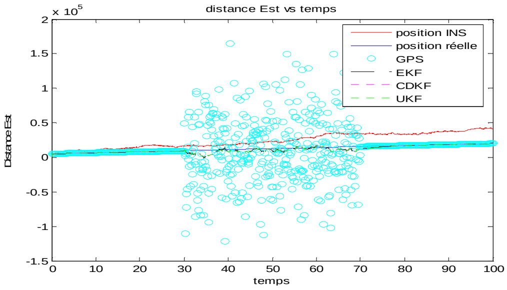

Fig.4. East distance in meter (m) during time(s)

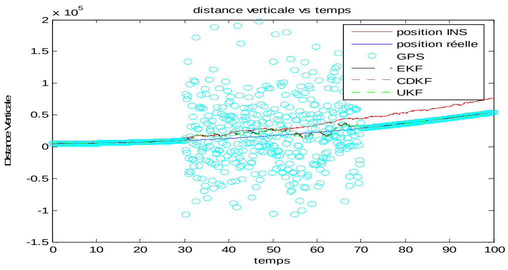

Fig.5. Vertical distance in meter(m) during time (s)

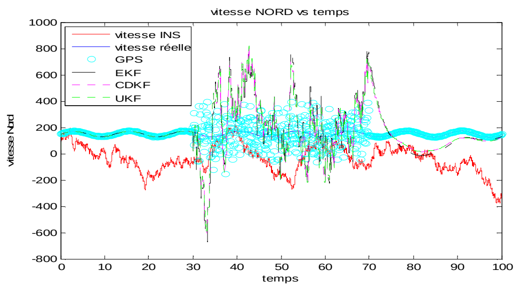

Fig.6. North velocity estimation (m/s) during time (s)

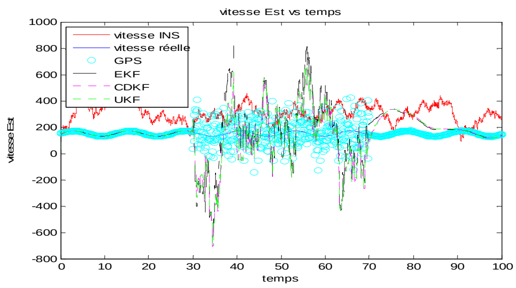

Fig.7. East velocity estimation (m/s) during time in second (s)

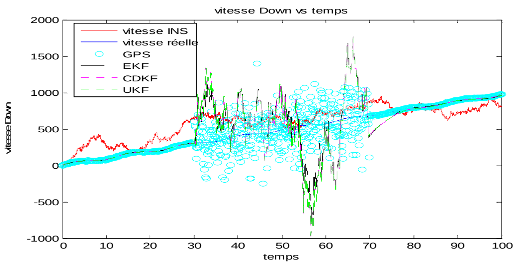

Fig.8 .Down velocity estimation(m/s) during time in second (s)

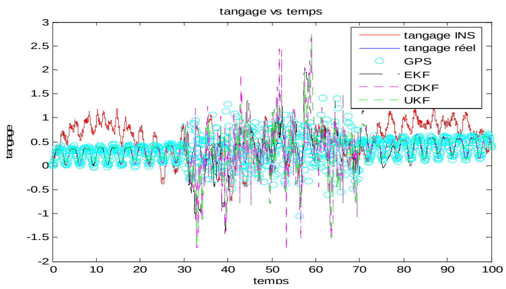

Fig.9 .pitch angle estimation in degrees (°) during timle in second (s)

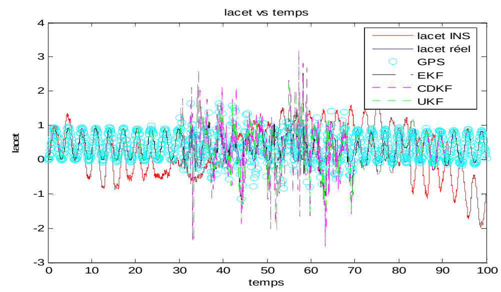

Fig.10. Yaw angle estimation in degrees (°) during time in second (s)

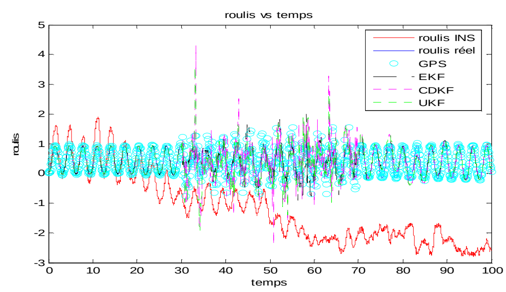

Fig.12. Roll angle estimation in degrees (°) during time in second (s)

6. CONCLUSION

After testing the various algorithms to estimate a nonlinear model, we can conclude that all these algorithms have worked well and gived good results without real difficulties with the exepted UKF with three parameters to set properly. It is therefore worth noting that it would be preferable to implement a CDKF that UKF since it shows the performance and competing does not parameters. but hardly a time of execution is little more. For the velocity , SPKF offer better solutions then EKF in general conditions, but when SA was introduced , we observed a duration of time to return on the true values of the estimate state vector. Really , the SPKF in this case and using this model don't perform significantly the EKF because the non linearity is only present in the dynamic state equation, witch is used only to make the prediction step , so in the future work , these algorithms will be used and applied in INS/GPS integration using a non linear measurement model to determine and verify the theoretical results comparing means and variances of each estimator.

REFERENCES

Bo Tang; Cui Pingyuan; Chen Yangzhou, *(2005)*; Sigma-Point Kalman Filters for GPS Based Position Estimation Information, *Communications and Signal ProcessingFifth International Conference* on 0-0 0 Page(s):213 – 217.

Cho Seong Yun; Wan Sik Choi (2006) ;Robust positioning technique in low-cost DR/GPS for land navigation; *Instrumentation and Measurement, IEEE Transactions* on Volume 55, Issue 4, Page(s):1132 - 1142 .

Crassidis, J.L (2006) Sigma-point Kalman filtering for integrated GPS and inertial navigation., *Aerospace and Electronic Systems, IEEE Transactions on Volume* 42, Issue 2, Page(s): 750 – 756

Haykin Simon , editor. *Kalman Filtering and Neural Networks* (2001)., chapter 7 - The Unscented Kalman Filter, E. A. Wan and R. van der Merwe, pages 221–280. Adaptive and Learning Systems for Signal Processing, Communications, and Control. Wiley.

Kim.J,(2004). Autonomous Navigation for Airborne Applications. PhD thesis, Australian Centre for Field Robotics, The University of Sydney.

Norgaard. M, N. Poulsen, and O. Ravn. (2000) New Developments in State Estimation for Nonlinear Systems. *Automatica*, 36(11):1627–1638.

Savage.P.G.(1998) Strapdown Inertial Navigation Integration Algorithm Design Part 1: Attitude Algorithms. *Journal of Guidance, Control, and Dynamics*, 21(1):19–28.

Sukkarieh.S, ,(1999) “Aided Inertial Navigation Systems for Autonomous Land Vehicles,” PhD thesis‘, Australian Centre for Field Robotics, The University of Sydney.

Van der Merwe.R, E. Wan, and S. J. Julier (2004). Sigma-Point Kalman Filters for Nonlinear Estimation and Sensor-Fusion: Applications to Integrated Navigation. In *Proceedings of the AIAA Guidance, Navigation & Control Conference*, Providence, RI.