Dynamic Stabilization and Control of Material Flows in Networks and its Relationship to Phase Synchronization

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***Abstract*— We study the self-organization and optimization of conflicting material flows on complex networks as it may take place in the case of vehicular traffic or the supply of goods in a production network. A decentralized control is used to approach a demand-driven switching between "on" and "off" states of the flow in a particular direction at intersections or merges represented by nodes in the corresponding networks. Whereas intrinsic oscillatory instabilities of material flows in networks usually have negative effects on the performance of the overall system, these self-organized oscillations allow to optimally use the available transportation capacity of the network. Under rather general conditions, our control approach leads to phase synchronization of the switching dynamics at the respective nodes, which is studied using a new framework for measuring the strength and homogeneity of frequency locking in networks of oscillatory components.**

I. Introduction

Network structures can be found in numerous complex systems in nature and society. In many of these networks, the flows of material or information between the respective nodes represent the essential dynamics of the system [1]. This applies in particular to traffic, production, telecommunication, supply, or biological networks [2][3]. However, such flows are typically characterized by a large number of interacting transportation processes. Therefore, the aim of an efficient organization and control of the network dynamics is to minimize the total amount of time required for all these processes. Typically, such an optimization is difficult and demanding, as the topology of the underlying networks is composed of a potentially large number of merges and intersections at which there are conflicts between the flows on different routes [4][5]. To avoid physical collisions, these flows have to be controlled by devices like traffic lights, which lead to an oscillatory switching between "service" and "no-service" states for different links at the respective node. The operation strategy of these devices is decisive for maximizing the system performance.

One traditional strategy for the optimization of material flows is a central control of the flows in the whole network. However, in large networks, there is a large amount of (particularly delayed) information which makes a central organization of all flows a complicated problem. In contrast to this, decentralized control strategies [5][6] are much more flexible to react to local changes of the current state of the system and may therefore lead to an even better performance than centralized ones.

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Unlike the controlled oscillations which are "triggered" by the action of traffic lights and similar devices, material flows in many real-world networks show already an intrinsic oscillatory behavior. There are different possible reasons for this kind of dynamical behavior, including a varying external demand or supply of material (forced oscillations) as well as symmetry breaking bifurcations or instabilities due to the presence of feedback loops [7] (self-sustained oscillations). In the latter case, the instabilities of the stationary system may have severe consequences for the overall performance of the entire network. The resulting oscillations may be further amplified by phenomena like the Bullwhip effect in production systems, which may finally lead to a highly irregular system behavior [7].

In this paper, we demonstrate how a suitable decentralized control can be used to reach controlled oscillations of material flows at the intersections within a transportation network. For this purpose, we study if the self-organized oscillations yield an optimal control of the material flows, and try to understand the importance of phase synchronization (a) between neighboring nodes and (b) within the entire network for an optimization of the overall capacity of the system. In Sec. 2, we present a basic model for a self-organized control of conflicting material flows. Possible approaches to phase synchronization analysis in complex networks are summarized in Sec. 3. In Sec. 4, we present some examples for the occurrence of frequency locking of the material flow dynamics in the presence of periodic external forcings. Subsequently, the occurrence of self-sustained phase synchronization phenomena in networks in the presence of a decentralized control is studied in Sec. 5. Finally, we discuss the general relationship between the self-organized optimization of material flows in networks and the occurrence of phase synchronization.

II. Self-Organized Network Flows

To describe the behavior of material flows at intersections, we assume that the effects of acceleration or deceleration near intersections are small enough to be negligible (corresponding to an adiabatic speed adjustment). Moreover, all links in the considered networks have an infinite (or at least sufficiently high) storage capacity, i.e., there is no possibility of complete congestion in our simplified model.

In this contribution, we will use the following notations: N_i is the amount of queued material on the incoming link

i. A_i and O_i are the respective actual arrival and departure flows at the upstream and downstream ends of the queue on this link, respectively. They are bounded by the maximum flow $\hat{Q}$. For simplicity, we assume all flows to be continuous (the consideration of discrete packages or vehicles [8] does not lead to a fundamentally different behavior under rather general flow conditions).

