# HYSTERESIS MODELING OF A CLASS OF RC-OTA HYSTERETIC-CHAOTIC GENERATORS

**Leonardo Acho, Yolanda Vidal**
CoDAlab, Departament de Matemàtica Aplicada III,
Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona,
Universitat Politècnica de Catalunya,
Comte d'Urgell,187, 08036 Barcelona, Spain
Email: {leonardo.acho,yolanda.vidal}@upc.edu

**Abstract**
A class of RC-OTA hysteretic-chaotic generators has been proposed from the electronics point of view. Hysteresis is captured by an electronic realization. To extend its applicability, hysteretic mathematical modeling is an important issue. Here, a new hysteretic model is proposed. With this model, we realized a well known hysteretic-chaotic attractor using Simnon.

**Key words**
Hysteresis; chaos; modeling.

## 1 Introduction

During recent years, many chaotic generators have been proposed. Some of them are based on hysteresis feedback [Storace, and Parodi,1998], [Nakagawa, and Saito, 1996]. However, these oscillators have been designed using electronic realization. Nevertheless, hysteresis modeling is an important issue in mechanical and structural systems [Ismail, Ikhouane, and Rodellar,2009]. Some hysteresis models are developed using physical laws. Meanwhile, others are heuristic ones. Moreover, Ref. [Li, and Meng, 2007] reported chaotic behavior in structures with hysteresis, in which hysteresis is governed by the well-known Bouc-Wen model. However, this Bouc-Wen model is not appropriate for the class of RC-OTA chaotic oscillators because it has more parameters than needed. Here, we propose a new dynamic-hysteretic model that is appropriate for capturing the hysteresis behavior for a class of RC-OTA hysteretic-chaotic oscillators designed, for instance, in Ref. [Nakagawa, and Saito, 1996]. Using Simnon, we reproduce the chaotic attractor obtained by Ref. [Nakagawa, and Saito, 1996].

The structure of the paper is as follows. Section two presents the mathematical model of a class of RC-OTA hysteretic-chaotic oscillators. This model is taken from Ref. [Nakagawa, and Saito, 1996]. Our new dynamic-hysteretic model is presented in Section three along with simulation results. Final comments and future work are stated in Section four.

## 2 RC-OTA hysteretic chaotic systems

According to Ref. [Nakagawa, and Saito, 1996], a dimensionless dynamic model of a class of RC-OTA hysteretic-chaotic oscillator is given by:

$$\ddot{x} - 2\delta\dot{x} + x = ph(x), \qquad (1)$$

where $\delta$ and $p$ are the system parameters. The hysteresis function $h(x)$ is shown in Fig. 1. This system presents chaos with $\delta = 0.05$, and $p = 1$ [Nakagawa, and Saito, 1996].

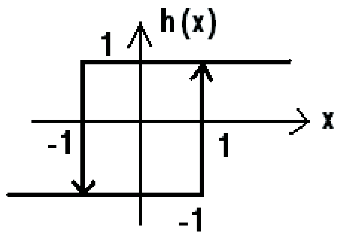


Figure 1. Normalized hysteretic function.

## 3 Hysteresis modeling

Hysteresis behavior is recognized as a system with memory. One way to capture hysteresis is by using a dynamic system. For instance, the new hysteretic system:

$$\dot{z} = \alpha(-z + bsgn(x + asgn(z))), \tag{2}$$

can reproduce the hysteretic behavior shown in Fig. 2, where $a$ and $b$ are the hysteresis parameters. The speed transition between $b$ and $-b$ is governed by the positive parameter $\alpha$; $sgn(\cdot)$ is the signum function. For instance, if $a = b = 1$ and $\alpha = 10$, the system (2) is:

$$\dot{z} = 10(-z + sgn(x + sgn(z))). \tag{3}$$

Using $x = 10\sin(t)$ and $z(0) = 0$, the simulation results are shown in Fig. 3.

Next, we program a chaotic oscillator equivalent to (1):

$$\ddot{x} - 0.1\dot{x} + x = z \tag{4}$$
$$\dot{z} = 10(-z + sgn(x + sgn(z))). \tag{5}$$

Fig. 4 shows the simulation results. The obtained chaotic attractor is the same as that shown in [Nakagawa, and Saito, 1996, Fig. 6].

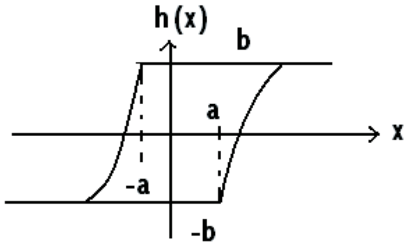


Figure 2. Hysteretic behavior.

As an example of the applicability of the proposed hysteresis model to chaos modeling, consider the simple state-space hyper-chaos realization:

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= z_1 + z_2 + 0.1x_2 - x_1,\\
\dot{z}_1 &= 10(-z_1 + sign(x_1 - 1 + sign(z_1)) + 0.5),\\
\dot{z}_2 &= 10(-z_2 + sign(x_1 + 1 + sign(z_2)) - 0.5).
\end{aligned}$$

The simulation results of this hyper-chaotic system are shown in Fig. 5. Fig. 6 shows $x_1(t)$ and $x_2(t)$. With a small change in the initial condition of $x_1$ from zero to $0.001$, the results displayed on Fig. 7 are obtained. We

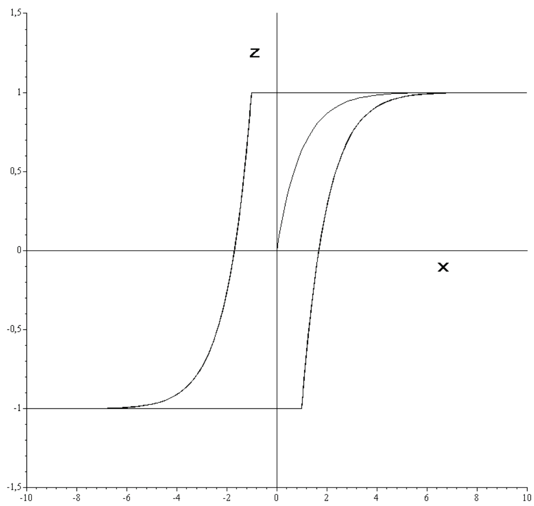


Figure 3. Simulation result.

