Robust features of tokamak plasma stabilization systems

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Abstract—**This paper describes new controller synthesis approach based on DK-iterations procedure. The technique allows providing a compromise between dynamic and robust features of closed-loop system. The approach is described as a computational algorithm using MATLAB common procedures. An application to MAST tokamak plasma shape control problem is presented.**

I. Introduction

A crucial problem concerning achieving conditions necessary for thermonuclear reaction in tokamaks is the magnetic plasma confinement problem. Plasma is a very complex physical object that is affected by variety of external electrotechnical structures and unpredictable intrinsic behavior.

Engineers widely use linearized mathematical models for control system design. Such models approximately describe currents dynamics of plasma and all conducting structures in tokamak around some certain plasma equilibrium state [1]. Obviously, these models essentially depend on the equilibrium chosen. Nevertheless, technical realization aspects require these model variations not to affect designed control laws. It means that a single controller should stabilize plasma at different scenario points. Furthermore, this controller should take into account a variety of real discharge conditions that often remain unmodelled.

Cited features require designing controllers that have robust features, i.e. they must be fully functional in the case when nominal and actual plant model differs.

II. Fundamentals

Consider closed-loop system shown on figure 1. Here $\mathbf{T}(s,\mathbf{\Delta})$, $\mathbf{K}(s)$ and $\mathbf{H}(s,\mathbf{K},\mathbf{\Delta})$ are transfer functions of a plant, controller and closed-loop system $\mathbf{e}=\mathbf{H}(s,\mathbf{K},\mathbf{\Delta})\mathbf{d}$ correspondingly. Symbol $\mathbf{\Delta}$ denotes the modeling uncertainty, which is known to belong to the given set $\mathbf{\Delta}\in\mathbf{D}$.

Let us introduce the closed-loop characteristic polynomial $\delta(s,\mathbf{K},\mathbf{\Delta})$, which depends on controller selection and the specific uncertainty realization. Let n_{cl} denote its degree and $\delta_i=\delta_i(\mathbf{K},\mathbf{\Delta})$, $i=\overline{1,n_{cl}}$ – its roots, which must be located in the left half-plane C^- for the serviceable control system.

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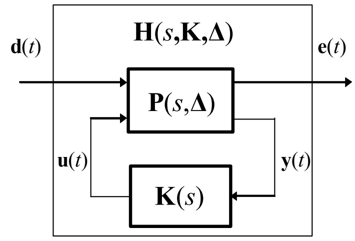

Fig. 1. Closed-loop control system.

It is said that closed-loop system $\mathbf{e}=\mathbf{H}(s,\mathbf{K},\mathbf{\Delta})\mathbf{d}$ is robust stable with respect to uncertainty $\mathbf{\Delta}$, if the condition $\delta_i(\mathbf{K},\mathbf{\Delta})\in\mathbf{C}^-$, $i=\overline{1,n_{cl}}$ holds for all $\mathbf{\Delta}\in\mathbf{D}$. In this case controller $\mathbf{u}=\mathbf{K}(s)\mathbf{y}$ is said to be guaranteeing the robust stability of a closed-loop system.

Let consider certain functional $J=J(\mathbf{H}(s,\mathbf{K},\mathbf{\Delta}))$ which characterizes system performance. This functional maps uncertainty set $\mathbf{D}$ on a set $\mathbf{I}=J(\mathbf{H}(s,\mathbf{K},\mathbf{D}))\in R^1$ for any given controller.

It is said that closed-loop system $\mathbf{e}=\mathbf{H}(s,\mathbf{K},\mathbf{\Delta})\mathbf{d}$ has robust performance feature if it is robust stable with respect to uncertainty $\mathbf{\Delta}$ and $\mathbf{I}\subset\Re\subset R^1$, where $\Re$ denotes the permissible range of concerned functional values. In this case controller $\mathbf{u}=\mathbf{K}(s)\mathbf{y}$ is said to be guaranteeing robust performance of a closed-loop system.