In the following, we will describe a basic model for the simplest case of two intersecting unidirectional links where the possibility of turning is not explicitly taken into account. The evolution of the different flows through an intersection is controlled by a permeability function γ whose shape and parameters are responsible for a switching between conditions where material flows are only possible in one of the different directions. This permeability function represents a traffic light in the case of traffic networks. In order to implement a demand-dependent self-organized control of the flows, the difference between the currently delayed material on both incoming links, $\Delta N = N_2 - N_1$, and the difference between the respective outflows, $\Delta O = O_2 - O_1$, are taken into account. Moreover, there has to be a sharp switching between conditions with $(\gamma_1, \gamma_2) = (1, 0)$ and $(0, 1)$ such that values of $\gamma_i \notin \{0, 1\}$ do practically not occur or are restricted to some short, deterministic time interval ("amber light").

Helbing *et al.* [5] have already proposed a permeability function which fulfills the above requirements and leads to a preference of those links on which either (a) a higher number of material is already accumulated or (b) a large outflow takes place. To further simplify the shape of the permeability function, one may use the following setting:

$$\gamma_1(t) = \begin{cases} 1, & E(t) > \Delta E/2 \\ 0, & E(t) < -\Delta E/2 \\ \left(\frac{E(t)}{\Delta E} + 1\right), & \text{else} \end{cases} \tag{1}$$

and $\gamma_2(t) = 1 - \gamma_1(t)$ where $E(t) = \Delta N(t) + \alpha \Delta O(t)$. Here, α determines the relative weight given to the current outflows with respect to the amount of waiting material, and ΔE is the width of the transition region which determines the switching period. If we are not interested in finite time intervals necessary for switching, we may further modify this setting such that a switching between the incoming links 1 and 2 (2 and 1) takes place immediately when $E(t)$ falls below the value of $-\Delta E/2$ (or rises above $\Delta E/2$). The resulting hysteresis effect (see Figure 1) gives rise to a certain inertia of switching, i.e., if the conditions on both links are comparable ($E(t) \approx 0$), there is a stronger tendency to remain in the current state than to switch into the other one.

With the definition of a suitable permeability function $\gamma_i(t)$ as described above, the temperal evolution of the amount of delayed material $N_i(t)$ and the outflow $O_i(t)$ are described by the following set of equations:

$$\frac{d}{dt} N_i(t) = A_i(t) - O_i(t) \tag{2}$$

$$O_i(t) = \gamma_i(t) \ \max\left\{A_i, \ \hat{Q} \ \Theta(N_i)\right\}, \tag{3}$$

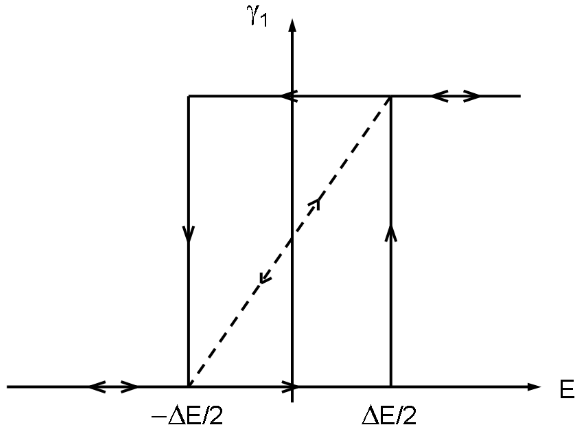

Fig. 1. Self-organized switching between two intersecting material flows on link 1 (permeability function $\gamma_1(t) = 1, \ \gamma_2(t) = 0$) and 2 ($\gamma_1(t) = 0, \ \gamma_2(t) = 1$) in dependence on the evolution of the quantity $E(t)$ which balances the differences between the actual outflows $\Delta O(t)$ and the total amounts of waiting material $\Delta N(t)$ on each link.

where $\Theta(\cdot)$ is the Heavyside function.

In the case of bidirectional traffic or more than two intersecting pairs of links, the above formalism can be easily generalized. For this purpose, the terms in ΔN and ΔO have to be replaced by sums over the respective values for all incoming links for which the current permeability is 0 or 1, respectively.

