Shell Model of Open-Loop Optical Control for Atomic Beam Focusing in Momentum Space

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***Abstract*— We propose the simple shell model to describe the focusing of cool atomic beam interacting with open-loop modulated optical field.**
This model can be apply to form efficient splitting effect in momentum space.

I. Introduction

In nano-lithography the optical control of atomic motion is one of the main problems. In principle, we can make such a control for atom dynamics because there is an exchange of momentum between the atoms and the optical fields. The momentum exchange can be created with practical devices: atomic mirrors and atomic beam splitters which are main elements of atomic interferometer. The most interesting possibility here is to obtain the splitting of the initial atomic wave packet coherently into two main momentum components only by controllable way. It is needed both for increasing of atomic interferometer sensitivity and for the creation of periodic nano structures by atomic wave packet lithography [1].

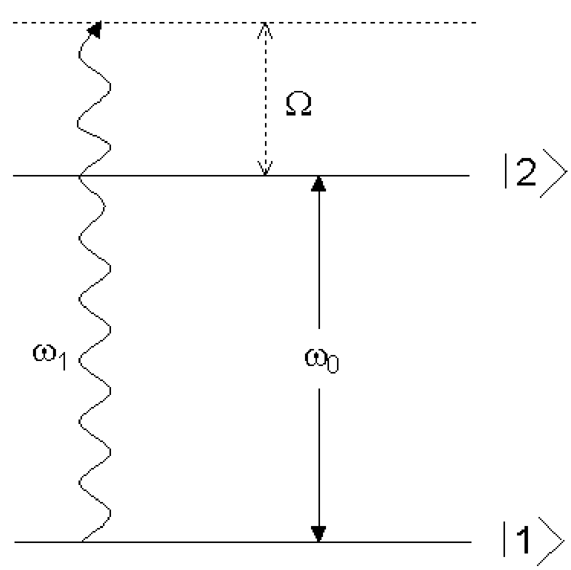
The principal opportunity to split the beam in the momentum space was demonstrated in [2], [3]. To achieve the effective splitting, we apply the scheme of open-loop control, or feed-forward control, i.e. a control signal depends only on the time. Our control goal is to obtain the large angle splitting for the initial wave packet after some time of the interaction between the atoms and the field of modulated standing wave.

II. Physical Background and Mathematical Model for Beam Splitting in Momentum Space

Atom lithography is an active field now a days. The resolution of an optical lithography technology is limited by diffraction, which for the case of deep ultraviolet light approaches 200 nm. The progress of recent device technology requires smaller patterning of 10 nm size. However, when one tries to make very small devices, the resolution of the resistance is limited by the spread of the secondary electron in an electron beam lithography, as well as in X-ray lithography.

The ability to generate ultracold atoms using lasers has opened up new possibilities. The long de Broglie wave of cold atoms makes possible an interferometric manipulation with atomic wave packets, which is designed by an optical standing wave. In this case, atoms can be controlled directly to form a desired pattern. To produce the pattern with high resolution, we need to split the wave packet into two coherent momentum components only. For the model with only two states (i.e. an approximation of two level atom states), we have to split the population of the lower state (because the population of the exited state usually loses the coherency very fast by spontaneous decay) in several momentum components (in an ideal case – only two). At the same time to form the pattern with small step we need to control the scale of splitting between two main coherent components in momentum space. Therefore, an atomic beam splitter is the main element for the practical realization of nano-scale lithography with the controlled step by coherent scattering of an atomic wave packet.

Let's consider now a two level atom in a far detuned standing wave with the intensity modulated in time as $I = I_0 f(\varepsilon, \Delta \cdot t, \phi_0)\cos(kx)$, where ε is an amplitude, Δ is the frequency of the modulation, and ϕ_0 is an initial modulation phase.



The standing wave with the frequency ω_1 applies between two states of atom system, where the state 1 is ground and the state 2 is the exited one. Here, ω_0 is the frequency of atom transition and the difference $\omega_1 - \omega_0$ is the detuning.