III. Model Uncertainty Specifications for Tokamaks

Consider a problem of uncertainty specification in tokamak control system design. As an example we will consider linear models which were obtained for Gutta tokamak [2] using the PET-code [1]. For certain scenario point several linear models of a different order were constructed. Figure 2 and 3 shows singular plots and frequency response from PF2 coil to plasma current for all constructed models. It is easy to see that essential model variations are expressed in the frequency range ~[30 - 5000] rad/s. In order to formalize such uncertainty it is convenient to use input multiplicative uncertainty model as shown on figure 4. Here uncertainty $\mathbf{\Delta}$ is norm-bounded $\|\mathbf{\Delta}\|_\infty\le 1$ and the weighting matrix $\mathbf{W}_{unc}$ defines permissible uncertainty frequency range.

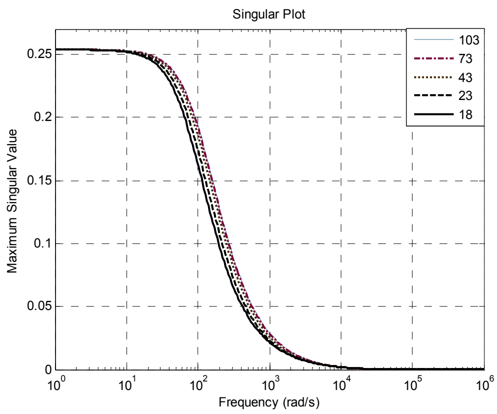

Fig. 2. Singular plots for Gutta models.

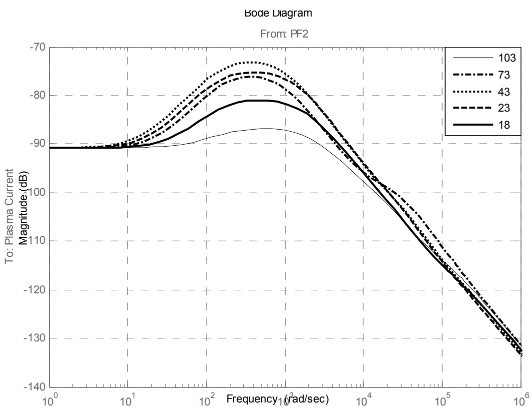

Fig. 3. Frequency responses from PF2 voltage to plasma current for Gutta models.

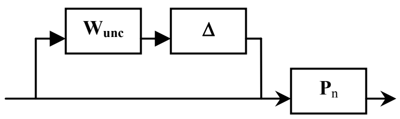

Fig. 4. General input multiplicative uncertainty model.

IV. Plasma Shape Controller Synthesis for MAST Tokamak

Consider MAST tokamak linear model [3]. Model equations could be expressed in the standard state-space form as follows

$$\begin{aligned} \dot{\mathbf{x}}_f &= \mathbf{A}_{hv}\mathbf{x}_f + \mathbf{B}_{hv}\mathbf{v} \\ \mathbf{y}_f &= \mathbf{C}_{hv}\mathbf{x}_f + \mathbf{D}_{hv}\mathbf{v} \end{aligned}, \qquad (1)$$

here $\mathbf{x}_f \in E^{50}$, $\mathbf{v} \in E^5$, $\mathbf{y}_f \in E^{12}$.

Controller synthesis design for MAST tokamak control system is based on a controller representation as a sum of the following components [4,5]

$$\mathbf{v} = \left(\mathbf{W}_v(s) + \mathbf{W}_{sh}(s) + \mathbf{W}_{ip}(s)\right) \cdot \mathbf{y}_f , \qquad (2)$$

where $\mathbf{W}_v(s)$, $\mathbf{W}_{sh}(s)$, $\mathbf{W}_{ip}(s)$ are transfer matrices of a plasma vertical position, plasma shape and plasma current controller correspondingly.

For plasma shape controller design we consider semi-closed-loop system (1) with vertical controller $v_5 = y_2 + 20\dot{y}_2 + y_{11}$ and plasma current controller $v_1 = 10000 y_{12}$ [4,5]. Control vector components v_2, v_3, v_4 (i.e. voltage on P3, P4, P5 coils) are fully available for plasma shape control along with $y_4, y_5, y_6, y_8, y_9, y_{10}$ components of the measurement vector (i.e. z_x, r_{in}, r_{out} deviations and P3, P4, P5 coil currents).

