# VARIABLE SAMPLING RATE IN NETWORKED CONTROL SYSTEM

*Da Xu, Chih-Chung Chen*

Institute of Automatic Control Engineering
Technical University Munich
Munich, Germany
da.xu@mytum.de, chen@lsr.ei.tum.de

*Dragan Obradovic, Bernhard Lang*

Information and Communications
Corporate Technology, Siemens AG
Munich, Germany
{dragan.obradovic,bernhard.lang}@siemens.com

**ABSTRACT**

In this paper we propose a variable sampling rate approach for networked control system. During control procedure we regulate sampling rate in terms of system behavior intensity. Input increment is used as indicator to reflect the system intensity. A regulator is placed at the controller side, with full-state feedback it can predict the system behavior in the future and determine the next sampling instant according our regulation rule. We implement our approach both over ideal network and average modeled dropout network. By simulation we show that by using our approach we realize more effective usage of network bandwidth than using constant sampling rate. We also present simulation method, wherein we can find regulation rule that optimize the networked control systems.

## 1. INTRODUCTION

During the past two decades, the use of data network in control applications is rapidly increasing. Connecting control systems over communication networks eliminates the restrictions of traditional point-to-point control architectures and offers many advantages in terms of low cost installation and maintenance, and reconfigurability. In spite of the benefits, the communication network exhibits characteristics which degrade control system performance. These characteristics include discretization, quantization effect, time-delay, and data loss. Thus, the challenges arise, to design networked control systems, which should take into account more factors than traditional control systems.

A proper message transmission protocol is necessary to guarantee the network quality of service (QoS). There are a wide variety of different commercially available control network, such as ControlNet, DeviceNet, Profibus, WorldFIP, emerging Ethernet and Wireless. In the work of Lian, et al.[1] three classes of control networks are compared for their performance. In the further work [2], the impact of network architecture on control performance NCS, and design considerations related to control quality of performance as well as network quality of service. One of these considerations is the determination of an acceptable working range of sampling periods in an NCS. The trade-off problem between control performance and network traffic was addressed. Messages with smaller sampling periods also generate high network traffic load. The high network traffic load could increase the possibility of data loss or the waiting time for message contention and induce longer time delays, and control performance may be degraded.

Despite the significant improvements in communication network performance, the limitation of available bandwidth is still the first difficulty for many applications. In order to reduce the network traffic in NCS there have existed basically two classes of ways: compressing or reducing the size of data transferred at each transaction by sophisticated coding or quantization techniques [3]; or minimizing the frequency of transfer of information between the sensor and the controller/actuator. However, the characteristics of network traffic generated by networked control systems determines that the latter approach seems more promising with respect to the resulting traffic reduction, because the protocol overhead is the dominant component of the induced network traffic volume. In the work of L.A. Montestruque et al.,[4], Model-Based Networked Control Systems were introduced, which uses an explicit model of the plan to reduce the network bandwidth requirements. Otanez et.al [5] proposed a deadband approach, a restriction on the ability of an NCS node to send its information, to reduce network utilization and improve bandwidth utilization. One major disadvantage is that the deadband control framework is only suitable for the system with slowly time-varying states such as manufacturing systems, chemical processing plants, because an agent with deadband control adjusts its transmission rates on its own state[6].

The goals of our research are two folds: to reduce the data packets transmitted over the network by a network control systems and meanwhile the control performance is guaranteed. In achieving our goals, the variable sampling rate approach is proposed. Our paper is organized as follows: In section 2 we will give the model of NCS and assumptions in our study. In section 3 we will introduce the principle of our

