DECENTRALIZED MODEL PREDICTIVE CONTROL BASED ON I/O PLANT MODEL

Róbert Krasňanský and Alena Kozáková
Slovak University of Technology in Bratislava
Slovak Republic
robert.krasnansky@stuba.sk, alena.kozakova@stuba.sk

Abstract

The paper deals with the decentralized model predictive controller design. The unconstrained Generalized Predictive Control (GPC) design approach is considered to design local controllers within the Equivalent subsystems methodology. According to ESM, the original multivariable plant is diagonalized by generating so called equivalent subsystems, for which local controllers are tuned independently. Resulting local controllers constitute the decentralized controller and are implemented on the real plant. Plant-wide closed-loop stability and performance under the decentralized controller are guaranteed if local controllers provide stability and required performance of equivalent subsystems.

The proposed approach has been verified on a case study - decentralized GPC design for a two-input two-output laboratory plant.

Key words

Decentralized control, model predictive control, Generalized Predictive Control, frequency domain, RST controller

1 Introduction

Complex systems with multiple inputs and multiple outputs (MIMO) are usually controlled using multi-loop or decentralized controllers. Compared to centralized control, decentralized control is characterized by several important benefits such as hardware simplicity, operation simplicity as well as design simplicity and reliability improvement; due to them, decentralized control design techniques still remain popular among control strategies.

The Equivalent Subsystems Method (ESM) [Kozáková, et al., 2009; Rosinová and Kozáková, 2012] is a Nyquist based frequency domain technique for decentralized controller design. According to it, interactions are taken into account through a chosen characteristic locus of the matrix of interactions used to modify frequency responses of decoupled subsystems; these modified frequency responses are called equivalent subsystems. Local controllers are designed for individual equivalent subsystems independently using any single-input single-output (SISO) method, frequency-domain methods are preferred (Bode design, Neymark D-partition) and the recently developed Sine wave method [Bucz et al., 2012]. It has been proved [Kozáková et al., 2009; Rosinová and Kozáková, 2012] that if closed-loop stability of individual equivalent subsystems under the respective local controllers is guaranteed then the full closed-loop system is stable as well.

Implementation of the ESM for the decentralized controller (DC) design with local GPC controllers as proposed in this paper provides new perspectives to further development of the ESM-based approach. Generalized Predictive Control e.g. [Camacho and Bordons, 2004; Rossiter, 2004] is one of the most popular and successfully implemented Model Predictive Control (MPC) algorithms.

There are many papers on decentralized MPC design and implementation. In [Richards and How, 2004] a decentralized MPC algorithm is presented for systems consisting of multiple subsystems with independent dynamics and disturbances but with coupled constraints. A plug-and-play MPC based on linear programming is proposed in [Riverso et al., 2013]. An extension of the GPC algorithm to a multivariable case by designing several SISO controllers and compensation for interactions is presented in [Linkens and Mahfouf, 1992], in [Vesely and Osusky, 2013] a frequency domain robust model predictive controller with hard input constraints is addressed. The paper by [Shah and Engell, 2010] presents a systematic approach that relates MPC tuning to linear control theory; in particular a systematic tuning of the prediction horizon and the cost function weights are provided to achieve desired closed-loop pole and zero locations for the unconstrained case. Results presented in this paper can be extended for the MIMO case and be applied in the decentralized GPC design proposed in this paper as well.

In this paper, the GPC algorithm is applied within the ESM framework. Closed-loop stability analysis is based on the transformation of the GPC controller into the polynomial RST structure [Landau, 1998], which also useful for implementation of local unconstrained GPC's.

The paper is organized as follows: Section 2 presents problem formulation and theoretical background for the decentralized control design technique developed in Section 3. In Section 4, theoretical and experimental results illustrating effectiveness of the proposed approach are provided. Conclusions are drawn at the end of the paper.

2 Problem Formulation and Preliminaries

Consider a complex MIMO plant in the standard feedback loop (Fig. 1) consisting of the plant transfer function matrix $G(s) \in R^{m \times m}$, and a decentralized controller $R(s) \in R^{m \times m}$, where w, u, y, d, e are vectors of reference, control, output, disturbance and control error of compatible dimensions.

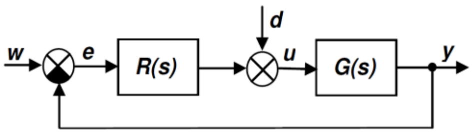

Figure 1. Stadnard feedback configuration

The plant transfer function matrix $G(s)$ can be decomposed according to Fig. 2

$$G(s) = G_d(s) + G_m(s) \qquad (1)$$

where $G_d(s) = \text{diag}\{G_{ii}(s)\}_{m \times m}$, $\det G_d(s) \neq 0$ is the transfer function matrix of decoupled subsystems and $G_m(s) = G(s) - G_d(s)$ matrix of interactions between them.