We will assume that the beam from an atom source propagates along z-axis and crosses the optical wave, standing along x-axis, by right angle. The spontaneous emission from

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the upper level in this system can not be neglected. After some time t of the interactions between the atoms and the field of the standing wave, the initial atomic wave packet is spllited in few coherent momentum components.

Dynamics of the atom in the modulated standing wave is described with non-stationary Schroedinger equation for the wave function $\Psi(r,t)$ of the two level atom:

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t} = \hat{H}\Psi(r,t), \tag{1}$$

where $\hat{H}$ is a Hamiltonian which takes into account both the atom movement along the standing wave and the dipole interaction between the atom and the optical field. For sufficiently large detuning, when it is much larger than Rabi frequency and the natural width of the atomic transition, $\Omega >> R_0, \Gamma$ (where R_0 is the Rabi frequency, Γ is the natural width of the atomic transition), the excited state 2 can be adiabatically eliminated. As a result, we obtain the equation for the amplitude of the probability of the ground state $\Psi_1(x,t)$:

$$\begin{aligned} i\hbar\frac{\partial\Psi_1(x,t)}{\partial t} &= -\frac{\hbar^2}{2m}\triangle_{xx}\Psi_1(x,t) + \\ &+\frac{R_0^2}{\Omega}[f(\varepsilon,\Delta\cdot t,\phi_0)]^2\cos^2(kx)\Psi_1(x,t)\ , \end{aligned} \tag{2}$$

where m is the atom mass.

After the Fourier transform the same equation in the momentum space is given by:

$$\begin{aligned} i\frac{\partial\Psi_1(p,\tau)}{\partial\tau} &= (p^2+R^2)\Psi_1(p,\tau) + \\ &+\frac{R^2}{2}\left[\Psi_1(p+2,\tau)+\Psi_1(p-2,\tau)\right]\ , \end{aligned} \tag{3}$$

where

$$R^2 = \frac{R_0^2}{2\Omega\omega_R}\left[f(\varepsilon,(\Delta/\omega_R)\cdot\tau,\phi_0)\right]^2$$

$\omega_R = \hbar k^2/2m$ is called a recoil frequency, $\tau = \omega_R t$.
Here we normalised atom momentum along x-axis to $\hbar k$ and the other quantities (the interaction time, the Rabi frequency and the detuning) we normalised to the recoil frequency ω_R. We have to point out that equations (2)-(3) are valid in the approximation when both the changing of atom momentum along z-axis and the initial value along x-axis can be neglected.

III. Shell Model for the Splitting Process: Parametric Control

To explain the effect of splitting in the momentum space we start from the case of parametric control with a constant R. We invent a complex shell model for Ψ_1-function. Initially the atomic beam has a Gaussian distribution centered at $p = 0$. Thus, from the structure of the RHS (3) we can expect the non-zero meanings of Ψ_1 functions to be concentrated in the neighbourhoods of the points $p = 2n$, where $n = 0, \pm1, \pm2, \ldots$. Then we can predict the continuous dependency on p with the discrete number n:

$$\Psi_1(p+2n,\tau) = y_n(\tau).$$

Dynamics Eq. (3) can be re-written in the form:

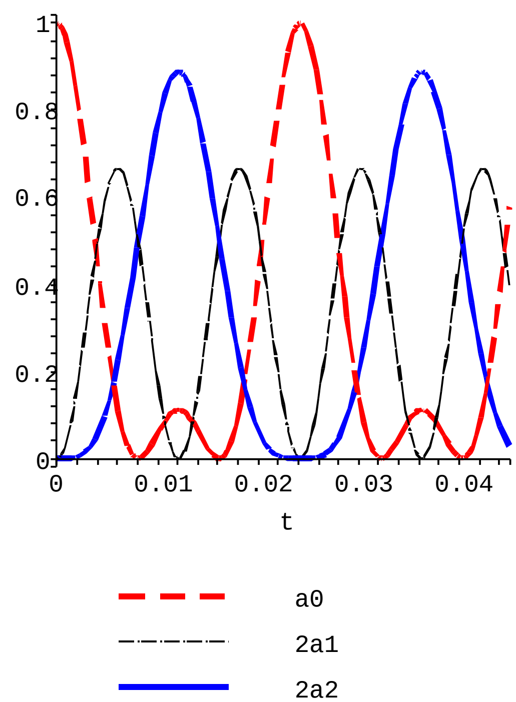
$$\begin{aligned} i\frac{dy_n(\tau)}{d\tau} &= (4n^2+R^2)y_n(\tau) + \\ &+\frac{R^2}{2}\left[y_{n-1}(\tau)+y_{n+1}(\tau)\right] \end{aligned} \tag{4}$$

with the initial conditions: $y_0(0) = 1\ \ ;\ \ y_{n\neq 0}(0) = 0$.

Now we want to limit our shell number. In the case of five shells, we will omit the coefficients 4 and 16 in RHS of Eq.(4), because the numerical meaning of R^2 is about 300 (i.e. $R^2 >> 4$ and 16), and we have the following system of equations.

$$\begin{aligned} \iota\frac{dy_\circ(\tau)}{d\tau} &= R^2\Big(y_\circ(\tau)+y_1(\tau)\Big) \\ \iota\frac{dy_1(\tau)}{d\tau} &= R^2\Big(y_1(\tau)+\frac{y_\circ(\tau)}{2}+\frac{y_2(\tau)}{2}\Big) \\ \iota\frac{dy_2(\tau)}{d\tau} &= R^2\Big(y_2(\tau)+\frac{y_1(\tau)}{2}\Big) \end{aligned} \tag{5}$$

with initial conditions: $y_\circ(0) = 1,\ \ y_1(0) = y_2(0) = 0$.
We demand for the elder shells: $y_{\pm3} = y_{\pm4} = \ldots\ldots \equiv 0$, for any moment τ.
The solution of system (5) for constant R is given in [3].
The splitting effect for the model of 5 shells in the case of constant R is shown in the following figure.



However, if the number n of a shell is increased such that $4n^2 >> R^2$ (i.e. for $R \simeq \sqrt{300}$ we have $n >> 10$), then R^2 in (4) can be excluded as a small parameter, and for the elder shells

$$i\frac{dy_n(\tau)}{d\tau} \simeq 4n^2 y_n(\tau) \quad (n >> 10)\ . \tag{6}$$

This function is almost independent of the neighbour shells and it has the solution

$$y_n(\tau) \simeq \mathrm{e}^{-4in^2\tau} y_n(0) \quad . \tag{7}$$

But $y_n(0) = 0$ for any $n \neq 0$, thus, the elder shells do not participate in the re-distribution of the initial Gaussian population. Thus, the simple parametric control with the fixed R is not enough to split the beam efficiently. Another scheme of time-dependent R (corresponding to the most general open-loop control) should be applied.

IV. Shell Model for the Splitting Process: Time Dependent R

Now we solve the system (5), and the solution is:

$$y_\circ(\tau) = e^{-\iota\theta}\Big[\frac{1}{3} + \frac{2}{3}cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]$$
$$y_1(\tau) = \frac{-\iota}{\sqrt{3}} e^{-\iota\theta}\Big[sin\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]$$
$$y_2(\tau) = e^{-\iota\theta}\Big[\frac{-1}{3} + \frac{1}{3}cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]$$

where

$$\theta = \int R^2(\tau) d\tau \tag{8}$$

The corresponding population amplitudes of the shells 0 , ± 1 and ± 2 are given by:

$$\begin{aligned} a_\circ(\tau) &= y_\circ(\tau) y_\circ^*(\tau) = \frac{1}{9} + \frac{4}{9}\Big[cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]^2 + \\ &+ \frac{4}{9}\Big[cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big] \\ a_1(\tau) &= y_1(\tau) y_1^*(\tau) = \frac{1}{3}\Big[sin\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]^2 \\ a_2(\tau) &= y_2(\tau) y_2^*(\tau) = \frac{1}{9} + \frac{1}{9}\Big[cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big]^2 - \\ &- \frac{2}{9}\Big[cos\Big(\frac{\sqrt{3}}{2}\theta\Big)\Big] \end{aligned} \tag{9}$$