This work was supported by CT, Simens AG

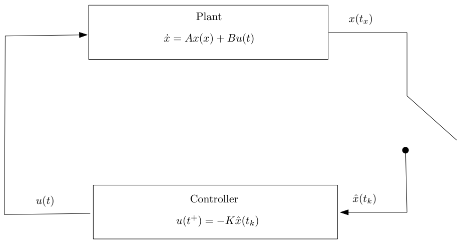



**Fig. 1**. System model of a networked control system

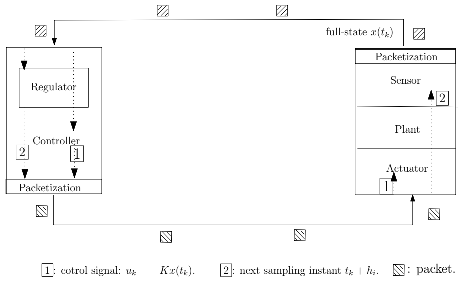



**Fig. 2**. Structure of networked control system with variable sampling rate

approach. Stability, performance evaluation and optimization problem will be also addressed. In section 4 we will implement our approach over ideal network and dropout network. We will simulate our approach in a SISO system and the simulation results are presented and analyzed. In the last section we will conclude our work and discuss some future works to be done in our research.

## 2. ASSUMPTION AND MODEL

### 2.1. Assumptions

We model our system as a sampled-data system, and the plant behaves as a continuous time system. The network resides between sensor and controller as well as controller and actuator, Fig.1. The data generated by sensor or controller at a time instant $t_k$ will be encapsulated into single packet. No processing delay and waiting delay for encapsulation and are also assumed. The following assumptions are also used throughout our work: (1)All sensor nodes are time-driven. (2)All controller nodes are event-driven. (3) All actuator nodes are event-driven.

### 2.2. NCS Model

For simplicity we only consider linear SISO control system as described in the following:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

with the full-state feedback control $u(t) = -Kx(t)$. Therefore, the closed-loop continuous system is given by

$$\dot{x}(t) = (A - BK)x(t). \tag{2}$$

Now the feedback control loop is closed through a communication network. The full-state information is sent in one packet, as shown in Fig.1. The system equation can be written as:

$$\begin{aligned}\dot{x}(t) &= Ax(t) - BK\hat{x}(t), t \in [t_k, t_{k+1})\\ \hat{x}(t_k^+) &= x(t_k), k = 0, 1, ...,\end{aligned} \tag{3}$$

where $\hat{x}(t)$ is a piecewise continuous and only changes value at $t_k$, and $t_k$, $k = 0, 1, \ldots$, is the sampling instant. This equation implies a zero-order hold data reconstruction strategy.

And we define $h$ be the interval between successive sampling. In our study it is nothing else but the transmission interval. Define the sampling interval at $t_k$ as:

$$h_k = t_{k+1} - t_k.$$

In our approach the sampling interval $h_k \in \mathcal{H}$, and $\mathcal{H}$ is a predefined set,$\mathcal{H} \doteq \{h_1, h_2, \ldots, h_s\}$. From the digital control theory we know:

$$\begin{aligned}x(t) = (e^{A(t-t_k)} - K\int_0^{t-t_k} e^{As}B\,ds)x(t_k),\\ t \in [t_k, t_{k+1}).\end{aligned} \tag{4}$$

## 3. VARIABLE SAMPLING RATE APPROACH IN NCS

### 3.1. Principle

Our approach is inspired from the Variable Bit Rate (VBR) strategy [7], which is widely used in audio and video compression and transmission. We place an external regulator at the controller side, see Fig.2, whose task is to select and tell sensor node the next sampling instant $t_k + h_i$, $h_i \in \mathcal{H}$, when the full-state feedback $x(\hat{t_k})$ arrives. This information will be attached together with control signal in a packet. It won't increase the packet size too much, e.g. if $s = 2$ (it is the case we consider in the following), we only need one additional bit in the packet.

Since sampling is the quantization in time, we could use high resolution quantization when the plant behaves fast and intensive, which means using small $h_i$ in set $\mathcal{H}$, in order that the trajectory will be smoothed. We will use *input increment* as our index to reflect the intensity of system behavior, defined as:

$$\Delta u_k = |u(t_{k+1}) - u(t_k)|, \tag{5}$$

where $t_k$ and $t_{k+1}$ are two successive sampling instant. With full-state feedback the regulator can predict the next input increment and distinguish the system intensity in future. Then it decides the next sampling instant. We will describe more in detail later, how regulator works by predicting next input increment.

