Feedback Error Correction Method of Creating Pseudo-Pure States for NMR Quantum Computers

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In this paper, a feedback error correction method of creating pseudo-pure states for large scale nuclear magnetic resonance quantum computers (NMR-QC) is proposed that is aimed at creating the initial states asymptotically within a specified error tolerance by using a spatial averaging method. The present method enables almost complete error correction and even enhances the NMR signal compared with the initial state. An implementation of this method was successfully demonstrated on a five-qubit NMR-QC.

Keywords: Quantum Computer, NMR, Pseudo-Pure, Error Correction, Feedback

INTRODUCTION

For NMR quantum computer, pseudo-pure states are generally used as initial states instead of pure states, which it is very hard to create. However, as the number of qubits increases, it becomes difficult to prepare even pseudo-pure states due to increase of the errors which are caused by imperfection of gate operations, decoherence and dissipation effects during the long process. In this work an efficient method of producing pseudo-pure states for large number qubit system is proposed, which is aimed at creating the states within a specified tolerance by using a few types of two-qubit gates, and it is shown that the present method enable to correct the errors almost completely and enhance the NMR signal intensities to a certain extent in the initialization process itself. The experimental demonstrations have been also performed successfully with a five-qubit NMR quantum computer.

There are predominantly two ways to prepare pseudo-pure state from the thermal equilibrium state. One is known as the temporal averaging method [1], which can be expressed as a sum of unitary transformations. The other is the spatial averaging method [2], which can be expressed as a series of unitary and non-unitary transformations. For the former method the total number of steps increases exponentially with the number of the qubits, so it is ineligible for large-qubit system. On the other hand, for the latter method it could be expected that the number of steps is proportional to the number of the qubits, abandoning analytic exact solution, which will not make sense practically for large-qubit system due to the enormous errors.

Preparation Method of Pseudo-Pure States with Error Correction

Let us next introduce a type of two-qubit gate, a *controlled-transfer* gate of angle θ, which is defined by the following transformation for diagonal density matrices.

$$diag(a,b,c,d) \Rightarrow diag\left(a,b,\frac{c+d+(c-d)\cos\theta}{2},\frac{c+d-(c-d)\cos\theta}{2}\right) \qquad (1)$$

This operation can be implemented as a controlled-rotation gate of angle θ followed by a magnetic gradient pulse. It can be shown that a sequence of controlled-transfer gates of angle $\pi/2$, $CT_{ij}(\pi/2)$, where i indicates the control-qubit and j indicates the target qubit, can create pseudo-pure states within a given tolerance, and the required number of gates and the signal intensity are proportional to the number of the qubits assuming that decoherence and dissipation effects can be ignored. Figure 1 shows the required number of the *controlled-transfer* gates required to achieve initialization within a tolerance of 1%, where the tolerance means the normalized standard deviation. From Figure 1, it can be seen that below six-qubit, the number of the gates increases as a second order polynomial, while it is proportional to the number of qubits above six-qubit. The linearity of the convergence can be understood from the fact that each of the gates averages out half of the diagonal elements at a time. The signal intensities increase with the number of qubits, n, as $\{n+n/(2n-1)\}/2$ compared to one of the initial states, because the energy of the ground state decreases proportionally with n.

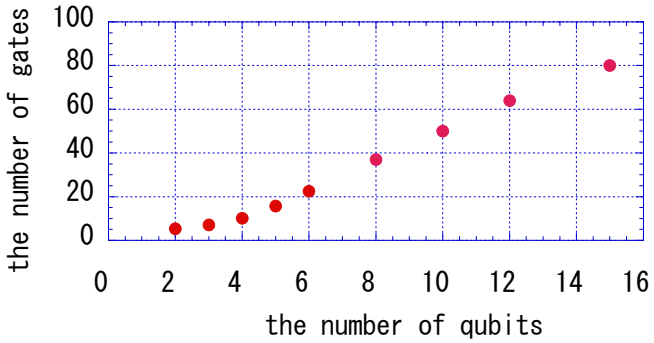

FIGURE 1. The required number of gates to obtain a tolerance of 1% with the number of qubits.