III. Detection and Quantification of Phase Synchronization in Networks

A variety of different measures can be computed to detect signatures of phase synchronization or, more general, phase coherence between *two* oscillating systems j and k [9][10], including the standard deviation and normalized Shannon entropy of phase differences [11], the mutual information between both phases [12], and the mean resultant length $r_{jk} = \left|\left\langle e^{i\Delta\phi_{jk}(t)}\right\rangle_t\right|$. Among these quantities, r_{jk} is the probably most important parameter as a measure of the dispersion of phase differences originated in circular statistics. All approaches however quantify "only" the *mutual* synchronization of *pairs* of oscillators, whereas a characterization of the *joint* synchronization of an *ensemble* of $N > 2$ oscillating systems requires that the states of all components are taken into account.

As an alternative to the consideration of averages of bivariate synchronization measures, the concept of synchronization cluster analysis (SCA) has been recently introduced [13][14][15]. In this framework, subsets of oscillating systems are interpreted as a synchronization cluster if they exhibit a higher mean phase coherence between each other than with the remaining oscillators. The original SCA approach assumes the existence of only a single cluster whose strength r_C can be iteratively computed [13][14], which however may not be the case for complex networks. A subsequent extention of SCA which takes this problem into account analyzes the eigenvalues λ_j and eigenvectors $\vec{v}_j$ of the complete matrix $R = (r_{jk})$ of pairwise synchronization indices [15]. In this generalized SCA, the number of eigenvalues $\lambda_i > 1$, N_c, serves as an indicator for the number of synchronization clusters, whereas their actual values can be interpreted as the respective cluster strengths. This interpretation however leads to the conceptual problem that even in a non-synchronized system, the method detects a number of synchronization clusters although the notion of a synchronization cluster is rather meaningless here.

Whereas SCA characterizes phase synchronization which is by definition heterogeneously distributed, the use of averages of bivariate synchronization indices corresponds to the assumption of a homogeneous distribution. In order to distinguish between these two cases, the eigenvalue decomposition approach [15] may be extended by a subsequent analysis of the whole range of eigenvalues of the bivariate synchronization matrix R. For this purpose, the LVD (linear variance decay) dimension density may be considered which quantifies the average degree of collectiveness in the behavior of a multi-component system [16]. Instead of the eigenvalues λ_j themselves (which are normalized here to unit sum), the LVD dimension approach models the decay of the associated remaining variances $\nu_j = 1 - \sum_{k=1}^{j-1} \lambda_k$ (up to a given fraction of the total variance) by an exponential function $\nu_j = e^{-j/\delta_{LVD}N}$. The coefficient δ_{LVD} of this model measures the collectiveness of the dynamics, i.e., the strength and homogeneity of phase synchronization.

IV. Frequency Locking by Periodic Demands

In the case of vehicular traffic controlled by a traffic light, the outflows $O_i(t)$ on the different links are a periodic function of the phase angle of the corresponding permeability function with a period T_s [6]. If two or more intersections form a traffic network, this periodicity also holds for the inflows $A_i(t)$ into the queues at the neighboring intersections. Consequently, for a single crossing of two material flows, this specific situation can be modeled by a suitable periodic modulation of the different inflows $A_i(t)$. Lämmer *et al.* [6] have already suggested that in this situation, a suitable decentralized control can be used to achieve phase synchronization of the switching at different intersections within a traffic network. In the following, we will study two practical examples for implementations of such decentralized control strategies. It will be shown that in the cases considered, periodic variations of the external demand lead to frequency locking phenomena, which may be understood as a phenomenon similar to phase synchronization in the presence of a unidirectional coupling.