### 3.2. Stability

We can regard the transmission interval of NCS as time varying, because the sampling interval is variable. In [8], Walsh et al. have have derived bounds on the MATI (maximum allowable transfer interval) such that the resulting system is stable. But the resulted bound is too conservative to be of practical use. W. Zhang has proposed in [9] better methods to find the bound on the time-varying transmission interval. The following lemma guarantees stability of NCS described by Equation (3):

**Lemma 1 (Stability of NCSs)** *The NCS described by (3) in uniformly asymptotically stable if there exists a continuous differentiable, locally positive definite function* $V : R^n \rightarrow R_+$ *and functions* $\alpha$, $\beta$, $\lambda$ *of class K such that for all* $x \in B_r$

$$\alpha(\|x\|) \leq V(x) \leq \beta(\|x\|), \tag{6}$$

*and*

$$\Delta V_k \doteq V(x(t_{k+1})) - V(x(t_k)) \leq -\gamma(\|x(t_k)\|), \quad k = 0, 1, \ldots \tag{7}$$

**Proof**: See [9].

Lemma 1 is only concerned with the Lyapunov function's decreasing at sampling instants; it doesn't require the Lyapunov function to be strictly decreasing over time, $\dot{V}(x(t)) < 0$. Based on Lemma 1 two theorems [9] have been derived to find an upper bound, $h_{suff}$, on $h_i$, for $i = 1, ..s$, which it is sufficient that the networked system is still exponentially stable. Let $h_{true}$ denote the true bound on $h_i$, which means it is necessary and sufficient condition. By using the theorems[9] we can find some bound $h_{suff} \leq h_{true}$, therefore it is a sufficient condition. So we just ensure that $h_i \leq h_{suff}$, for $i = 1, 2, \ldots s$, the system is still stable, when we use any $h_i$ in set $\mathcal{H}$ during control procedure. For more detail see [9].

### 3.3. Performance Evaluation and Optimization

Generally two criteria are used to evaluate control system design and performance. IAE is the integral of the absolute value of the error and ITAE is the integral of the time multiplied by the absolute value of the error. Their mathematical formulas are as follows:

$$\begin{aligned} IAE &= \int_{t_0}^{t_f} |e| \; dt, \\ ITAE &= \int_{t_0}^{t_f} t \, |e| \; dt, \end{aligned} \tag{8}$$

where $e$ is the error between the actual and reference trajectories. $t_0$ and $t_f$ are the initial time and final times of the evaluation period in continuous time. In our study we will use IAE as our control performance index.

In the side of network, we consider network utilization. Average transfer rate is used to evaluate the network traffic, which is defined as:

$$R_{av} = \frac{\text{total number of packets sent}}{\text{running time}}. \tag{9}$$

Because the controller is event-driven, the average transfer rate in both down-link (controller and actuator) and up-link (sensor and controller) is equivalent, in the following the average transfer rate means explicitly the feedback rate in up-link. Obviously, if data is sampled at constant $R$ Hz, $R_{av} = R$.

Commonly the optimization problem in NCS can be divided into two classes of constrained optimization problems as follows:

- *Given a network traffic* $R$*, minimize the control error or control cost* $E$*.*
- *Given a control error* $E$*, minimize the network traffic load* $R$*.*

We can consider network traffic and control performance in NCS simultaneously. In oder to realize such joint optimization an unconstrained Lagrangian cost function is used, which is combine both $R$ and $E$.

$$\min J = E + \lambda R, \tag{10}$$

where $\lambda$ is weight used to change the emphasis of the network traffic and control error on $J$, in a graphic depiction $-\frac{1}{\lambda}$ can be thought as slope of lines of constant $J = D + \lambda R$. The cost $J$ is used as new performance metric of NCS. The optimization problem will be formulated as the minimization of this cost function.

## 4. IMPLEMENTATION

In this section we will implement our approach over ideal network and packet dropout network.

