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ACCELERATION OF THE HEAVY IONS IN THE FLOW OF THE ELECTRONS

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Abstract
The process of ion flow extraction from plasma is one of the most important objects being interesting while creating effective electrojet engine. Due to this fact a self-consistent problem of ion acceleration in electron cloud is considered in this research with additional electron flux taken into account. The kinetic description in the case of collision absence is used. Electron flow is described by unconventional solution of kinetic equation which dependent not only on the energy integral. It is shown that cold ion can be accelerated to the energy exceeding the electron temperature, i.e. ion velocity can exceed ion-acoustic velocity.

Key words
Kinetic equation, Poisson equation, ion flow, electron flow.

1 Introduction

To study the real processes of heavy ion extraction from plasma the lot of researches is carried out which propose the different extraction models. In work [Riemann, 1991] it is shown that plasma is left by ions with the speeds exceeding ion-acoustic velocity. In practice temperature of electrons T significantly more than temperature of ions $T_i (T >> T_i)$ the number of the accelerated ions is exponential small hence the ion current and a traction of the electrojet engine are small too. We will note, however, the work [Kovalenko, Chernyshev, and Chikhachev, 2011a] studying acceleration of a thin ionic beam. In this work it is shown that the velocity of ions can surpass ion-acoustic velocity when the beam radius is changed. In work [Sternberg and Godyak, 2007] the condition of the accelerated stream cold ion flux existence in the cloud of hot electrons is found. In particular, in [Sternberg and Godyak, 2007] it is shown that the transitional layer in plasma vacuum system infinite. Current of electrons is equal to zero. Work [Kovalenko and Chikhachev, 2013] studies conditions for the ion flux moving in the electron layer perpendicularely to the direction of the electron flux. In this work the maximum energy which ions can gain in a layer, is equal the electron temperature T, however current of ions is not exponential small. It should be noted here that experimental works exist studying the various aspects of the problem of the power electrojet engine development, as example the work [Ermilov and Kovalenko, *at al.*, 2008] that studies the traction characteristics such engine here. In the real work the consecutive kinetic description of ion-electronic system in the presence of a nonzero stream of electrons is used. Thus function of distribution of electrons depends not only on motion integrals.

2 Problem Definition

We will describe particles by means of collisionless kinetic equations for both electrons and ions considering the problem for the sake of simplicity as one-dimensional. For particles by means of the kinetic equitation takes the form of:

$$\frac{p}{m}\frac{\partial f}{\partial x} + e\frac{d\Phi}{dx}\frac{\partial f}{\partial p} = 0 \qquad (1)$$

where m is the mass of electron, e is the charge, Φ is a potential, x is a coordinate, p is amomentum, f is a particle distribution function. Equation (1) always has a solution

$$f = \Psi(H) = \Psi\left(\frac{p^2}{2m} - e\Phi(x)\right),$$

where Ψ - arbitrary function. This solution is characterized by particle zero flux Γ_e along axis x : $\Gamma_e = \int\limits_{-\infty}^{+\infty} \frac{p}{m} dp\Psi(H) = 0$ because of antisymmetry of integrand. At that density

$$n_e = \int\limits_{-\infty}^{+\infty} dp\Psi(H) = 2\int\limits_{-e\Phi(x)}^{+\infty} \frac{m\Psi(H)dH}{\sqrt{2m(H + e\Phi)}}.$$

If an exponential energy distribution is used $\Psi = \kappa_0 \exp(-\frac{H}{\varkappa T})$, the density has the form of:

$$n_e = 2\kappa_0 \sqrt{\frac{2\pi\varkappa T}{m}} \exp\left(\frac{e\Phi}{\varkappa T}\right), \qquad (2)$$

where $\varkappa$ is Boltsmans constant.