Let us first study the case of material flows across an intersection as one part of a traffic system in the framework of our model described in Sec. 2. For simplicity, we will assume that the inflow in one direction is constant in time, whereas that in the second direction varies periodically. As it is shown in Figure 2, for some distinguished interval of modulation period T, the switching period T_s of the self-organized control locks to this external demand period in a

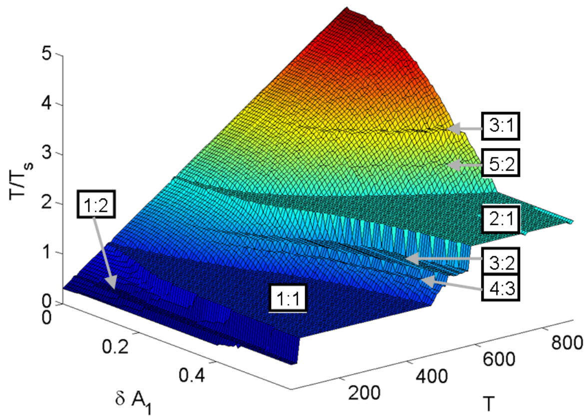

Fig. 2. Locking of the self-organized switching period T_s to the modulation period T as a function of the amplitude δA_1 of the sinusoidal modulation of $A_1(t) = \langle A_1 \rangle (1 + \delta A_1 \sin(2\pi t/T))$ for average incoming flows $\langle A_1 \rangle = 0.5$ and $A_2(t) \equiv 0.3$. A similar behavior can be observed for other parameter combinations as well as for an inflow $A_1(t)$ which is periodically switched on and off.

1:1 way. If the amplitude of modulation increases, the width of this locking window increases as well. Moreover, there are other windows of non-trivial n:m frequency locking for which a similar behavior can be found. The detailed position and width of these locking intervals is determined by the parameters α and ΔE of the permeability function $\gamma_i(t)$ and the average inflows $\langle A_1 \rangle$ and $\langle A_2 \rangle$. In general, the choice of these parameters yields a naturally preferred switching frequency in the case of constant inflows, whose existence gives rise to the non-trivial locking intervals.

The material flows in production networks or, more general, spatially extended logistic systems, may show some structural similarities to flows of vehicular traffic. However, there is one fundamental difference: In the case of traffic networks, the demand for service at one intersection is determined by the "production" of flow at the neighboring intersections, which corresponds to a push system in the context of production and logistics. In contrast to this, the action of networks of production and logistics can often be described as a pull system where the demand at one node itself initiates inflows by ordering the corresponding goods. Hence, whereas the basic structure of traffic and production networks may be similar, the dynamics on these networks may differ considerably.

Nonetheless, frequency locking phenomena of the flows at different nodes can also take place in production networks with a periodic demand or supply. Scholz-Reiter *et al.* [17][18] studied a model in which the production rate Q of manufacturers is successively adjusted to a variable optimum value which is determined by the current demand and the amount of available products. It has been shown that in this model, a temporally varying demand and a finite adaptation time τ^{-1} lead to different lockings between production rate and demand, which depend on the respective values of τ.

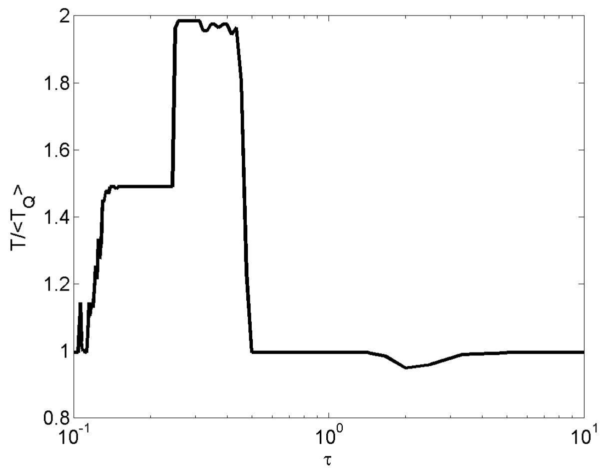

Fig. 3. Locking of the mean period $\langle T_Q \rangle$ of the production rate Q to the period T of the external demand oscillations in dependence of the adaptation rate τ. The model and the specific parameter setting are described in more detail in [17][18]. In particular, the values $\tau = 0.2$ and $\tau = 1$ lead to a 3:2 and 1:1 locking, respectively, as already found by Scholz-Reiter *et al.*. In between, there is a range of adaptation times which lead to a 2:1 locking.