### 4.1. Over ideal Network

In this subsection we assume that there is no time delay or packet dropout in the network. We give a reference signal $r(t)$ so the closed-loop system is presented instead of (2)as:

$$\dot{x}(t) = (A - BK)x(t) + Br(t), \tag{11}$$

the input of the plant is $u(t) = r(t) - Kx(t)$. The sampling interval set consists of only two values, $\mathcal{H} \doteq \{h_1, h_2\}$, where $h_1 > h_2$. In order to guarantee stability of the system $h_1 < h_{suff}$ must be satisfied. A threshold $H$ is defined to determine which sampling rate should be used. We assume that the regulator have accurate knowledge about the plant. At time $t_k$, which denotes the time period from initial time $t_0$ to the $k_{th}$ sampling instant $t_k$, the regulator receives the sensed full-state $x_k$ from plant, and it can calculate the next input increment defined as (5) if the next sample step is $t_k + \bar{h}$ by using (3) and (11):

$$\begin{aligned}\Delta u_k &= |u(t_{k+1}) - u(t_k)| \\ &= |[r(t_{k+1}) - Kx(t_{k+1})] \\ &\quad + [r(t_k) - Kx(t_k)]| \\ &= |[r(t_k + \bar{h}) - Kx(t_k + \bar{h})] \\ &\quad + [r(t_k) - Kx(t_k)]| \\ &= |[r(t_k + \bar{h}) - r(t_k)] \\ &\quad + K(I - \bar{\Phi} + \bar{\Gamma}K)x(t_k) - K\bar{\Gamma}r(t_k)|,\end{aligned} \tag{12}$$

where $\bar{\Phi} = e^{A\bar{h}}$,$\bar{\Gamma} = \int_0^{\bar{h}} e^{As}B\,ds$, and $\bar{h} = \min\{h_1, h_2\}$. Here we calculate the next input increment under the assumption that the next sampling interval is the smallest one in set $\mathcal{H}$, and compare it with threshold $H$, so that we could guarantee that, the actual input increment of two successive sampling instants with interval $\bar{h}$ is always greater than threshold $H$. Our first regulation rule is:

**Regulation Rule 1** *At time* $t_k$,

- *If* $\Delta u_k \geq H$, *the next sample instant is* $t_k + h_2$;
- *If* $\Delta u_k < H$, *the next sample instant is* $t_k + h_1$;

where $\Delta u_k$ is predicted by (12). When data is sampled with interval $h_2$, the input increment is greater than threshold: $\delta u \geq H$ will be always satisfied, however, it cannot guarantee $\delta u > H$. Here we let $\delta$ to denote the actual input increment. In this regulation rule, there are three parameters: $h_1$, $h_2$, and $H$. For simplicity we first assume the two sampling period is already given and fixed, $h_1 = \frac{1}{25}$ s, $h_2 = \frac{1}{80}$ s.

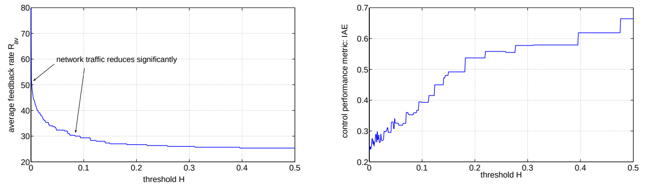
We consider a second-order SISO system with the following systems matrix as our numeric example:

$$A = \begin{bmatrix} -25 & 0 \\ 1 & 0 \end{bmatrix}; B = \begin{bmatrix} 32 \\ 0 \end{bmatrix}; C = \begin{bmatrix} 0 & 32.51 \end{bmatrix} \tag{13}$$

We set the full-state feedback gain $K = C$. Our reference is $r(t) = u(t) + 0.5u(t-1) - 1.5u(t-2)$, and running time $t_f = 3$ s.

By varying threshold we depict impact of the value of threshold $H$ over network traffic and control performance in Fig.3. It shows that as threshold H increases, average feedback rate will decrease monotonically and control error will increase (not monotonically always). It's very noticeable that even with very small threshold the network traffic will be reduced.

The relationship between network traffic and control performance of varying $H$ is presented in Fig.4. We also illustrate this relationship when constant sampling rate is used,