To describe the non-zero electron flux we will use equation solution (1) that is not only function of the integral of the motion H. Equation:

$$f = \sigma\left(p - \sqrt{2m(C_0 + e\Phi)}\right)\Psi(H). \qquad (3)$$

Here $\sigma(x)$ is the Heaviside function, $C_0 \geq -e\Phi(x)$ for any x. It's easy to make sure that equation (3) corresponds to equation (1). After differentiation in accordance with equation (1) we will get:

$$\delta(p - \sqrt{2m(C_0 + e\Phi)}) \frac{de\Phi}{dx}\left(1 - \frac{p}{\sqrt{2m(C_0 + e\Phi)}}\right),$$

i.e. zero. Expression (3) determines non-zero electron flow:

$$\Gamma_e = \int\limits_{\sqrt{2m(C_0+e\Phi)}}^{\infty} dp \frac{p}{m}\Psi(H) = \int\limits_{C_0}^{\infty} dH\Psi(H).$$

In the case of exponential distribution $\Gamma_e = \kappa_0 \varkappa T \exp\left(-\frac{C_0}{\varkappa T}\right)$. So the electron density may be expressed by the next formulae:

$$n_e = \kappa_0 \sqrt{\frac{2\pi\varkappa T}{m}} \exp\left(\frac{e\Phi}{\varkappa T}\right) \times \left(1 - erf\left(\sqrt{\frac{C_0 + e\Phi}{\varkappa T}}\right)\right). \qquad (4)$$

Here $erf(x) = \frac{2}{\sqrt{\pi}} \int\limits_0^x \exp(-y^2)dy$ is an error function integral. In (3) the role of the multiplier $\sigma\left(p - \sqrt{2m(C_0 + e\Phi)}\right)$ is substantial - on the one hand, the shape of particle density changes, on the other hand particle current is not equal zero. Using of a such multiplier for the aims of the beam description gives us the possibility to study the new type of the beam equilibria. Combining the motion integrals and the multiplier in the form of step-function one can significantly affect on the balance of the system if the own magnetic field of the electron flux is taken into account (see [Kovalenko and Chikhachev, 2013]). Note, the kinetic equation solution with Heaviside function is used in paper [Lohder and Ulyanov, 2013] when studying the phenomena in the gas discharge.

By the similar way the ion flow can be described. Just like at expression (4) for the ion density we will get:

$$n_i = \kappa_i \sqrt{\frac{2\pi\varkappa T_i}{M}} \exp\left(\frac{-e\Phi}{\varkappa T_i}\right) \times$$

$$\times \left(1 - erf\left(\sqrt{\frac{C_1 - e\Phi}{\varkappa T_i}}\right)\right),$$

here M is the ion mass, T_i is the ion temperature. Using asymptotic decomposition for error function integral at $T_i \to 0$ we will get:

$$n_i = \kappa_i \sqrt{\frac{2\varkappa T_i}{M}} \exp\left(-\frac{C_1}{\varkappa T_i}\right) \sqrt{\frac{\varkappa T_i}{C_1 - e\Phi}}.$$

Upon that $\Gamma_i = \kappa_i \varkappa T_i \exp\left(-\frac{C_1}{\varkappa T_i}\right)$, whence it follows: $n_i = \Gamma_i \sqrt{\frac{M}{2(C_1 - e\Phi)}}$. Let us equate flow density as: $\Gamma_i = n_{0i}\upsilon_0, n_{0i}$ is an initial ion density, υ_0 is an initial ion velocity and express in terms of $C_1 = \frac{\upsilon_0^2 M}{2}$. It should be indicated that here ion description corresponds absolutely to hydrodynamic description of cold ion flow. So we get:

$$n_i = \frac{\Gamma_i}{\upsilon_i} = \frac{n_{0i}\upsilon_0}{\sqrt{\upsilon_0^2 - \frac{2e\Phi}{M}}} = \frac{n_{0i}\upsilon_0}{\upsilon_s \sqrt{\frac{\upsilon_0^2}{\upsilon_s^2} - u}}.$$

