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INTERMITTENCY INDUCED IN A BISTABLE MULTISCROLL ATTRACTOR BY MEANS OF STOCHASTIC MODULATION

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Abstract

In this work, numerical results of a nonlinear switching system that presents bistable attractors subjected to stochastic modulation are shown. The system exhibits a dynamical modification of the bistable attractor, giving rise to an intermit behavior, which depends of modulation strength. The resulting attractor converge to an intermittent double-scroll, for low amplitude modulation, and a 9-scroll attractor for a higher applied noise amplitude. A Detrended Fluctuation Analysis (DFA) applied to the x state variable, shows a perturbations robustness region, since the increase of noise does not present changes. Due to the applied noise, the final obtained system has higher randomness, compared with the original one. The understanding of the dynamical behavior of multiscrolls systems is highly important for advancing technology in communications, as well in memory systems applications.

Key words

Multiscroll, Intermittency, Dynamics, Bistability.

1 Introduction

In many nonlinear dynamical systems, intermittency is a common behavior, characterized by irregular burst that modify regular behavior [Manneville & Pomeau, 1979]. Different types of intermittency can be mentioned, e.g., type I and on-off intermittency are related with saddle-node bifurcations, type II and type III with Hopf and inverse period-doubling bifurcations, respectively, and crisis-induced intermittency with a crisis of chaotic attractors [Manneville & Pomeau, 1979; Hirsh, Nauenberg, & Scalpino , 1982; Hirsh, Huberman, & Scalpino; Hu, & Rudnick]. For systems that show multistable behavior, noise presence can be useful to influence interesting dynamics as hopping attractor [Kraut, & Feudel; Huerta-Cuellar et. al.; Pisarchik et. al.], as physical and natural phenomenons [Huerta-Cuellar et. al.; Pisarchik et. al; Gelens et. al.]. Also, noise can induce on-off intermittency in those systems that exhibit bistable behavior [Campos-Mejia, 2013]. A system with periodic potential in the high frequency regime could shows the occurrence of intermittency as the case of a pendulum, where the linear-response theory yields maximum frequency-dependent mobility as noise strength function [Saikia et al., 2011].

In the case of systems that generates intermittent activity, an analysis and characterization of the equilibrium points number in a Chua multiscroll system by applied noise was shown by [Arathi, Rajasekar, & Kurths]. In that sense, the generation of systems with scrolls in their phase space, such as the Lorenz and Chua systems [Lorenz, 1963; Chua, 1992], have been extensively studied from a dynamical point of view, being the Lorenz attractor a particular case with intermittent behavior [Manneville & Pomeau, 1979]. Over the past few years, the design and control of systems with multiple scrolls have been a subject of interest for the scientific community [Echenausía-Monroy, & Huerta-Cuéllar], having a great impact in their application, such as secure communication systems, neural modeling, generation of pseudo-random systems, and deterministic Brownian motion [Kwon et al., 2011; Yalcin et al., 2004; Huerta-Cuellar et al., 2014]. Recently a bistable multiscroll family has been presented and characterized [Echenausía et al., 2018]. In such system it is possible to obtain different behaviors by means of bifurcation parameter variation (ζ). By considering the results shown in [Echenausía et al., 2018], in this work a study on noise-induced intermittency between coexisting regimes of 1-scroll behavior, under the influence of external Gaussian noise, is carried out. The observed intermitent dynamics may be associated with on-off intermittency. The interest in intermittent fluctuations arises from its usefulness for both technical applications and fundamental research. There are some works that presents useful memory applications based on multiscrolls [Itoh & Chua, 2008; Pham et al., 2019], in this case the applied noise could be implemented to dynamically change the system memory properties.