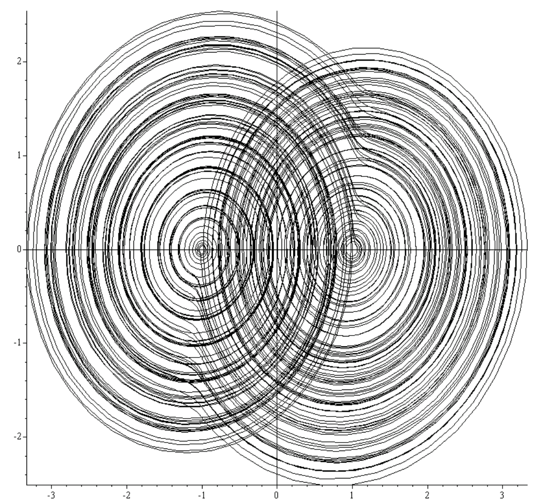

Figure 4. Chaotic attractor: $\dot{x}$ versus $x$ with initial conditions set to zero.

thus have a dynamic system that presents bounded trajectory solutions that is highly sensitive to initial conditions and whose signals are random-like. These are the properties of a chaotic system. Because of the non-differentiability of the proposed hysteretic model, there are some difficulties in calculating the Lyapunov exponents.

## 4 conclusions

A new hysteretic-dynamic model has been proposed. According to numerical experiments, this model is appropriate for a class of RC-OTA hysteretic-chaotic oscillators. With this model, further applications can be developed, such as synchronization and masking system design using chaotic signals [Acho, 2006]. One possible future work would be to study chaos in 3D hysteresis-based systems [Fengling , Jinhu, Xinghuo, and Guanrong, 2006] by using our proposed hysteretic

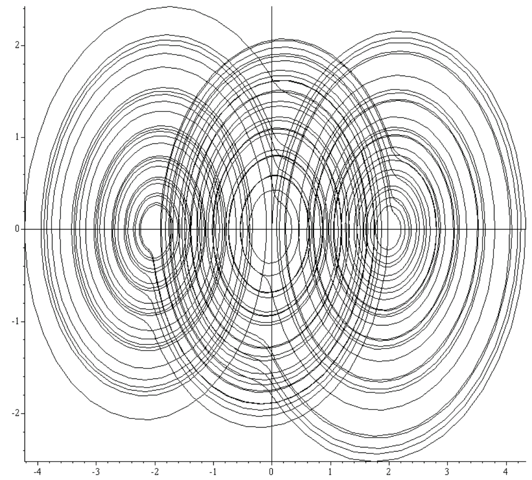

Figure 5. Chaotic attractor: $\dot{x}$ versus $x$ with initial conditions set to zero.

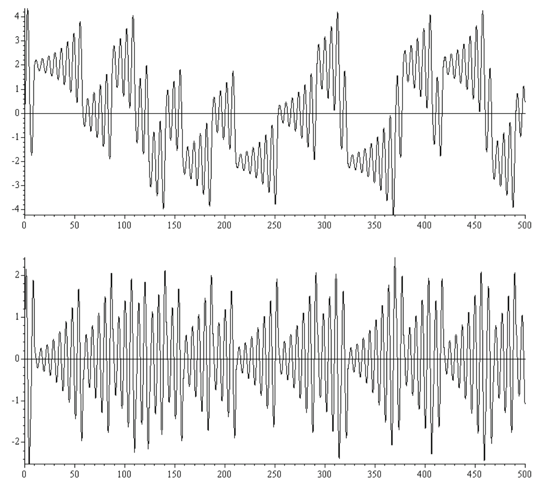

Figure 6. Simulation results: a) The top is $x_1(t)$, and b) the bottom is $x_2(t)$ both versus time (in seconds).

mathematical model, among others.

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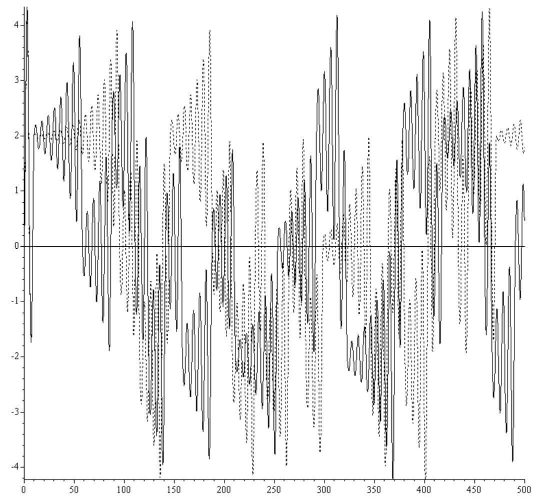

Figure 7. High sensitivity test on the initial conditions: a) solid line with $x_1(0) = 0$ and b) dotted line with $x_1(0) = 0.001$.


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