Here $\upsilon_s = \sqrt{\frac{\varkappa T}{M}}, u = \frac{e\Phi}{\varkappa T}$. Let us write down Poisson equation:

$$\frac{d^2\Phi}{dx^2} = \frac{e}{\varepsilon_0}(n_e - n_i) =$$

$$\frac{e}{\varepsilon_0}\left(n_{0i} \exp(u)(1 - erf\sqrt{\frac{C_0}{T} + u}) - \frac{n_{0i}\upsilon_0}{\upsilon_s\sqrt{\frac{\upsilon_0^2}{\upsilon_s^2} - u}}\right) \qquad (5)$$

where $n_{0e} = \kappa_0 \sqrt{\frac{2\pi m}{\varkappa T}}, \varepsilon_0$ is the "vacuum constant". After these dimensionless arguments should be introduced $t = \frac{x}{l_0}, l_0 = \sqrt{\frac{\varkappa \varepsilon_0 T}{e n_{0e}}}$, let us set $\frac{C_0}{\varkappa T} = \zeta_0$. We will get:

$$\frac{d^2u}{dt^2} = \exp(u(t))\left(1 - erf\sqrt{\zeta_0 + u}\right) - \frac{\nu_i}{\sqrt{\frac{\upsilon_0^2}{\upsilon_s^2} - u(t)}}. \qquad (6)$$

Here $\nu_i = \frac{n_{0i}}{n_{0e}} \frac{\upsilon_0}{\upsilon_s}$.

3 Results of Computational Solution

Let us set solution results of equation (6), where $\zeta_0 = 4, \nu_i = 0.08, \frac{v_0^2}{v_s^2} = 0.1$. In the function of initial condition let us set $u(0) = -4, \dot{u}(0) = 0$. Depenance of non-dimensional potential $u(t)/200$ (curve III) from non-dimensional co-ordinate is depictured in Fig.1. This solution has a periodical character. It is also represented qualities of functional dependence of ion density from co-ordinate (I) and electron density from co-ordinate (II). From the character of these dependencies we may conclude: ions accelerate away from point $t = -43.79$ (where $u = 0.016$) to point $t = 0$ (where $u = -4$). Electrons, on the contrary, decelerate in such motion. Electrons can accelerate during the motion in the opposite direction: from point $t = 0$ to point $t = -43.79$. Functional dependence of electron velocity from potential is depictured in Fig.2, curve II, curve I on this picture represented the dependence of ion velocity from potential. Directions of the velocities strictly opposite. Kinetic energy at the point $t = 0$ is determined by the value ζ_0. If this energy equals to $4.016T$. It is fourfold as much as ion temperature. The more is an absolute value ζ_0, the more is the value of kinetic energy ions at the exit from plasma area with the electrons described by distribution (3). This fact confirms the result of the work [5] in which is shown that in a layer of the electrons moving perpendicularly to a stream of ions, ions can reach the value of energy which is not exceeding electronic temperature. Flat gap is the electrode where $t = 0$ under potential which is equal to $-4T$ and the second electrode where $t = 43.7$ is under potential $0.016T$. Electron flow with energy more then $4T$ has to fall within flat gap from electrode with negative potential from the similar electrode ions with slow velocity enter in flat gap. These ions accelerate to kinetic energy $4T$.

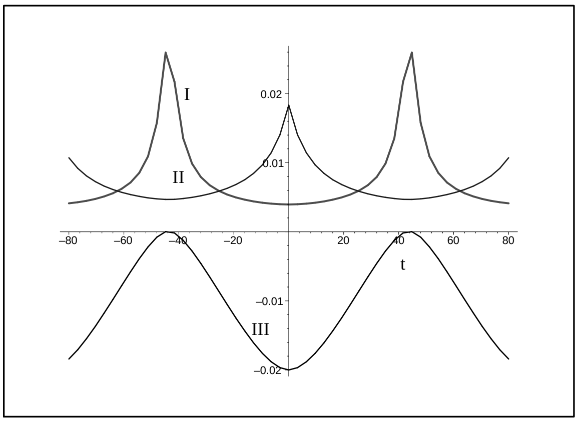

Figure 1. Co-ordinate ion density-relationship (curve I), Co-ordinate electron density-relationship (curve II) and co-ordinate potential (curve III).

It should be noted that the real consideration significantly differs from description of the system by means of the hydrodynamic equations. In the presence of a stream of electrons the pressure of electronic gas P is not equal to $P = n_e T$. The decision of the self consistent system of the hydrodynamic equations is provided in work [Kovalenko, Chernyshev, and Chikhachev, 2013].