As a result, after closing the nominal system with mentioned vertical position and plasma current controllers, and extraction of a controllable system we obtain the following system of 49 order

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{aligned}, \qquad (3)$$

here $\mathbf{x} \in E^{49}$, $\mathbf{u} \in E^3$, $\mathbf{y} \in E^6$. This model serves as the basis for synthesis of a plasma shape controllers in the form

$$\mathbf{u} = \mathbf{K}(s)\mathbf{y} .$$

V. LQG – Optimal Synthesis of Plasma Shape Controllers for MAST

One of widely using methods for synthesis of stabilizing controllers is LQG-optimization approach [6], which allows constructing a controller taking into account all dynamical performance requirements.

As it is known, realization of the LQG-approach is based on a minimization of the mean-square cost functional

$$I(\mathbf{K}) = \int_0^{\infty} \left(\mathbf{y}^T \mathbf{R} \mathbf{y} + c_0 \mathbf{u}^T \mathbf{Q} \mathbf{u}\right) dt , \qquad (4)$$

here $\mathbf{R}$, $\mathbf{Q}$ are weighting matrices, c_0 is a constant multiplier setting the tradeoff between controller's performance and control energy costs.

The weighting matrices $\mathbf{R}$ and $\mathbf{Q}$ along with c_0 multiplier are "tools" that allow us to achieve the desired dynamic behavior of a closed-loop system. In the field of tokamak plasma stabilization systems design such desired behavior is assayed by the results of simulated closed-loop system response to the external l_i and β-drops disturbances [1, 3].

From the results of analysis for system (3) LQG-optimal controller was obtained and simulated response to l_i and β-drops disturbances represented by figure 5.

The approach described allows us to achieve good stabilization performance, but it is not initially directed to guarantee the prescribed closed-loop robust specifications.

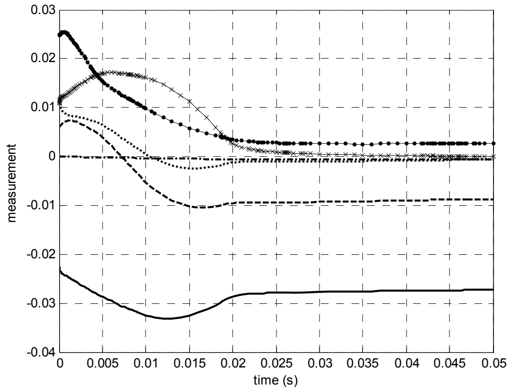

Fig. 5. l_i and β -drops response for closed-loop system with LQG-controller.

In accordance with the Gutta tokamak example described above, in order to analyze robust features for MAST tokamak model we chose weighting uncertainty matrix as $\mathbf{W}_{unc} = diag(W_d, W_d, W_d)$, frequency response of W_d is shown on the figure 6.

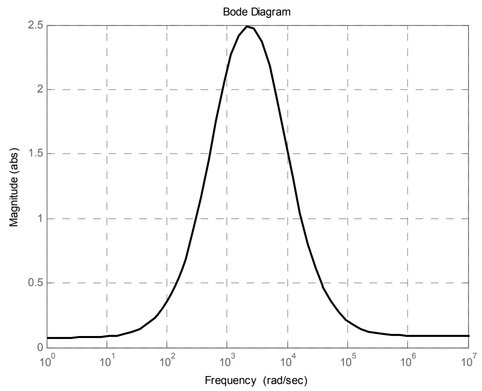

Fig. 6. $W_d(s)$ frequency response plot.

Using comparative technique for robust features, as described in [5], we construct the auxiliary transfer matrix

$$\mathbf{M}(s) = -\mathbf{K}(s)\left[\mathbf{I} + \mathbf{P}_n(s)\mathbf{K}(s)\right]^{-1}\mathbf{P}_n(s), \qquad (5)$$

where $\mathbf{P}_n(s)$ and $\mathbf{K}(s)$ are transfer matrices of a nominal plant and controller. As it is known [6,7], singular frequency response of the $\mathbf{M}(s)$ could be considered as a robust stability frequency margin for the corresponding closed-loop system. Note that in accordance with the small-gain theorem [7], if $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\mathbf{M}(j\omega)) < 1$ for all $\omega \in [0, \infty)$ then perturbed closed-loop system remains stable for any norm-bounded uncertainty $\mathbf{\Delta}(s)$ such that $\overline{\sigma}(\mathbf{\Delta}(j\omega)) \le 1$.