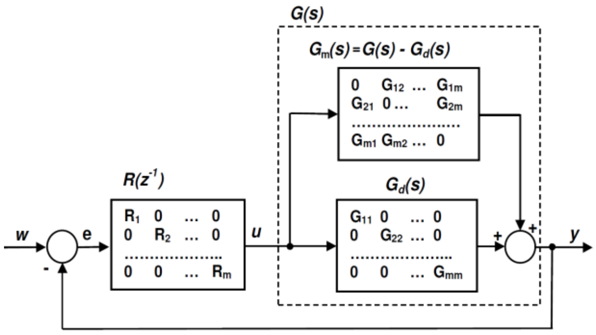

Figure 2. Standard feedback configuration with a decentralized controller

Denote the closed-loop characteristic polynomial (CLCP)

$$\det F(s) = \det[I + Q(s)] \qquad (2)$$

where $Q(s) = G(s)R(s)$ is the loop transfer matrix.

According to the Generalized Nyquist stability criterion the closed-loop system in Fig. 1 is stable if and only if for all $s \in D$

$$N[0, \ \det F(s)] = \sum_{i=1}^{m} N\{0, \ [1 + q_i(s)]\} = n_q \qquad (3)$$

where n_q is the number of open-loop unstable poles of $Q(s)$, D is the standard Nyquist D-contour in the complex plane, $N[0, \det F(s)]$ is the number of anticlockwise encirclements of $(0,0i)$ by $\det F(s)$; $q_i(s)$, $i = 1,\dots,m$ is the set of m eigenfunctions of $Q(s)$ defined as

$$\det[q_i(s)I - Q(s)] = 0 \quad i = 1,\dots,m \qquad (4)$$

The plant (1) is to be controlled using a decentralized (diagonal) controller (DC)

$$R(s) = diag\{R_i(s)\}_{m \times m} \qquad (5)$$

to guarantee closed-loop stability as well as a required plant-wide performance. The DC will be implemented as an unconstrained decentralized MPC within the framework of the ESM revisited in the next section.

2.1 Equivalent Subsystems Method

The Equivalent Subsystems Method is a Nyquist-based decentralized controller design technique for stability and guaranteed plant-wide performance, applicable for continuous- and discrete-time plants described by a transfer function matrix. [Rosinová and Kozáková, 2012].

The method is based upon the condition (3) and considers the diagonal controller (5) and the split system (1) in which the matrix of interactions has been replaced in the sense of the Hamilton-Cayley theorem by a diagonal matrix

$$P(s) = g_k(s) I_{m \times m} \qquad (6)$$

where $g_k(s)$ is either of the m eigenfunctions of $G_m(s)$ defined according to (4). The resulting equivalent system

$$G^{eq}(s) = G_d(s) + g_k(s)I, \quad k \in \{1, 2, \dots, m\} \qquad (7)$$

is a diagonal matrix of m equivalent subsystems $G^{eq}(s) = diag\{G_{ii}^{eq}(s)\}_{m \times m}$ generated as follows

$$G_{ik}^{eq}(s) = G_{ii}(s) + g_k(s), \ i = 1, 2, \dots m; \ k \in \{1, \dots m\} \qquad (8)$$

If transfer functions of decoupled subsystems $G_{ii}(s)$, $i = 1,..,m$ are viewed as eigenfunctions of $G_d(s)$, equivalent subsystems generated according to (8) can be denoted as „equivalent eigenfunctions" of $G(s)$, and related with corresponding "equivalent loci". Conditions for closed-loop stability under a DC are formulated in the next theorem [Rosinová and Kozáková, 2012].

Theorem 1 (Stability under DC)

The closed-loop in Fig. 1 comprising the plant (1) and a decentralized controller (5) is stable if and only if there exists $P(s) = g_k(s) I_{m \times m}$ such that

1) $\det[g_k(s)I - G_m(s)] = 0 \quad k \in \{1, \dots, m\}$;
2) closed-loop characteristic polynomials (9) of all equivalent subsystems (8) are stable

$$CLCP_i^{eq} = 1 + G_{ik}^{eq}(s)R_i(s) \quad (9)$$

Theorem 1 results from the equivalence between closed-loop characteristic polynomials of the original MIMO system (*CLCP*) and the diagonal equivalent system (*CLCP*^{eq}), both under a decentralized controller

$$\begin{aligned} CLCP(s) &= \det[I + G(s)R(s)] = \\ &= \det[I + G^{eq}(s)R(s)] = CLCP^{eq}(s) \end{aligned} \quad (10)$$