(a) H versus $R_{av}$ (b) H versus IAE

**Fig. 3**. Impact of varying threshold on network traffic and control performance.

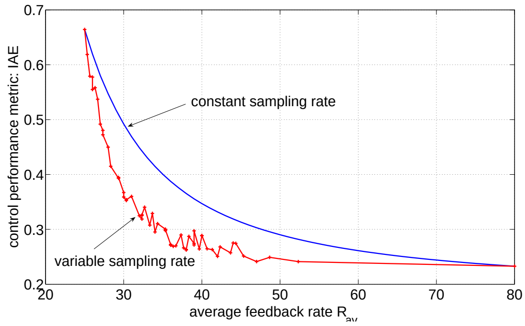



**Fig. 4**. Relationship between feedback rate and control performance

in which the system IAE is calculated at the constant rate equal to resulting average rate. We call this curve R-E curve, where R means rate, E means error. In Fig.4 we find these two curves have same edge points. Because zero threshold means that $\Delta u_k \geq H$ holds always, the feedback rate will be fixed at $\frac{1}{h_2}$; if $H$ is great enough to let $\Delta u_k < H$ be always satisfied, the data will be only sampled with interval $h_1$. The R-E curve of variable sampling rate lies under that of constant sampling rate, which means with the same traffic load, using variable sampling rate gains better performance than constant sampling rate; for the same desired control performance, with variable sampling approach the network utilization is smaller. So with variable sampling rate approach the network can be efficiently utilized.

### 4.2. Optimization

In this subsection the optimization problem will be considered. At first we have to take $h_1$ and $h_2$ take into account. We vary $\frac{1}{h_1}$ from 20 to 60, and $\frac{1}{h_2}$ from 40 to 120, both with increment of 5, and $\frac{1}{h_2} - \frac{1}{h_1} \geq 20$ (otherwise it is meaningless to use variable sampling rate ), for each sampling interval pairs $\{h_1, h_2\}$ threshold $H$ is varied as well. For each sampling interval pair we obtained one R-E curve, Fig.5. If these curves with different values of $h_1$ and $h_2$ are overlapped in some areas, at vertically lower and horizontally left points the NCS exhibit better performance than at other points.

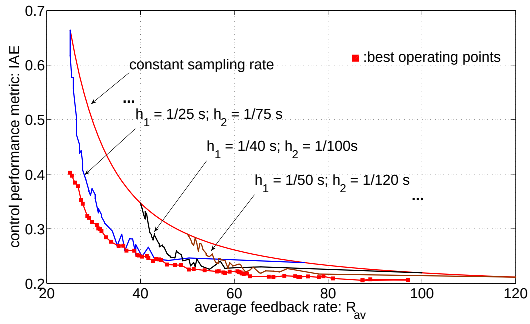



**Fig. 5**. Relationship between feedback rate and control performance at different values of $h_1$ and $h_2$

We select these points in all obtained curve. Working at these points imply that, the network can be most efficiently utilized: Under the same maximum network traffic constraint, they can obtain best control performance, while under the same maximum control error constraint, these points have minimal network traffic utilization. Because with higher feedback rate the system behaves more like continuous system, and the increase of feedback rate has smaller impact on the control error, the difference between our approach and constant sampling rate will be smaller with the increase of average feedback rate.

Now we use the Lagrangian cost function (10) to select the optimal best operating point. We set $\lambda = \frac{1}{500}$. Here we don't describe how to select $\lambda$ in detail, just assume that we have it from practice in advance. The system performance metric is $J = E + \frac{1}{500}R$. We can first use this cost function to select the optimal constant sampling rate for system (13). The optimal constant sampling rate is $R = 62$ Hz or packets/s, and $J_{min} = 0.3811$. For the above simulated threshold and sampling interval range, by using our approach, $J_{min} = 0.32572$ with threshold $H = 0.0112$ and $h_1 = \frac{1}{25}$ s and $h_2 = \frac{1}{125}$ s. The resulted average feedback rate is $R_{av} = 45.7$ packets/s.

## 5. OVER DROPOUT NETWORK

Now we implement our approach in a dropout network. We use the simplest fixed average model for packet dropout. $P$ is defined as the possibility of packet dropout. If the feedback packet from sensor is lost during transmission, new control signal won't be generated and sent to plant, neither the signal for next sampling instant of sensor. The sensor has two choices to determine the next sampling time: it will sample with the same interval as last one or with the smaller interval (in our case $h_2$).