Figure 7 shows singular plot $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\mathbf{M}(j\omega))$ for the closed-loop system with LQG-optimal controller. It is easy to see that small-gain theorem condition violated, hence we cannot guarantee that perturbed system will remain stable. It is possible to find norm-bounded uncertainty that cause closed-loop instability. Figure 8 represents impulse response for closed-loop system with the randomly sampled uncertainty. As is easy to see, destabilizing uncertainty indeed exists.

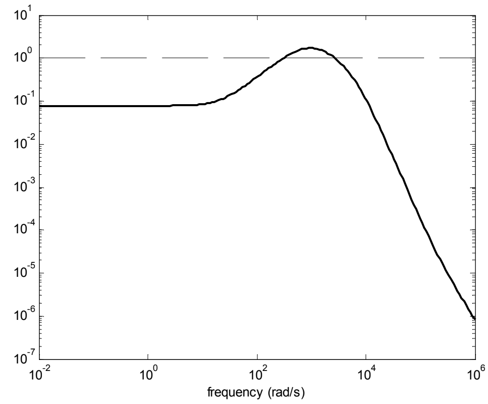

Fig. 7. Singular plot $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\mathbf{M}(j\omega))$ for closed-loop system with LQG-controller.

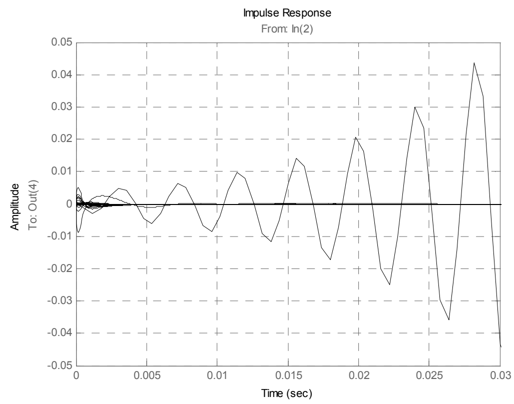

Fig. 8. Impulse response for randomly sampled uncertainty realizations.

VI. DK-ITERATION-BASED APPROACH

Taking into account weakness of LQG approach with the regard to robust requirements as explained above, it is possible to introduce special synthesis approach, which allows providing a compromise between required dynamic behavior and prescribed robust features.

The approach is based on μ-theory methods where we consider general LFT system as depicted on figure 9, where ***d*** denotes combined external disturbances vector, ***e*** is the error vector, $\boldsymbol{e_p}$ – weighted errors, ***u*** – control vector, ***y*** – measurements, $\mathbf{\Delta}_s$ – structured block-diagonal model uncertainty $\mathbf{\Delta}_s(s) = \begin{bmatrix} \mathbf{\Delta}_1(s) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Delta}_n(s) \end{bmatrix}$.

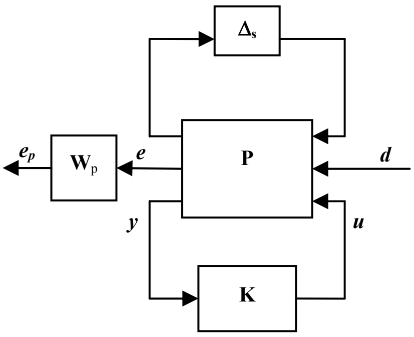

Fig. 9. LFT-characterization of uncertain closed-loop system.

Let $\mathbf{H}(s,\mathbf{K})$ denote transfer matrix of the closed-loop system from external disturbances $\boldsymbol{d}$ to weighted errors $\boldsymbol{e}_p$, and let $\mathbf{K}(s)\in\Omega$, where Ω is the set of all stabilizing controllers.

Uncertainty structure is defined by the chosen plant uncertainty model and robust performance problem specifications that express as a weighting matrix $\mathbf{W_p}(s)$.