As *CLCP*^{eq}*(s)* is a diagonal matrix of independent equivalent closed-loop polynomials

$$\begin{aligned} CLCP^{eq} &= diag\{CLCP_{ik}^{eq}(s)\}_{m\times m} = \\ &= diag\{1 + [G_{ii}(s) + g_k(s)]R_i(s)\}_{m\times m} \end{aligned} \quad (11)$$

then

$$\det F(s) = \prod_{i=1}^{m}[1 + G_{ik}^{eq}(s)R_i(s)] \quad (12)$$

and closed-loop stability is guaranteed if

$$N[0, \det F(s)] = \sum_{i=1}^{m} N\{-1, G_{ik}^{eq}(s)R_i(s)\} = n_q \quad (13)$$

The Equivalent Subsystems Method directly results from Theorem 1; it enables to design the decentralized controller as a collection of local controllers designed independently for individual equivalent subsystems. Local controllers independently tuned for stability and specified feasible performance of equivalent subsystems constitute the resulting decentralized controller guaranteeing stability and the specified performance plant-wide.

If a discrete-time DC controller is to be designed, discrete-time version of ESM is applied. First, the continuous-time plant is discretized using an appropriate sampling period *T;* given the discrete-time plant transfer function $G(z)$ where $z = e^{sT}$, its frequency domain properties can be studied by plotting all frequency response characteristics of $G(e^{j\omega T})$ by analogy with the continuous-time case. Besides choosing the sampling period *T* properly with respect to the plant dynamics it is necessary to keep in mind that a discrete frequency response is periodic with respect to the sampling frequency $\omega_s = 2\pi/T$ and thus can be represented only for frequencies up to half of the sampling frequency, i.e. $\omega \in <0; \omega_s/2>$; higher frequencies would be "wrapped" to some other frequency in the range. A proper choice of sampling period is crucial for achievable bandwidth and feasibility of the required phase margin [Lewis, 1992].

Actually, the ESM is a controller design framework within which local controllers are tuned for stability and specified feasible performance of each equivalent subsystem independently, and then implemented to real subsystems.

2.2 ESM-Based Decentralized Controller Design

The proposed design procedure has the following steps:

1. Discretization of the continuous-time plant *G(s)* using an appropriately chosen sampling period *T*, specification of the sampling frequency $\omega_s = 2\pi/T$ and of the feasible frequency range $\omega \in <0; \omega_s/2>$.
2. Partition of the discrete-time transfer function matrix *G*(*z*) into the diagonal and off-diagonal parts $G_d(z)$ and $G_m(z)$, respectively, calculation and plotting of characteristic loci g_i(z), *i* = 1,…,*m* of $G_m(z)$ for $z = e^{i\omega T}$, $\omega \in <0, \omega_s/2>$.
3. Choice of $g_k(z)$ for fixed $k \in \{1,\dots m\}$, generating and plotting discrete frequency responses of independent equivalent subsystems.
4. Design and implementation of local controllers $R_i(s)$, *i* = 1,…,*m* for all *m* equivalent subsystems.

Any SISO design is applicable yet due to the nature of equivalent subsystems, frequency-domain designs are preferred e.g. the Neymark D-partition method [Kozáková, Veselý and Osuský, 2009], standard Bode design [Rosinová and Kozáková, 2012], Quantitative Feedback Theory (QFT) design, Sine-wave method [Bucz et al., 2012] etc.

In the sequel, a procedure to design a decentralized MPC controller is developed. Local unconstrained MPC controllers are designed for equivalent subsystems and implemented on real subsystems to guarantee stability and plant-wide performance. In particular, the Generalized Predictive Control algorithm is considered since it is based on input-output models and most of the other MPC approaches are relatively easily derivable by its minor modifications [Shah and Engell, 2010]. The unconstrained optimization scheme is considered to be able to perform stability analysis.

2.3 Model Predictive Controller Design

In this chapter, a brief introduction to the main principles of MPC is recalled. Consider the plant described by the CARIMA model [Camacho and Bordons, 2004] in the following form:

$$A(z^{-1})y(t) = B(z^{-1})z^{-d}u(t) + \frac{C(z^{-1})}{\Delta}e(t) \quad (14)$$

where

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \dots + a_{n_a} z^{-n_a}, \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \dots + b_{n_b} z^{-n_b}, \end{aligned} \quad (15)$$

$C(z^{-1})$ is the noise filter polynomial, *u*(*t*), *y*(*t*) are the plant input and output, respectively, *e*(*t*) is a zero mean white noise, and $\Delta = 1 - z^{-1}$. For simplicity, consider that $C(z^{-1}) = 1$.