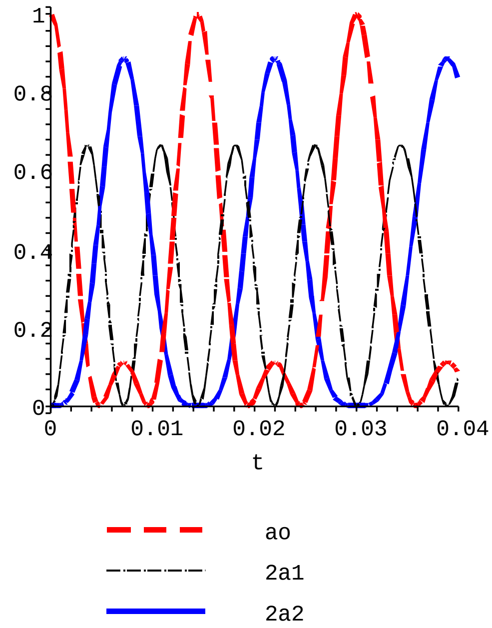
Surely, the normalization

$$a_\circ + a_{-1} + a_{+1} + a_{-2} + a_{+2} = a_\circ + 2a_1 + 2a_2 = 1$$

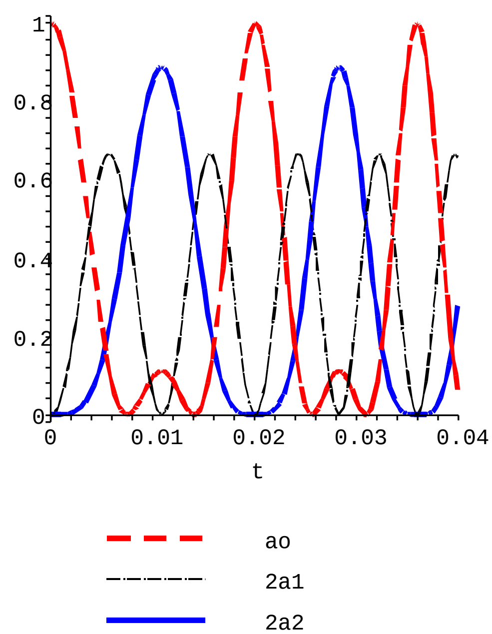
is satisfied for any moment t.

V. Different Values of R(τ)

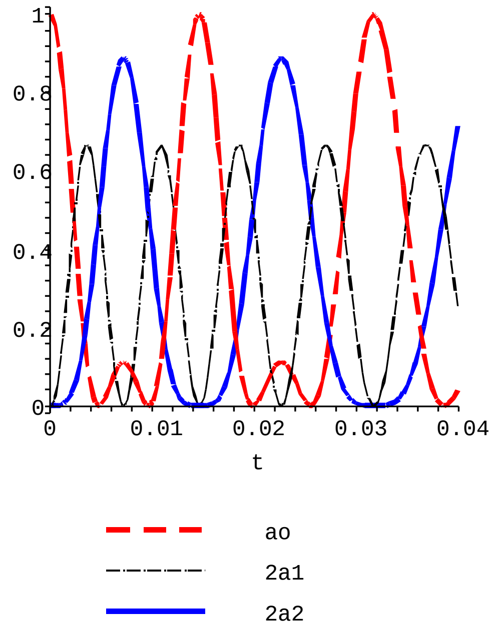
Now we will investigate different cases for different values of R(τ).
We present the typical behavior of the model for the case $R(\tau)^2 = R_\circ^2(1 + \epsilon cos(\Delta\tau))$ in the figure,
where $R_\circ \approx \sqrt{300}$, $\epsilon \approx 0.7$, $\Delta \approx 30$.