DK-iteration technique objective is to find stabilizing controller $\mathbf{K}(s)$ that minimizes structured singular value, which corresponds to the structured uncertainty $\mathbf{\Delta}_s$ [6-8].

$$\mu_{\mathbf{H},\mathbf{K}} = \sup_{\omega\in R}\left\{\mu_{\mathbf{\Delta}_s}[\mathbf{H}(j\omega,\mathbf{K})]\right\} \to \min_{\mathbf{K}\in\Omega}$$

The main disadvantage of the DK-iteration approach is that it does not allow providing robust/dynamic requirements compromise explicitly. Furthermore, obtained controllers often is of very high order. The latter problem could be avoided with the help of model order reduction techniques such as Schur balanced model reduction [9].

Let parameterize weighting matrix $\mathbf{W_p}$ and introduce the parametric set of weighting transfer matrices $\widetilde{\mathbf{W}}_\mathbf{p}(s,\xi)$ such that for any $\xi^*\in\left[\underline{\xi},\overline{\xi}\right]$ the corresponding robust performance problem has a solution via DK iteration procedure.

Taking into account model reduction of obtained controllers, we get parametric set of stabilizing controllers $\widetilde{\mathbf{K}}(s,\xi)$. Now choice of the parameter ξ^* value from the admissible interval will result in certain realization of controller transfer matrix $\mathbf{K}(s)=\widetilde{\mathbf{K}}(s,\xi^*)$, that will provide certain dynamic/robust compromise for closed-loop system $\mathbf{H}(s,\widetilde{\mathbf{K}})$.

For plasma shape controller synthesis for MAST tokamak let consider input multiplicative uncertainty model in accordance with figure 4. Let fix combined error vector in the form $\boldsymbol{e}=\begin{pmatrix}\widetilde{\mathbf{R}}\mathbf{x}\\ \widetilde{\mathbf{Q}}\mathbf{u}\end{pmatrix}$, where $\widetilde{\mathbf{R}}$ and $\widetilde{\mathbf{Q}}$ are constant matrices. For the robust performance problem instantiation we define parametric set of weighting matrices $\widetilde{\mathbf{W}}_p(s,\xi)$ as $\widetilde{\mathbf{W}}_p(s,\xi)=diag(diag(W_p(s,\xi)\ \ ...\ \ W_p(s,\xi))\quad E_{9x9})$, here $W_p(s,\xi)=\dfrac{f}{\xi}\cdot\dfrac{s+0.1\cdot\xi}{s+0.03\cdot 0.1}$, and $f=3\cdot 10^4$ is a constant multiplier. Using Schur balanced model reduction approach it is possible to reduce resulting controllers order (104) down to 10 without essential loss in dynamical features, as justified by figure 10. Introduced parameter ξ defines certain dynamic/robust compromise.

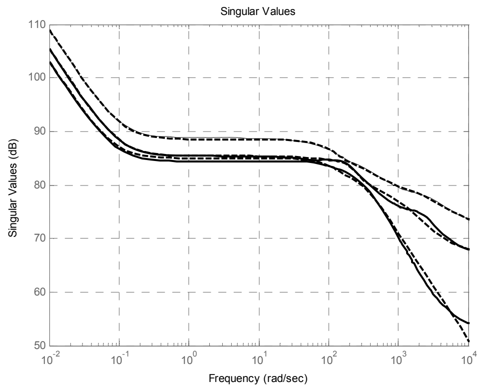

Fig. 10. Singular plots of the full DK-controller (solid) and the reduced one (dashed) for ξ = 1.

Figure 11 shows l_i and β-drops response of the closed-loop system with DK-controller obtained with $\xi=1$. As is seen, dynamic behavior of this controller is as good as it is for LQG-optimal controller.

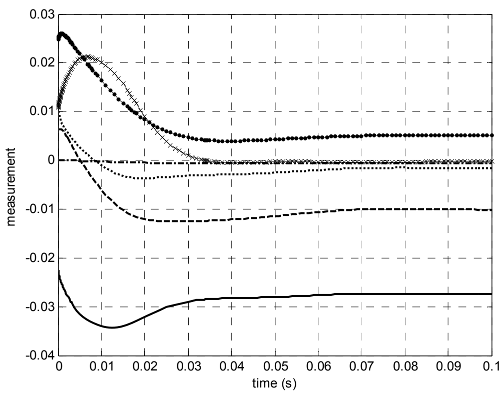

Fig. 11. l_i and β -drops response for DK-controller (ξ = 1).