To develop a prediction model, (14) can be rewritten as follows [Camacho and Bordons, 2004]:

$$\hat{A}(z^{-1})y(t+k) = B(z^{-1})\Delta u(t+k-1) + e(t+k) \qquad (16)$$

where $\hat{A}(z^{-1}) = \Delta A(z^{-1}) = 1 + A_1 z^{-1} + \ldots + A_{na+1} z^{-(na+1)}$. Since in (16) the control move $\Delta u(t)$ is considered, to obtain the plant input it is necessary to introduce an integrator

$$u(t) = \frac{1}{1-z^{-1}}\Delta u(t) \qquad (17)$$

Based on (16) the prediction model can be written in the following compact matrix/vector form

$$\Theta_A \tilde{y}_s(t) = \Theta_B \Delta\tilde{u}_s(t) + \Psi_B \Delta\tilde{u}_p(t) - \Psi_A \tilde{y}_p(t) \qquad (18)$$

where

$$\Theta_A = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ A_1 & 1 & 0 & \ldots & 0 \\ A_2 & A_1 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \ldots & \ldots & A_2 & A_1 & 1 \end{bmatrix}, \; \Psi_A = \begin{bmatrix} A_1 & A_2 & \ldots & A_n & A_{n+1} \\ A_2 & A_3 & \ldots & A_{n+1} & 0 \\ A_3 & A_4 & \ldots & 0 & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots \end{bmatrix} \qquad (19)$$

$$\Theta_B = \begin{bmatrix} b_1 & 0 & 0 & \ldots & 0 \\ b_2 & b_1 & 0 & \ldots & 0 \\ b_3 & b_2 & b_1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \end{bmatrix}, \quad \Psi_B = \begin{bmatrix} b_2 & b_3 & \ldots & b_{n-1} & b_n \\ b_3 & b_4 & \ldots & b_1 & 0 \\ b_4 & b_5 & \ldots & 0 & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots \end{bmatrix} \qquad (20)$$

$$\tilde{y}_s(t) = \begin{bmatrix} y(t+1|t) \\ y(t+2|t) \\ \vdots \\ y(t+N_y|t) \end{bmatrix}, \qquad \tilde{y}_p(t) = \begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-n) \end{bmatrix}$$
$$\Delta\tilde{u}_s(t) = \begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ \vdots \\ \Delta u(t+N_y-1|t) \end{bmatrix}, \quad \Delta\tilde{u}_p(t) = \begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \vdots \\ \Delta u(t-n) \end{bmatrix}. \qquad (21)$$

From (18), $\tilde{y}_s(t)$ is expressed as follows:

$$\tilde{y}_s(t) = \Lambda\Delta\tilde{u}_s(t) + \Gamma\Delta\tilde{u}_p(t) + \Omega\tilde{y}_p(t) \qquad (22)$$

$$\Lambda = \Theta_A^{-1}\Theta_B, \qquad \Gamma = \Theta_A^{-1}\Psi_B, \qquad \Omega = \Theta_A^{-1}\Psi_A \qquad (23)$$

The cost function to be minimized is in the form:

$$J(t) = \sum_{i=1}^{N_y}\left(\tilde{w}(t) - \tilde{y}_s(t)\right)^T \Pi_y \left(\tilde{w}(t) - \tilde{y}_s(t)\right) + \sum_{i=1}^{N_u} \Delta\tilde{u}_s(t)^T \Pi_u \Delta\tilde{u}_s(t) \qquad (24)$$

where

$$\tilde{w}(t) = \begin{bmatrix} w(t+1) \\ \vdots \\ w(t+N_y) \end{bmatrix} \qquad (25)$$

Π_y, Π_u are weighting matrices, N_y, N_u, $N_u \le N_y$ are the prediction and control horizons, respectively. Substituting (22) into (24) the cost function can be expressed after some manipulations as follows

$$J(t) = 2\left[-\tilde{w}^T(t)\Pi_y\Lambda + \Delta\tilde{u}_p^T(t)\Gamma^T\Pi_y\Lambda + \tilde{y}_p^T(t)\Omega^T\Pi_y\Lambda\right]\Delta\tilde{u}_s(t) + \Delta\tilde{u}_s^T(t)\left[\Lambda^T\Pi_y\Lambda\right]\Delta\tilde{u}_s(t) + \rho \qquad (26)$$

where ρ contains terms which do not depend upon $\Delta\tilde{u}_s(t)$ and hence can be ignored. If no constraints are considered, the solution has analytical form found from the condition

$$\frac{\partial J}{\partial \Delta\tilde{u}_s} = 0 \qquad (27)$$

The resulting GPC control law is defined by the first element of $\Delta\tilde{u}_s(t)$ [Rossiter, 2004]:

$$\Delta\tilde{u}_s(t) = P_r\tilde{w}(t) + D_k\Delta\tilde{u}_p(t) + N_k\tilde{y}_p(t) \qquad (28)$$

where

$$\begin{aligned} P_r &= e_1^T\left(\Lambda^T\Lambda + \Pi_y I\right)^{-1}\Lambda^T \\ D_k &= e_1^T\left(\Lambda^T\Lambda + \Pi_y I\right)^{-1}\Lambda^T\Omega \\ N_k &= e_1^T\left(\Lambda^T\Lambda + \Pi_y I\right)^{-1}\Lambda^T\Gamma \end{aligned} \qquad (29)$$

and $e_1^T = [I, 0, \ldots, 0]$.

In the unconstrained case when the control law and the controlled plant are linear, it is possible to derive the closed-loop characteristic equation and find its poles, and possibly to examine various frequency domain characteristics. The polynomial representation of the controller is used according to Fig. 3 where the predictive controller is described in the RST form [Landau, 1998].

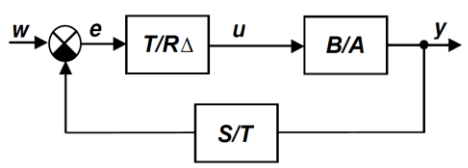

Figure 3. Feedback loop with the R-S-T controller

Digital filters R and S are designed to achieve the desired performance, and T is designed afterwards to achieve the desired tracking performance. Using (28) we can formulate the closed-loop system represented in the pole-placement structure [Camacho and Bordons, 2004]:

$$R(z^{-1})\Delta u(t) = T(z^{-1})w(t) - S(z^{-1})y(t) \quad (30)$$

Now, consider the CARIMA plant model (16) in the following form:

$$A(z^{-1})\Delta y(t) = B(z^{-1})\Delta u(t-1) + T(z^{-1})e(t) \quad (31)$$

Substituting (30) into (31) the closed-loop is obtained in the form:

$$\begin{aligned} &A(z^{-1})\Delta y(t) = \\ &= B(z^{-1})\left(\frac{T(z^{-1})}{R(z^{-1})}w(t) - \frac{S(z^{-1})}{R(z^{-1})}y(t)\right) + T(z^{-1})e(t) \end{aligned} \quad (32)$$

The output of the closed-loop system is then

$$\begin{aligned} y(t) = &\frac{B(z^{-1})T(z^{-1})z^{-1}}{R(z^{-1})A(z^{-1})\Delta + B(z^{-1})S(z^{-1})z^{-1}} w(t) + \\ &+ \frac{T(z^{-1})R(z^{-1})}{R(z^{-1})A(z^{-1})\Delta + B(z^{-1})S(z^{-1})z^{-1}} e(t) \end{aligned} \quad (33)$$

The closed-loop characteristic polynomial is:

$$R(z^{-1})A(z^{-1})\Delta + B(z^{-1})S(z^{-1})z^{-1} = 0 \quad (34)$$

Choosing $T(z^{-1})$ = 1 the resulting expressions for the polynomials $R(z^{-1})$and $S(z^{-1})$ are obtained

$$R(z^{-1}) = \frac{T(z^{-1}) + z^{-1}\sum_{i=N_1}^{N_y} k_i I_i}{\sum_{i=N_1}^{N_y} k_i} \quad (35)$$

$$S(z^{-1}) = \frac{\sum_{i=N_1}^{N_y} k_i F_i}{\sum_{i=N_1}^{N_y} k_i} \quad (36)$$

where k_i are elements of the first row of $(\Lambda^T\Lambda+\Pi_y I)^{-1}\Lambda^T$, I_i represents rows of Λ^T and F_i represent rows Ω^T.

3 Decentralized GPC controller design

This section presents main result of the paper - namely the decentralized GPC design implemented within the ESM according to the design procedure given in subsection 2.2. In particular, Step 4 (Design and implementation of local controllers) of the design procedure is explained next.

After completing steps 1-3 of the design procedure, frequency responses of individual equivalent subsystems are available; their discrete-time input/output transfer functions have to be identified to be able to perform local GPC designs according to subsection 2.3.

The resulting decentralized GPC consists of local independently tuned GPC's of equivalent subsystems. According to Theorem 1, stability of individual equivalent feedback loops under the local GPC's guarantees plant wide stability. In the unconstrained case, stability of equivalent feedback loops is verified using the RST representation of individual local GPC's.