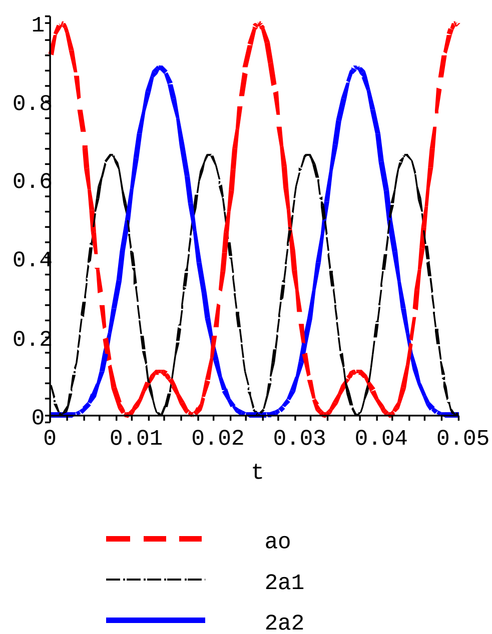
The behavior of the model for the case
$R(\tau)^2 = R_\circ^2(1 + \epsilon sin(\Delta\tau))$
is shown in the following figure,
where $R_\circ \approx \sqrt{300}$, $\epsilon \approx 0.7$, $\Delta \approx 30$.



The behavior of the model for the case
$R(\tau)^2 = R_\circ^2(1 + \epsilon cos^2(\Delta\tau))$
is shown in the following figure,
where $R_\circ \approx \sqrt{300}$, $\epsilon \approx 0.7$, $\Delta \approx 30$.

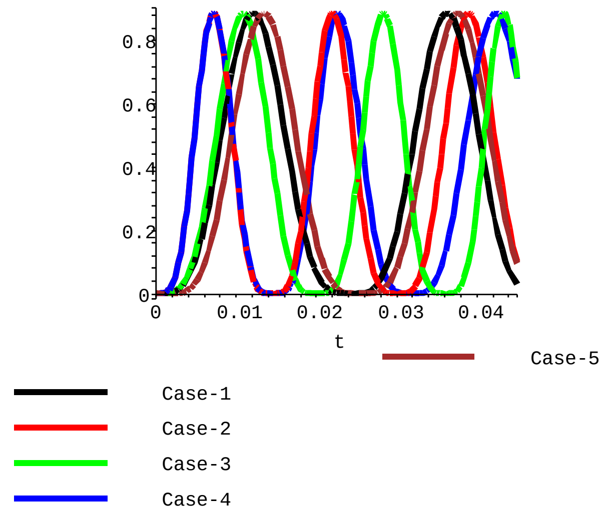


The behavior of the model for the case
$R(\tau)^2 = R_\circ^2(1 + \epsilon sign(cos(\Delta\tau)))$;
is shown in the following figure,
where $R_\circ \approx \sqrt{300}$, $\epsilon \approx 0.7$, $\Delta \approx 30$.



The following figure demonstrates the influence of the shape and the initial phase of the splitting effect for the following cases, when
Case-1: Parametric control (when $R = R_\circ$ is constant);
Case-2: $R(\tau)^2 = R_\circ^2(1 + \epsilon cos(\Delta\tau))$;
Case-3: $R(\tau)^2 = R_\circ^2(1 + \epsilon sin(\Delta\tau))$;
Case-4: $R(\tau)^2 = R_\circ^2(1 + \epsilon cos^2(\Delta\tau))$;
Case-5: $R(\tau)^2 = R_\circ^2(1 + \epsilon sign(cos(\Delta\tau)))$;
where $R_\circ \approx \sqrt{300}$, $\epsilon \approx 0.7$, $\Delta \approx 30$.