4 Three-Part System

Three-part system shall be understood as situation, emerging in the event when except electron flow and ions the open interval has electrons consisting a cloud with a zero average velocity. These particles are described by means of usual Maxwell distribution function $f \sim \exp(-H/\varkappa T)$. The density of this particles is proportional to $\exp(u)$.

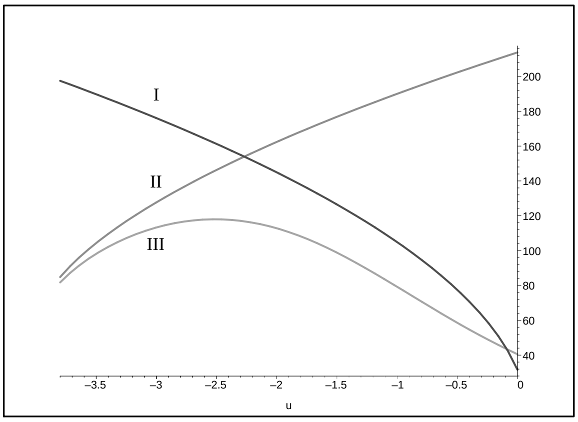

Figure 2. Dependences of the ions velocity on potential (curve I), dependences average velocity flow of the electrons on potential (curve II) and dependences average velocity of the electrons on potential when co-existent flow and cloud (curve III).

According to mentioned above let us add at right side of an equation (4) addend $0.01\exp(u)$. Then the equation for potential (6) turns into following expression:

$$\frac{d^2u}{dt^2} = \exp(u(t))\left(1.01 - erf\sqrt{\zeta_0 + u}\right) \quad (7)$$

$$-\frac{\nu_i}{\sqrt{\frac{v_0^2}{v_s^2} - u(t)}}$$

A minor addition leads to considerable changes in potential solution. Let us set: $\zeta_0 = 4, v_0^2/v_s^2 = 0.1, \nu_i = 0.01$. We also use initial conditions $u(0) = -3.8, \dot{u}(0) = 0$. Fig. 3 shows that electron cloud presence with zero average velocity hardly influences the behavior pattern of potential. The same figure demonstrates the change of electron density. In such case an additional maximum of electronic density appears at the same place where the maximum of ion density appears. The average electron velocity calculated with account of (because of electron cloud presence with zero average velocity) over density detects a potential for upward motion in the same direction that ion velocity (please see Fig. 4). It is possible to create a mechanism where a simultaneous acceleration in the same direction of electrons and ions takes place. Though in such case electron acceleration passes "in average" upon particle deceleration constituting electron flow. Small difference of potential, however, appears only at a small additive of the maxvellian electrons.

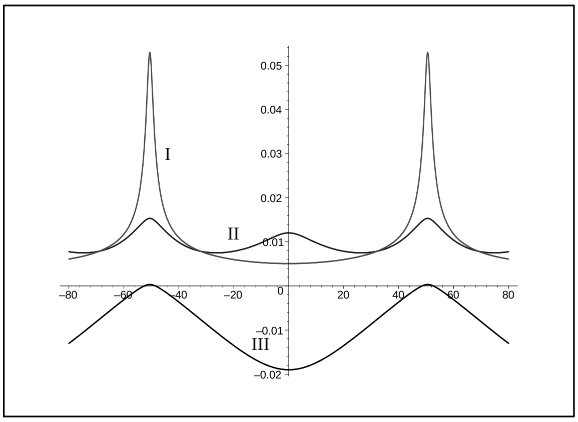

Figure 3. Co-ordinate ion density-relationship (curve I) and coordiante electron density-relationship (curve II). Co-ordinate potential-relationship (curve III) in case of flow presence and constituting in electron medium when density clouds $\sim 0.01\exp(u)$.