In case of decentralized GPC design, Step 4 of the ESM-based design procedure can be summarized as follows:

a) Identification of equivalent subsystems models based on their frequency responses.
b) Design and tuning of local GPC controllers for all *m* equivalent subsystems.
c) Verification of stability and performance of individual feedback loops of equivalent subsystems under local GPC's in the R-S-T form.
d) Implementation of the decentralized GPC on the real plant

The design procedure is illustrated on a case study.

4 Case Study

4.1. Plant description and modelling

A two input - two output (TITO) plant is considered, consisting of two interconnected cooperating DC motors (Fig. 4, Fig. 5). The plant inputs are armature voltages U_1, U_2, measured outputs are speeds of individual motors ω_1, ω_2 converted into voltage.

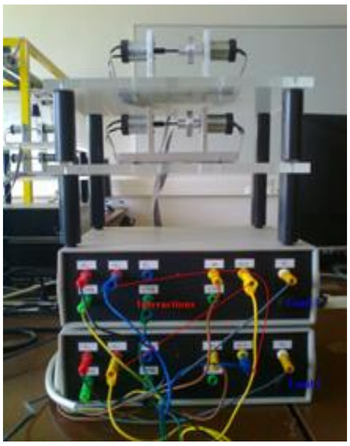

Figure 4. Laboratory plant – a two DC motor system

Each DC motor has its own adjustable load (0-100%). Interconnection of the DC motors brings interactions into the MIMO plant. Input and output signals are measured using the data acquisition card Advantech PCI 1711. Mathematical model of the plant was identified experimentally from step responses measured in the operating point specified by $U_1 = U_2$ = 3.5V, and the loads U_{L1} = 4V, U_{L2} = 3V.

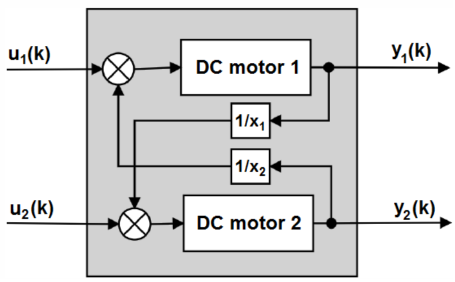

Figure 5. Block scheme of two interconnected DC motors

The discrete-time I/O model of the plant was identified in the following form:

$$\begin{bmatrix} Y_1(z^{-1}) \\ Y_2(z^{-1}) \end{bmatrix} = \begin{bmatrix} G_{11}(z^{-1}) & G_{12}(z^{-1}) \\ G_{21}(z^{-1}) & G_{22}(z^{-1}) \end{bmatrix} \begin{bmatrix} U_1(z^{-1}) \\ U_2(z^{-1}) \end{bmatrix} \qquad (37)$$

where

$$G_{11}(z^{-1}) = \frac{-0.006647z^{-1} + 0.01394z^{-2}}{1 - 1.855z^{-1} + 0.861z^{-2}}$$

$$G_{12}(z^{-1}) = \frac{-0.0001223z^{-1} + 0.0002967z^{-2}}{1 - 1.971z^{-1} + 0.9717z^{-2}}$$

$$G_{21}(z^{-1}) = \frac{-0.0001501z^{-1} + 0.0003441z^{-2}}{1 - 1.972z^{-1} + 0.9725z^{-2}} \qquad (38)$$

$$G_{22}(z^{-1}) = \frac{-0.006151z^{-1} + 0.01607z^{-2}}{1 - 1.821z^{-1} + 0.8292z^{-2}}$$

4.2 Decentralized GPC Design

To generate equivalent subsystems, characteristic loci $g_1(z)$ and $g_2(z)$ of interaction transfer function matrix $G_m(z)$ have been calculated; $g_1(z)$ was chosen to generate equivalent subsystems according to (8). Equivalent subsystems given by their frequency responses had to be identified to obtain discrete-time transfer functions.

Best fit model for both equivalent subsystems was the fourth-order Output Error (OE) structure polynomial model found using the Matlab System Identification Tool IDENT:

$$G_{ekv1}(z^{-1}) = \frac{-0.006782z^{-1} + 0.02761z^{-2} - 0.03465z^{-3} + 0.01382z^{-4}}{1 - 3.826z^{-1} + 5.49z^{-2} - 3.5z^{-3} + 0.8369z^{-4}} \qquad (39)$$

$$G_{ekv2}(z^{-1}) = \frac{-0.006287z^{-1} + 0.02876z^{-2} - 0.03835z^{-3} + 0.01589z^{-4}}{1 - 3.793z^{-1} + 5.392z^{-2} - 3.405z^{-3} + 0.806z^{-4}} \qquad (40)$$

Bode diagrams of generated equivalent subsystems and their OE polynomial models are compared in Fig. 6 and Fig. 7. Local MPC controllers were designed using the MPC technique described in subsection 2.3. Following design parameters were chosen: prediction horizon N_y = 6, control horizon N_u = 6, weights Π_{y1}, Π_{y2} = {(0.57, 1.58), (0.85, 3.50)} and the sampling period T = 0.1s.