We have two corresponding new regulation rules as follows:

**Regulation Rule 2:** *At time $t_k$, if the packet which contains sensed full state $x(t_k)$ is lost during transmission, sensor will sample data at $t_k + h_{last}$, where $h_{last} = t_k - t_{k-1}$; otherwise the sensor will follow the Regulation Rule 2.*

**Regulation Rule 3:** *At time $t_k$, if the packet which contains sensed full state $x(t_k)$ is lost during transmission, sensor will sample data at $t_k + \bar{h}$, where $\bar{h} = min\{h_1, h_2\}$; otherwise the sensor will follow the Regulation Rule 2.*

We illustrated the R-E curve of constant sampling rate and variable sampling rate using regulation rule 2 and 3 in Fig.6. The same system (13) was used. Because the packet loss occurs randomly, we will run simulation 100 times at each value of threshold and use the mean value. Other parameters used in this simulation are: the loss possibility $P = 10\%$, $\mathcal{H} = \{h_1 = \frac{1}{25}, h_2 = \frac{1}{125}\}$. The R-E curve of CR was obtained by varying sampling rate $R$, and the curve of VR was obtained by varying threshold $H$.

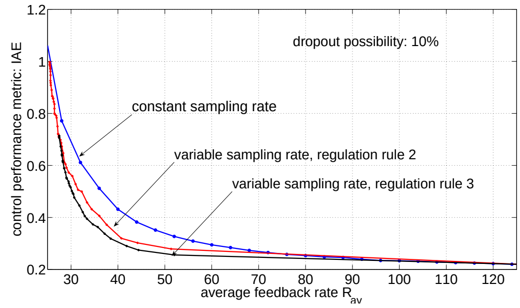



**Fig. 6**. Relationship between feedback rate and control performance over dropout network.

In Fig.6, the R-E curves of VR are still lower the R-E curve of CR. The resultant R-E curve by using Regulation Rule 3 is a bit lower than that by using Regulation Rule 2, because Regulation Rule 3 will use small sample interval to compensate the lost information caused by packet dropout. If the the threshold is great enough, the system will work only with $h_2$ regardless of applied regulation rules, so that both R-E curves of VR will approach the CR's curve with increase of $R_{av}$, and end at same point. If threshold is zero, with Regulation Rule 2 the system will work only with sampling interval $h_1$, so the left edge points of R-E curve with Regulation Rule 2 and CR are overlapped. But with Regulation Rule 3, if $H$ is zero, the sample interval will be switched to $h_2$, so the resultant average feedback rate $R_{av} > \frac{1}{h_1}$. We could use the same method as Chapter 4 to determine the best operating points, and optimal best operating point by minimizing Lagrangian cost function 10. We omit this part here.

In Fig.7 we illustrate how packet dropout impacts our variable sampling rate approach. The parameters of regulation rule is: $\mathcal{H} = \{h_1 = \frac{1}{25}, h_2 = \frac{1}{125}\}$, $H = 0.0012$. Higher packet dropout possibility leads to heavier network traffic and greater control error for both of regulation rules. The resulting average feedback rate of Regulation Rule 2 is smaller than that of Regulation Rule 3, at same dropout possibility ($P < 35\%$), and Regulation Rule 3 benefits control performance. When $P > 35\%$, with same dropout possibility Regulation Rule 2 has higher network traffic than Regulation Rule 3. Although in Regulation Rule 2 $h_{last}$ is used in the case of packet dropout, but sometimes $h_{last} = h_1$ will lead that when the packet arrives, regulator decides to use more smaller $h_2$ inverval to compensate the used $h_2$, and such situation is more critical at high dropout possibility.

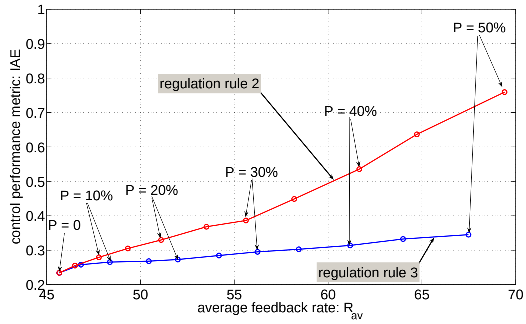



**Fig. 7**. The impact of packet dropout possibility over variable sampling rate approach, $\mathcal{H} = \{h_1 = \frac{1}{25}, h_2 = \frac{1}{125}\}$, $H = 0.0012$