Using the robust features comparative analysis technique [5], we construct the parametric set of transfer matrices according to (5)

$$\widetilde{\mathbf{M}}(s,\xi)=-\widetilde{\mathbf{K}}(s,\xi)\left[\mathbf{I}+\mathbf{P}_n(s)\widetilde{\mathbf{K}}(s,\xi)\right]^{-1}\mathbf{P}_n(s).$$

Figure 12 shows singular plots $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\widetilde{\mathbf{M}}(j\omega,\xi))$ for three DK-based controllers obtained for ξ= 0.2, 1, 2. It is apparent that for these three controllers $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\widetilde{\mathbf{M}}(j\omega,\xi))<1$ for all $\omega\in[0,\infty)$, hence corresponding closed-loop systems will preserve stability for any uncertainty from the admissible range defined by weight $\mathbf{W}_{unc}(s)$.

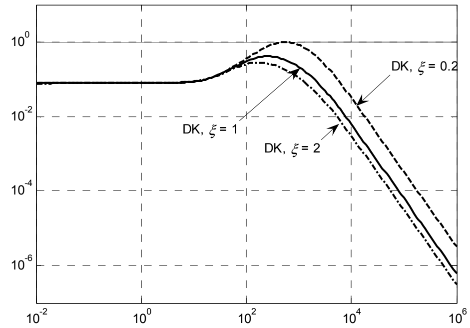

Fig. 12. Singular response $\overline{\sigma}(\mathbf{W}_{unc}(j\omega)\widetilde{\mathbf{M}}(j\omega,\xi))$ for DK-controllers (ξ = 0.2, 1, 2).

Frequency robust stability margins for three obtained DK-controllers along with LQG-optimal controller are shown on figure 13. Figure 14 shows l_i and β-drops responses for these controllers (R_{out} control point). It is clear that DK-based controller obtained for ξ=1 has essential superiority regarding robust features of the closed-loop system in the operating frequency range having the close dynamic behavior comparing to LQG-controller at the same time. Decreasing ζ parameter value to 0.2 leads to the controller better than LQG-controller regarding stabilization rate and robust features in the frequency band $\sim[600, 3000]\ rad/s$, but robust stability margin deteriorates at higher frequencies.

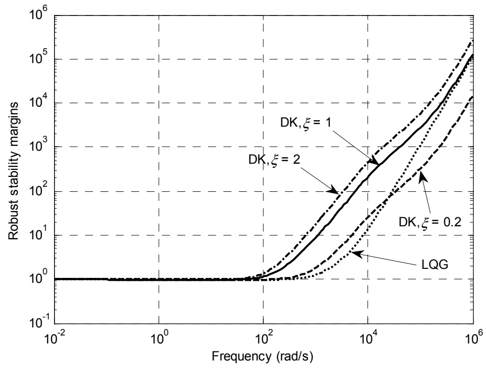

Fig. 13. Robust stability frequency margins.

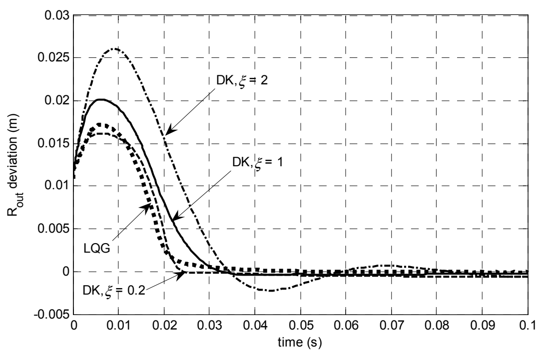

Fig. 14. l_i and β-drops response (control point R_{out}).

VII. Conclusions

New controller synthesis approach based on DK-iteration procedure is presented. The technique described rests on a performance weighting matrix paramertization proposal that allows goal seeking of the compromise controller with respect to dynamic/robust behavior of the closed-loop system. The approach realized as a computational algorithm based on MATLAB-Simulink environment.

The described technique used for plasma shape control design for MAST tokamak. The results obtained lead us to conclusion that the approach allows achieving a compromise between excellent dynamical behavior inherent to LQG-optimal synthesis technique and robust features typical of DK-synthesis that guarantees closed-loop stability for perturbed system in the prescribed frequency range.

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