Parameters of the decentralized predictive controller in the R-S-T form are summarized in Table 1.

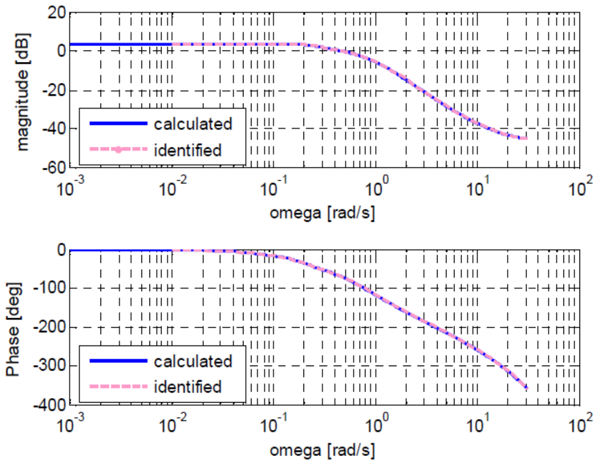

Figure 6. Bode plots of the equivalent subsystem $G_{11}^{eq}(i\omega)$ and its model $G_{ekv1}(z^{-1})$

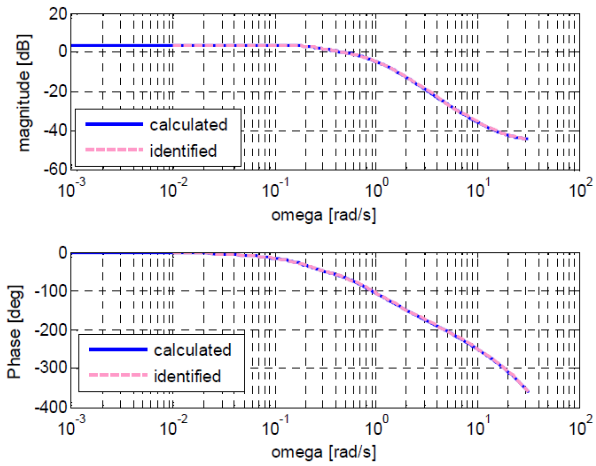

Figure 7. Bode plots of the equivalent subsystem and $G_{21}^{eq}(i\omega)$ its model $G_{ekv2}(z^{-1})$

Table 1. Controllers Parameters

R_i	Parameters		
	$R(z^{-1})$	$S(z^{-1})$	$T(z^{-1})$
R_{11}	$1 - 0.9992z^{-1}$ $- 0.0007846\, z^{-2}$	$3.477 - 11.33z^{-1} + 14.15z^{-2}$ $-7.993z^{-3} + 1.718z^{-4}$	1
R_{22}	$1 - 0.9991z^{-1}$ $- 0.0008891\, z^{-2}$	$2.638 - 8.405z^{-1} + 10.32z^{-2}$ $- 5.754z^{-3} - 1.222z^{-4}$	1

4.3 Experimental Results

Experimental results on the real plant under the designed decentralized controller are shown in Fig. 8 and Fig. 9.

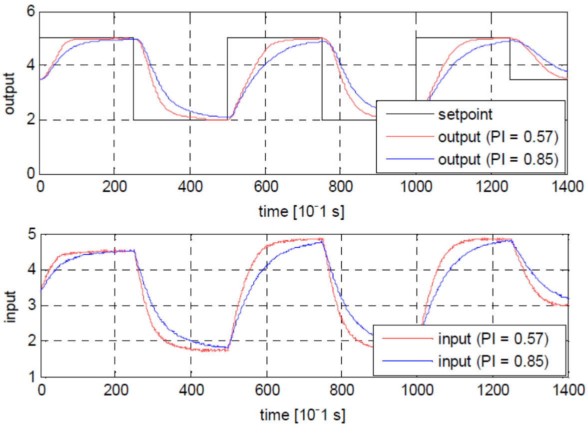

Figure 8. Time responses of the real plant output y_1 and control input u_1 for different values of Π_{y1}, Π_{y2}