For large values of τ, the production rates of downstream producers are adjusted fast enough such that there will be no significant shortage of goods. As a consequence, the production rates of all manufacturers oscillate with the same frequency, which equals the frequency of the external market demand. Decreasing the adaptation rate τ causes shortages, which may lead to a doubling of the oscillation frequency of the production rates (2:1 locking, see Figure 3). If τ is further decreased, this shortage becomes increasingly significant, such that the production rate is dramatically reduced. Since this case corresponds to a very slow adaptation, it will take a long time until Q reaches its average level again. Because of the reduced consumption of this producer, shortages will occur less frequently, which leads firstly to a 3:2 locking and, for even smaller values of τ, to a 1:1 locking again.

V. Phase Synchronization due to Self-Organization of Material Flows

All phenomena described in the previous sections do not correspond to phase synchronization, as the observed frequency locking is only due to a periodic external forcing, but does not refer to two distinguished self-sustained oscillators in the system. In the following, we will see that the self-organized switching between flows in different directions in our material flow model gives rise to similar locking phenomena, even if the corresponding actual demand for transportation capacity is assumed to be constant in time.

In order to investigate successive frequency adaptation and locking in traffic networks, Lämmer *et al.* [6] studied a paradigmatic model, in which the oscillatory switching of the nodes was represented by a set of locally coupled phase oscillators whose frequencies we adapted themselves up to a maximum value given by the minimum cycle time. If a network of such oscillators is considered, there is indeed a successive adjustment of the oscillation frequencies of the different nodes which finally leads to a state with a complete frequency locking among all oscillators in the network. Hence, this phase synchronization leads to a decentralized coordination of the individual switching on a network-wide scale. However, there is still no proof that the dynamics of the phase oscillator model with adaptive frequencies gives a proper representation of the self-organized switching of material flows on a corresponding network. In the following, we will therefore study how such a system would behave in the case of the decentralized control strategy presented in Sec. 2.

In order to keep the number of parameters of the analyzed model in a suitable range, we make some simplifications:

1) We study a regular grid network, which consists only of nodes of degree $k = 4$ (i.e., with four in- and outgoing links which connect neighboring pairs of nodes).
2) The distance between neighboring intersections is kept constant, which means that the travel time t_d between each pair of connected nodes is assumed to be the same for all links without considering effects due to the presence of a queue.
3) Left-hand turning is forbidden, whereas a fraction p of the flow can turn to the right. For simplicity, this fraction is taken to be the same for all links.
4) The sum of the maximum inflows into the two possible directions of the intersecting roads is sufficiently smaller than the maximum link capacity. This prerequisite is necessary to avoid an unlimited congestion of the links in the considered network[1].

With these assumptions, the dynamics of the queue at the downstream end of link i is determined by the following equations:

$$\begin{aligned} \frac{dN_i}{dt} &= A_i(t) - O_i(t) && (4)\\ A_i(t) &= \sum_{j \neq i} \alpha_{ji} O_j(t - t_d) && (5)\\ O_i(t) &= \gamma_i(t) \max\{A_i(t), \hat{Q}\Theta(N_i(t))\}, && (6) \end{aligned}$$

where α_{ji} is the fraction of the flow on link j which is turning to link i ($\alpha_{ji} = p$ for right-hand turning, $\alpha_{ji} = 0$ for left-hand-turning, and $\alpha_{ji} = 1 - p$ otherwise).

In order to evaluate the presence of phase coherence within the considered ensemble of nodes with the methods described in Sec. 3, an appropriate definition of a phase variable is necessary. Without loss of generality, we define the initial phase $\phi_j = 0$ for node j corresponding to the time of the first switching of its permeability function. In a similar way, $\phi_j = (n - 1)\pi$ then corresponds to the time of the n-st switching at this node. Between these switching times, the

[1]For convenience, we start all simulations with a state where there is no flow on the links connecting the nodes of the networks. All flows are induced by an external supply of material to every node at the boundary of the network. This setting leads to a slow synchronization process, which can therefore be studied more easily.