2 Theory

Consider a set of deterministic nonlinear differential equations, with chaotic behavior, defined as in [Campos-Cantón et al., 2010].

$$\dot{\chi} = A\chi + B, \tag{1}$$

where $\chi = [x_1, x_2, x_3]^T \in \mathbb{R}^3$ is the state variable, $B = [b_1, b_2, b_3]^T \in \mathbb{R}^3$ stands for a real constant vector. The behavior of the system is defined by the eigenvalues of the matrix $A \in \mathbb{R}^{3\times3}$ which is given as a linear operator as follows:

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}. \tag{2}$$

Discarding any combination of eigenvalues that is not characteristic of hyperbolic-saddle-node points, that is, the system is consistent with the Unstable Dissipative System type I (UDS I), defined in [Campos-Cantón et el., 2012; Ontañon et al., 2012].

A switched system is implemented, constituted by a set of equations in form of *eq. (1)*, in order to change the dynamical behavior which is governed by a switching law, S_i with $i = 1, ..., n$, and $n \geq 3$. Each system S_i has a domain $\mathcal{D}_i \subset \mathbb{R}^3$, containing the equilibrium $\chi_i^* = -A_i^{-1}B_i$. Then, the switching law governs the SW dynamics by changing the equilibria from χ_i^* to χ_j^*, $i \neq j$, when the flow $\Phi^t : \mathcal{D}_i \longrightarrow \mathbb{R}^3$ crosses from the $i - th$ to the $j - th$ domain.

In order to generate multiscrolls dynamics, *eq. (1)* must to accomplishes with the UDS I definition. A UDS type I corresponds to equilibrium points with a real negative eigenvalue (λ_1) with two complex conjugated with positive real part (λ_2, and λ_3), with $\Sigma(\lambda_{1,2,3}) < 0$, and a UDS type II is defined by equilibrium points with a real positive eigenvalue with two complex conjugated with negative real part, with $\Sigma(\lambda_{1,2,3}) < 0$ [Campos-Cantón et al., 2010].

Multiscroll generation by means of series of saturated functions results in a better way to control the generated fixed points, than with Piece Wise Linear (PWL) systems [Lü et al., 2004; Lü et al., 2006]. This paper is based on a multiscroll generator system aimed by the jerk equation, with the implementation of a Saturated Non Linear Function (SNLF), as switching law, resulting in the system described by *eq. (3)*.

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -\alpha_{31}x - \alpha_{32}y - \alpha_{33}z + \alpha_{31}f(x; k, h, p), \end{aligned} \tag{3}$$

been $f(x; k, h, p)$ the SNLF, defined as:

$$f(x; k, h, p) = \sum_{m=-p}^{p} f_m(x; k, h), \tag{4}$$

for which $k > 0$ is the slope of the saturated function series, $h > 2$ is the saturated delay time of the saturated function, defined by the op-amp switching speed, p is a positive integer, $m = 1, 2, \ldots, n$, where n defines the number of scrolls to generate. The function segmentation is defined as follows:

$$f_m(x; k, h) = \begin{cases} 2k & \text{if } x > mh + 1, \\ k(x - mh) + k, & \text{if } x - mh \leq 1, \\ 0 & \text{if } x < mh - 1, \end{cases} \tag{5}$$

$$f_{-m}(x; k, h) = \begin{cases} 0 & \text{if } x > mh \pm 1, \\ k(x \pm mh) - k, & \text{if } x \pm mh \leq 1, \\ -2k & \text{if } x < mh - 1, \end{cases} \tag{6}$$

The SNLF contemplated for this study is constructed based on [Echenausía et al., 2018], studied with a bifurcation parameter ζ. This ζ parameter works as an individual control gain for the nonlinear function. This allows the possibility to analyze the bifurcation diagrams for a defined set of parameters in the model and generates more than one single attractor. The modified system is described by:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -\alpha[x + y + z - \zeta f(x; k, h, p)], \end{aligned} \tag{7}$$

where $0 < \alpha < 1$ is a system parameter that modifies the equilibria stability, and ζ is the bifurcation parameter. This modification allows to analyze the behavior