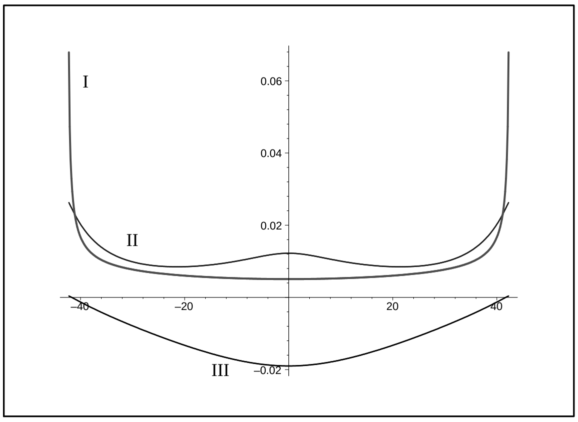

Figure 4. Coordinate ion density-relationship (curve I) and coordiante electron density-relationship (curve II). Coordinate potential-relationship (curve III) in case of flow presence and constituting in electron medium when density clouds $\sim 0.02\exp(u)$.

If we add to the right part (4)composed $0.02\exp(u)$ instead of $0.01\exp(u)$ one can find that the character of the decision will sharply change (see Fig. 4). The decision isn't periodic now, there are positive values that results in divergence of the right part and the ionic stream is locked by the own charge. The curve III in Fig. 2 represents dependence of the electron velocity on potential in this case dependence isn't monotonous velocity has an extremum, i.e. there is an area of values of potential where there is an acceleration both the ions and the electrons.

5 Conclusions

This research studied the behavior of ion and electronic collisionless system with self-consistent electric field. Electrons can be represented as particle flow and cloud flow with zero average velocity and relatively high temperature. Cold ions can be accelerated to energy exceeding temperature of electrons, i.e. their velocity can exceed ion-acoustic velocity if there an electron flow with high directing velocity. As maximum potential difference over a particular period of the time is determined by the value $C_0 = \zeta_0 \varkappa T$, the main problem is to create electron flow characterized with high directed velocity. If $\zeta_0 = 4$ drift velocity shall constitute $\sim 2v_{Te}$. In the case of both electrons and clouds exist in the system, there is an area of values of parameters at which there is a simultaneous acceleration of electrons and positively loaded ions.

The technique of the description of the self-coordinated system of the real work can be useful to the solution of more complex challenge — studying of the electronic and ion bunch limited in the cross direction.

Problems of the real work were studied also in papers [Kovalenko, Chernyshev, and Chikhachev, 2011b], [Chikhachev, 2013].

References

Chikhachev, A.S. (2013) Ion-electronic ensemble in electric field. *Uspehi prikladnoy Fiziki*, vol. 1, no. 2, pp. 189–193.

Ermilov, A.N., Kovalenko, Yu.A., at al. (2008) Experimental investigations of a model breadboard of Hall thruster. *High Temperature*, vol. 46, no. 4, pp. 535–541.

Kovalenko, Yu.A., Chernyshev, T.V., and Chikhachev, A.S. (2011) Acceleration of heavy ions in the quasyneutral regime. *Technical Physics*, vol. 56, no. 5, pp. 725–727.

Kovalenko, Yu.A., Chernyshev, T.V., and Chikhachev, A.S. (2011) Acceleration of heavy ions at equality of streams of ions and electrons. *Izvestiya Akad. Nauk*, ser. "Energetika", no. 4, pp. 24–28.

Kovalenko, Yu.A, and Chikhachev, A.S. (2013) Dynamics of an ion flow in an electron layer. *Technical Physics*, vol.58, no. 5, pp. 770–773.

Kovalenko, Yu.A., Chernyshev, T.V., and Chikhachev, A.S. (2013) Acceleration of an ion-electron flow in a plane gap. *Technical Physics*, vol. 58, no. 4, pp. 608–611.

Lohder, Ya.I., and Ulyanov, K.N. (2013) Theory of the negative anode drop in low-pressure discharges. *High Temperature*, vol. 51, no. 1, pp. 7–16.

Riemann, K.-U. (1991) The Bohm criterion and sheath formation. *J.Phys.D: Appl. Phys.* vol. 24, pp. 493–519.

Sternberg, N., and Godyak, V. (2007) The Bohm plasma-sheas model and the Bohm criterion revisted. *IEEE Transaction on Plasma Science*, vol. 35, no. 5, pp. 1341–1349.