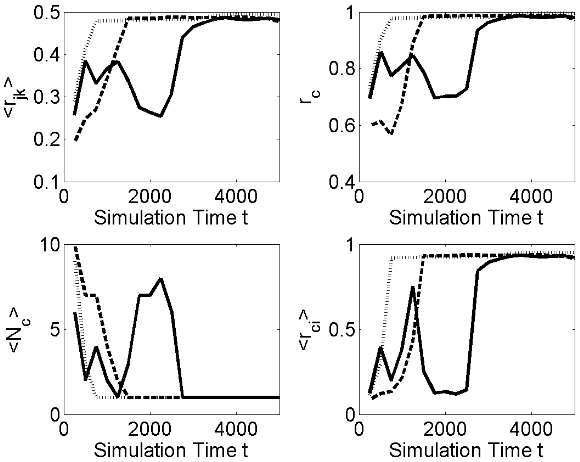

Fig. 4. Multivariate phase synchronization indicators calculated from the switching intervals at the respective nodes in a regular 5×5 grid network for disjoint windows of 250 time steps obtained from one simulation: average value of pairwise mean resultant lengths $\langle r_{ij} \rangle$, cluster strength r_c, number of clusters N_c, and average cluster strength $\langle r_{ci} \rangle$ (normalized by the total number of nodes). The maximum link capacity has been set to $\hat{Q} = 1.0$ for all roads in the network with travel times $t_d = 30$ and turning probabilities $p = 0.05$. The external inflows have been initially taken from a normal distribution with mean $\langle A \rangle = 0.3$ and $\sigma_A = 0.05$ and then kept constant over the entire simulation. The permeabilities have been randomly initialized. For the parameters of the function $E(t)$ which determines the permeability (see Eq. (1)), we have used $\alpha = 0.1$ and $\Delta E = 10$ (solid lines), 20 (dashed lines), and 50 (dotted lines). The behavior of the different measures clearly indicates a successive transition towards a state with a significant degree of phase synchronization.

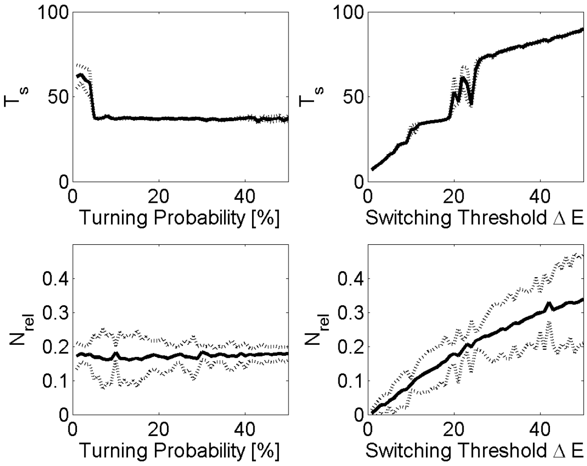

Fig. 5. Dependence of the asymptotic switching period T_s (upper panels) and the relative amount of waiting material N_{rel} (defined as the ratio of the absolute amount of queued material and the total amount of material within the network, lower panels) on the turning probability p (left, $\Delta E = 20$) and the width of the transition region ΔE (right, $p = 0.05$) for the same network as in Fig. 4. Dotted lines indicate error bars corresponding to the standard deviation of T_s taken over all nodes of the network ($\langle T_s \rangle \pm \sigma_{T_s}$) and the temporal variance of N_{rel} ($\langle N_{rel} \rangle \pm \sigma_{N_{rel}}$), respectively.

phase variable is defined by linear interpolation. Although this definition leads to an increase of the phase which may be periodically modulated if the "on" and "off" times for one specific direction are not symmetric, in the long-term limit, these variables may be used for a phase synchronization analysis.

In Figure 4, the transition from random initial conditions to a phase synchronized network is illustrated for a regular symmetric grid with 25 intersections. After a suitably long time, the network reaches a state for which the different multivariate phase synchronization indices become almost stationary. This behavior indicates that the permeabilities of all intersections switch with almost the same frequency, which corresponds to a phase-synchronized system. A detailed inspection shows that the corresponding transition is accompanied by a successive decrease of the relative amount of waiting material within the network, whose average value becomes minimal when the synchronized state is reached. (Note, however, that this quantity still oscillates with the period of the self-organized switching.)