However, since the length of the sequence becomes much longer than in the temporal averaging method as the number of qubits is increased, it will fail to create pseudo-pure states even for a several-qubit system. Decoherence and dissipation during the gate operations affect the convergence and result in a lower limit of the convergence. The calculation results in a lower convergence limits shows that the limits increase proportionally with the number of qubits, where decoherence and dissipation effects are evaluated using the transverse and longitudinal relaxation rates, which were determined by experiments for a certain molecule. Although this situation seems to strand us, it would not be difficult to envision that a cooperative use of *controlled-transfer* gates of different angles would enable the convergence to accelerate or would enable the error correction. The calculation results when two types of *controlled-transfer* gates and *controlled-not* gates are used shows that the

convergence speeds are considerably improved, and the lower limits of the convergence are improved by a factor of ten overall by using the three gate types.

Evaluation of Decoherence and Dissipation Effects

To embed an error correction function into the initialization sequences with a classical computer, the effects of decoherence, dissipation and imperfection of pulse operations on the density matrix have to be predicted as accurately as possible. The intensities of the NMR signals observed with a $\pi/2$ pulse after applying n units of *controlled-transfer* gate, $CT_{ij}(\pi)$, to the thermal equilibrium state can be obtained from expression (1) by using the Bloch equations. The results are given in Table 1, where R_1 and R_2 are the longitudinal and transverse relaxation rates. In Figure 2, the experimental results with a certain molecule and the theoretical results including expected decoherence and dissipation effects with R_1 and R_2 are shown. From the figure it can be seen that by using parameters, p_1 and p_2 in expression (2), the effects of decoherence and pulse imperfection are well described.

$$R_1' = e^{-p_1 \frac{\Delta t}{T_1}}, \quad R_2' = e^{-\left(\frac{\Delta t}{T_2}\right)^m} - p_2 \tag{2}$$

TABLE 1. The signal intensities of each qubit after identical *controlled-transfer* gates of angle π are applied repeatedly, where the signal intensity in the thermal equilibrium state is assumed to be 1.

	signal intensity of the control qubit		**signal intensity of the target qubit**	
n	**state of the target qubit**		**state of the control qubit**	
	Up	**Down**	**Up**	**down**
odd	$2(R_1^n-1)/(\sqrt{R_1}+1)+1+R_2^n$	$2(R_1^n-1)/(\sqrt{R_1}+1)+1-R_2^n$	R_2^n	$-R_2^n$
even	$2(R_1^n-1)/(\sqrt{R_1}+1)+1$	$2(R_1^n-1)/(\sqrt{R_1}+1)+1$	R_2^n	R_2^n