Thus the splitting effect is shown in the following figure, which contains the elder shells of all the cases.



VI. Numerical simulation results for open-loop control with harmonical modulation

Now let us consider the two level atom in a far detuned standing wave with an intensity, which is modulated in time harmonically as

$$I = I_0(1 + \varepsilon\cos(\Delta t))^2\cos^2(kx),$$

where ε is the amplitude and Δ is the frequency of the modulation.

We assume also that an initial wave function $\Psi_1(p, \tau = 0)$ has Gaussian profile with the width δp:

$$\Psi_1(p, \tau = 0) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{p^2}{(\delta p)^2}\right] . \tag{10}$$

We remind that now we use the dimensionless time $\tau = \omega_R t$.

The following figures show the numerical solution of an equation for amplitude of the probability of ground state $|\Psi_1(p, \tau)|^2$ in momentum representation for the cases unmodulated and modulated standing wave. We assume that initial wave packet has the width equals $\delta p = 0.5\hbar k$ and $\varepsilon = 0.8$, $\Delta/\omega_R = 29$. As we can see from these pictures, the scattering result strongly depends on the amplitude modulation existing in this system. If for an unmodulated case, it is well-known scattering picture observed (Fig. a), when an initial wave packet is splitted into a number of momentum components.

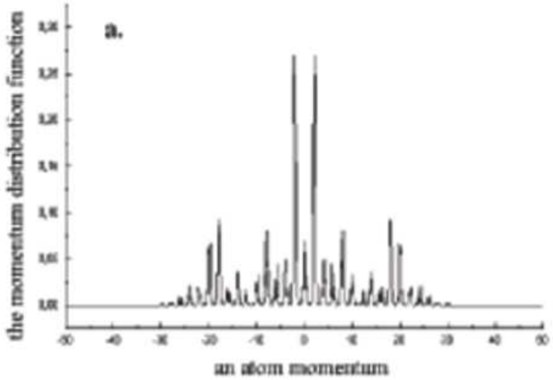


FIGURE . The dependence of the distribution function on an atom momentum for an interaction time $\tau_{\rm int} = 0.567$. The unmodulated standing wave with the dimensionless Rabi frequency $R_0 = (320)^{1/2}$.

However, for modulated standing wave the scattering picture is changing dramatically and two main momentum components centered on $\pm 40\hbar k$ can be observed (Fig. b).

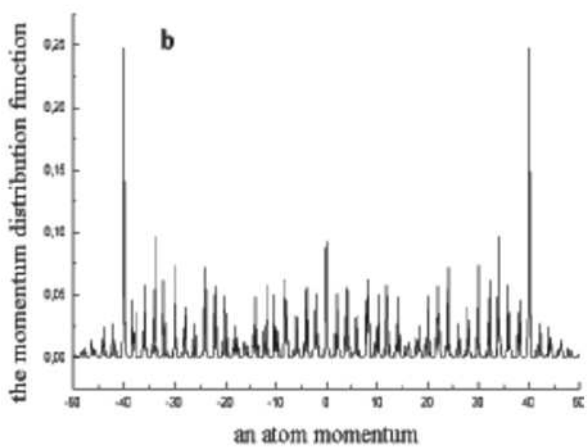


FIGURE . The dependence of the distribution function on an atom momentum for an interaction time $\tau_{\rm int} = 0.567$. The modulated standing wave with the dimensionless Rabi frequency $R_0 = (280)^{1/2}$.

Such behaviour of the momentum components is due to specific parametric resonance, which occurs in this system by the well defined amplitude and frequency modulation.

VII. CONCLUSIONS

In this paper we concentrated on the possibility to split an atomic wave packet in standing wave with modulated amplitude because this beam-splitter has a number of advantages by comparing with others.

The first one is the simplicity for an experimental realization because it is quite easy to obtain the time modulation of intensity with any shape.

The second advantage is that the scale splitting of an atomic wave packet can be controlled by changing the values of both an amplitude and the frequency of the modulation.

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