In addition to these general findings, Figure 4 suggests that the parameter ΔE has a considerable influence on the time scale required for synchronizing the network: For low values, the corresponding transition may be discontinuous and requires much more time than for high parameter values. The detailed behavior of the synchronization indicators suggests that there is already a certain initial degree of synchronization from the beginning of the simulation, which is due to the fact that an initial switching period is prescribed by the travel time t_d between each pair of intersections, while there are no initial flows in the interior of the network. The specific setting of a regular grid with symmetric distances also leads to a spatially homogeneous synchronization process, which is reflected by almost constant values of the LVD dimension density δ_{LVD} (not shown in Figure 4) and the fact that the average bivariate phase synchronization indices and the measures from synchronization cluster analysis show an almost identical behavior.

In order to achieve a better understanding of the factors which determine the performance of our control (for example, the relative amount of waiting material), the degree of phase synchronization in the system, and the average switching frequencies, we have investigated the influence of the turning probability p and the width of the transition region ΔE on the corresponding quantities. On the one hand, no systematic influence of both parameters on the asymptotic values of the phase sychronization indicators was found. On the other hand, the average switching period shows a clear dependence on both p and ΔE, which is illustrated in Figure 5, whereas the relative amount of waiting material depends only on ΔE in a monotonously increasing way.

Concerning the average switching period, one can clearly distinguish two regimes in dependence on the turning probability: If only very few vehicles turn to the right, the periodicity is rather large (regime I), whereas there is a sharp transition towards significantly smaller values of T_s (regime II), if the turning probability p exceeds a certain threshold value. In this regime II, the actual value of p does not play an important role for the switching period. It is likely that the detailed location of the transition point depends on the other parameters of the considered system as well, in particular, ΔE, $\langle A \rangle$, and t_d. At least the first dependence is reflected by the upper right panel of Figure 5: Whereas one may expect a continuous increase of the switching period T_s with ΔE, for a non-vanishing value of the turning probability, there are some rather abrupt transitions whose exact location and sharpness may vary between different simulation runs. In particular, for $\Delta E \approx 20 \ldots 25$, the jump from values between 35 and 40 to values of above 60 does probably reflect the transition from regime II to regime I. The dynamical mechanism of this transition and a detailed determination of the corresponding stability boundary will be subject to future studies.

VI. Conclusions

We have studied a decentralized control approach for the regulation of intersecting material flows in complex networks, which occur in many real-world systems like vehicular traffic or production systems. In particular, in the case of traffic networks, our approach may be considered to represent a self-organized traffic light control which allows a large throughput with a relatively low amount of queued material. We have demonstrated that the self-organized switching of the permeability of the respective directions at the different intersections leads to a phase synchronized dynamics, which supports recent suggestions [6].

It has to be underlined that our self-organized control does not require any periodicity of the external demand or supply to reach a state with phase synchronized switching of the permeabilities. However, it has still to be studied whether a comparable degree of phase synchronization can still be achieved in the case of spatially heterogeneous model parameters (like varying travel times or turning probabilities), an instationary external supply of (or demand for) material, or an irregular network. In general, even under such more realistic conditions, our decentralized control may allow to optimize the flows in a way that maximizes the degree of spatio-temporal organization.

In the presented study, we have focused our attention to a proof of the presence of phase synchronization in decentrally controlled material flows. However, several open questions are still remaining. For example, if the studied model is generalized to asymmetric grid networks where the travel times t_d are not the same on every link, the spatial heterogeneity might play a crucial role for the possibility of synchronization and the optimization of material flows. In addition to the already mentioned phase transition between two different switching regimes, the dependence of (i) the time required to approach the synchronized state and (ii) the asymptotic values of the phase synchronization measures on the size of the network and the distribution of the inflows should be further studied to determine the respective influences of all relevant model parameters. Finally, when trying to adapt our results to more general networks, where some of the nodes have a degree of $k > 4$, different strategies are probably required to schedule and optimize the flows in the different possible directions.

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