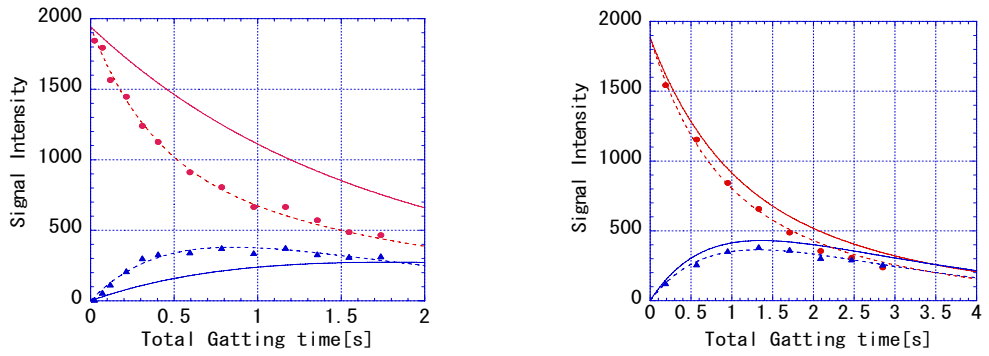

Figure 2. The signal intensities of a control-qubit after applying odd numbers of *controlled-transfer* gates of angle π to the thermal equilibrium state. (a) The experimental results when the shortest gates are applied, the duration time per gate is 29ms. (b) The experimental results of the longest gate, the duration time per gate is 190ms. The filled-circles indicate that the target-qubit state is |0>, and the filled-triangles indicate that the target-qubit state is |1>. The solid lines are the theoretical values obtained in the presence of T_1 and T_2 relaxation. The broken lines indicate the theoretical values when p_1 and p_2 are used as fitting parameters.

Producing of Initialization Sequence and Experimental Results

The sequence to prepare pseudo-pure state is produced by more than two processes. The optimization of the sequences of gates is performed with an annealing method. The first step is to create a basic sequence, which is prepared by appending one gate at a time in order that the standard deviation from the ideal state at this point becomes smallest including effects of the errors. The second step is to optimize the basic sequence, which is performed by examining and changing one gate at a time in order to minimize the standard deviation at the end of this sequence.

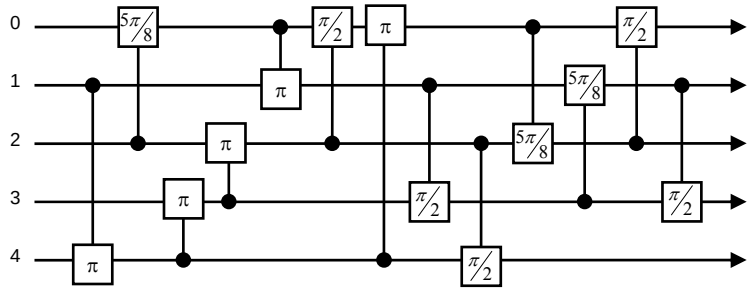

Figure 3. The quantum diagram of the optimized initialization sequence for a five-qubit NMR-QC, where three angles of controlled-transfer gates are used. The standard deviation from the ideal state is 1.6%.

To investigate the performance of the optimized sequence, initialization experiments are performed on a five-qubit NMR quantum computer with a 2,3,4,6-tetrafluoroaniline molecule, where four fluorine nuclear spins and a hydrogen nuclear spin were used as qubits. These experiments were taken under magnetic field of 9.4T and the sample was kept at a temperature of 308K. The diagonal elements before and after the initialization are shown in Figure 4. From the figure, it is seen that the experimental results agree well with the theoretically expected values. Although the population of the zero state would be reduced almost by half due to the decoherence effects, the signal intensity is increased approximately 1.25 times.

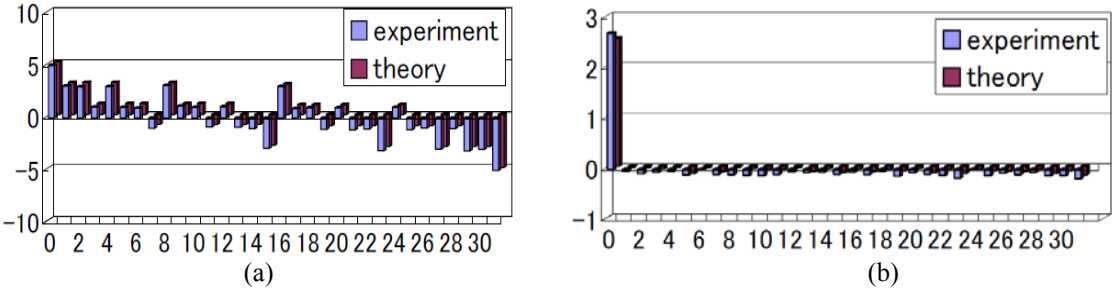

Figure 4. The theoretical and experimental results of the deviation density matrices' diagonal elements for both the thermal equilibrium state (a) and after initialization (b).

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