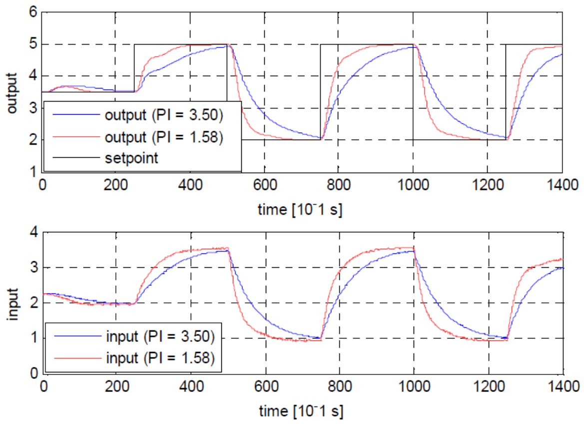

Figure 9. Time responses of the real plant output y_2 and control input u_2 for different values Π_{y1}, Π_{y2}

From the closed-loop characteristic polynomials of individual equivalent subsystems and of the full system under the decentralized controller obtained according to (34) stability was verified.

Maximum moduli of discrete closed-loop poles of the full system and the individual equivalent subsystems, respectively, are:

For the weights $\Pi_{y1} = 0.57$, $\Pi_{y2} = 1.58$:

$$|p|_{max} = 0.9861$$

$$|p_1|_{max} = 0.9882$$

$$|p_2|_{max} = 0.9872$$

For the weights $\Pi_{y1} = 0.85$, $\Pi_{y2} = 3.50$:

$$|p|_{max} = 0.9861$$

$$|p_1|_{max} = 0.9886$$

$$|p_2|_{max} = 0.9886$$

Obtained theoretical and experimental results prove that the local GPC controllers independently designed for each equivalent subsystem which constitute the resulting decentralized controller guarantee the closed-loop plant-wide stability.

5 Conclusion

In this paper a novel approach to the decentralized GPC controller design within the ESM design framework has been proposed. The main advantage of this approach is a diagonalization of the original plant by generating a diagonal matrix of equivalent subsystems. Thus, local predictive controllers of individual equivalent subsystems can be designed and tuned independently; stability of equivalent closed-loops guarantees plant wide stability. Important points in the design procedure are model identification from frequency responses of equivalent subsystems, and stability analysis based on polynomial control structure of the unconstrained GPC control algorithm (RST form). The proposed decentralized GPC design approach was verified in a real-world laboratory plant. Presented theoretical and experimental results have proven effectiveness of the proposed control design strategy.

The future research will deal with guaranteeing desired plant-wide performance by systematic tuning of the GPC design parameters (prediction horizons, weights), as well as with the problem of guaranteeing stability under the decentralized GPC with constraints.

Acknowledgements

The work has been supported by the Slovak Scientific Grant Agency of the Ministry of Education, Science, Research and Sports of the Slovak Republic, Grant No. 1/1241/12, and the Slovak Research and Development Agency, Grant No. APVV-0772-12.

References

Bucz, Š., Kozáková, A., Veselý, V. (2012). Easy tuning of PID controllers for specified performance. *IFAC Conference Advances in PID Control*, Brescia (Italy), paper No. 112.

Camacho E. F. and Bordons C. (2004). *Model Predictive Control*. - 2.ed. London: Springer Verlag.

Kozáková A., Veselý V., Osuský J. (2009). A new Nyquist-based technique for tuning robust decentralized controllers. *Kybernetika*, 45 (1), 63-83.

Landau, I.D. (1998). The R-S-T digital controller design and application. *Control Engineering Practice*, 6, 155-165.

Lewis, F.L. (1992). *Applied optimal control & estimation: digital design & implementation*. Texas Instruments Inc.

Linkens,D. A. and Mahfouf , M. (1992). *Multivariable generalised predictive control using feedforward (GPCF) for anaesthesia*. International Journal of Control, vol.56, no.5, pp.1039-1057.

Richards, A. and How. J. (2004). In *American Control Conference*, Boston, Massachussets.

Riverso, S., Farina, M. and J. Ferrari-Trecate (2013). Design of plug-and-play model predictive control: an approach based on linear programming. *52nd IEEE Conference on Decision and Control*, Florence, Italy.

Rosinová, D., Kozáková, A. (2012): Robust decentralized PID controller design. In *Introduction to PID controllers – theory, tuning and application to frontier areas,* Part 3 (Multivariable Systems – Automatic Tuning and Adaptation), Chapter 6, 133–168. InTech, Rijeka, Croatia.

Rossiter J. A. (2004). *Model-based predictive control: a practical approach*, CRC Press, 2004

Shah, G. and Engell S. (2010). Tuning MPC for desired closed-loop performance for SISO systems. *18th Mediterranean Conference on Control and Automation*. Marrakech, Morocco.

Veselý, V. and Osuský, J. (2013). Robust multivariable generalized predictive control design. *Int. Journal of Innovative Computing, Information and Control*, 9(8), 3377-3390.