If we consider constrained optimization, Regulation Rule 3 realizes better performance than rule 2, Fig.6. But if we use unconstrained optimization, which means we take both take network traffic and control performance into account, the situation is difference. In Fig.7 Regulation Rule 3 have smaller control error but at the cost of increasing the total average feedback packet rate. So if we think the network traffic utilization is important in unconstrained optimization, when there is low dropout possibility, Regulation Rule 2 is better, e.g $\lambda = \frac{1}{20}$, $J_{rule2} = 2.669$, $J_{rule3} = 2.684$

## 6. CONCLUSIONS

In this work we have proposed variable sampling rate approach, wherein the sampling rate is varied in terms of system behavior intensity. We implemented our approach both over an ideal network and a dropout network and proposed corresponding regulation rule. The simulation results showed that by using variable sampling rate we utilize the network more efficiently and obtain better control performance. We presented a optimization strategy for network control system, by taking both network traffic and control performance into account. And we found the optimal best operating points for our approach by using this strategy.

In our work we have considered a SISO system and proposed the corresponding regulation rule. New approach may be developed for the MIMO system. The new and accurate system intensity index should be used. We have used heuristic method in our study. Most of our results was obtained by simulation and some ideas are still waiting for more strongly theoretic support. Analytical method should be developed. The most possible way to analytical method is use optimal control theory and we should convert the optimization into numerical solvable question.

In future we could implement our approach in other network mode like two state Markov chain or network simulator, so we could more characteristics about our approach. Some interesting aspects should be taken into account, e.g impact of sampling rate over packet dropout [10, 11]. Implementation in real environment is also expected.

*The partial contribution to this work of Sandra Hirche is gratefully acknowledged.*

# References

[1] Feng-Li Lian, James R. Moyne, and Dawn M. Tilbury, "Performance evaluation of control networks: Ethernet, controlnet, and devicenet," *Control Systems Magazine, IEEE*, vol. 21, no. 1, pp. 66–83, Februray 2001.

[2] Feng-Li Lian, J.Moyne, and D.Tilbury, "Network design consideration for distributed control systems," *Control Systems Technology, IEEE Transactions on*, vol. 10, no. 2, pp. 297–307, March 2002.

[3] N. Elia and S.K. Mitter, "Quantization of linear systems," in *Proceedings of the 38th IEEE Conference on Decision and Control.*, December 1999, pp. 3428 – 3433.

[4] L.A. Montestruque and P.J. Antsaklis, "Model-based networked control systems: necessary and sufficient conditions for stability," in *10th Mediterranean Conference On Control And Automation*, Juli 2002.

[5] Paul G. Otanez, James R. Moyne, and Dawn M. Tibury, "Using deadbands to reduce communication in networked control systems," in *American Control Conference*, May 2002, vol. 4, pp. 3015 – 3020.

[6] Feng-Li Lian, J.K. Yook, D.M Tilbury, and Moyne.J, "Network architecture and communication modules for guaranteeing acceptable control and communication performance for networked multi-agent systems," in *Industrial Informatics, IEEE Transactions on*, Feburary 2006, pp. 12–24.

[7] S. Elramly, S.G. Foda, and M. El-Shafie, "Continuous variable sampling rate, application on speech," in *2nd IEEE Symposium on Computers and Communications (ISCC '97)*, 1997, p. 189.

[8] Gregory C. Walsh, Hong Ye, and Linda Bushnell, "Stability analysis of networked control systems," in *Proceedings of the American Control Conference*, San Diego, California, USA, June 1999.

[9] W Zhang, *Stability Analysis of Networked Control Systems*, Ph.D. thesis, Electrical Engineering and Computer Science Dept., Case Western Reserve Univ.,, May 2001, http://dora.cwru.edu/msb/pubs/wxzPHD.pdf.

[10] Paul A. Kawka and Andrew G. Alleyne, "Stability and feedback control of wireless networked systems," in *American Control Conference*, Portland, OR, USA, June 2005.

[11] N.J. Ploplys, P.A. Kawka, and A.G. Alleyne, "Closed-loop control over wireless networks," *Control Systems Magazine, IEEE*, vol. 24, no. 3, pp. 58 – 71, June 2004.