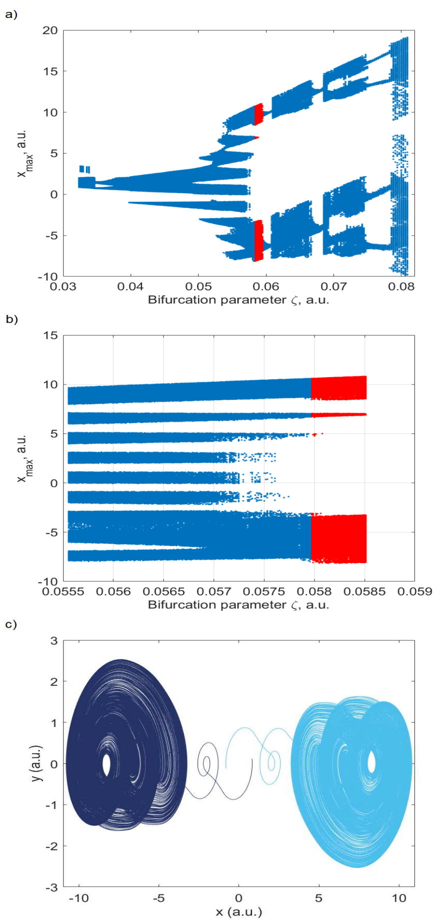

Figure 1. (a) Bifurcation diagram that shows monostable and bistable behavior for $\zeta < 0.0570$, and $\zeta > 0.0570$, respectively, corresponding to *eq. (7)*. (b) Zoom of the interest region for $\zeta = 0.0585$, red fringe corresponds to a bistable multiscroll attractor. (c) Phase space (x, y), of the bistable attractor, negative part (dark blue) which corresponds the initial conditions $[0.1, 0.1, 0.1]$, same ones used in this work.

of the system through numerical simulation in a better way. The system shown in *eq. (7)*, can change behavior from 1, 3, 5, 7, and 9-scrolls monostable attractor, and generate six bistable 1-scroll attractors, by means of bifurcation parameter variation.

In this work, an additive random value is applied as additive modulation of z state of *eq. (7)*, which is generated from the Box-Müller method defined as follows [Box & Müller, 1958]:

$$\begin{aligned} \omega_0 &= \sqrt{-2lnU_1}cos(2\pi U_2), \\ \omega_1 &= \sqrt{-2lnU_2}cos(2\pi U_1), \end{aligned} \tag{8}$$

for which U_1 and U_2 are random values from interval of -1 to 1, ω_0 and ω_1 are independent variables with standard deviation equal to 1, and changes its values for each simulation step.

3 Methodology and Results

From the dynamical system in *eq. (7)*, with a switching law that creates a 9-scroll attractor, a bifurcation diagram constructed, generated by a gradual change of the bifurcation parameter ζ, and a fixed parameter $\alpha = 0.45$, is shown in Figure 1(a). A value of α parameter higher than 0.45 causes a more restrictive dynamical behavior of the system [Echenausía et al., 2018]. Figure 1(b), shows a zoom of the region to analyze with $\zeta = 0.0585$, which generates a bistable behavior.

Noise addition to the *eq. (7)* can stimulate jumps between each of the bistable states. This bistable region forms a double potential well, for which the system response only have one state each time. With the noise amplitude increasing is possible to change the average residence times of the system until get an equilibrium of jumps between the two states. Without loss of generality, the equation system with the added noise is as follows:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -\alpha[x + y + z - \zeta f(x; k, h, p)] + N\eta, \end{aligned} \tag{9}$$

where N is the noise amplitude, and η represents the added noise, generated by the Box-Müller method. The noise addition is made for each itteration of the numerical system, in order to have a perturbation for every simulation time.

Next results are obtained by numerical simulations, by using *eq. (9)* in the bistable region (Figure 1(a), red fringe), $\zeta = 0.0585$, and fixed initial conditions $(x = 0.1, y = 0.1, z = 0.1)$, which generates the behavior shown in Figure 1(c), dark blue color. Figure 2, shows some temporal series of the observed behavior, where it can be seen the temporal evolution of the system. When noise is increased, jumps between the two possible initial states appears, then it is possible to observe that the residence time in each state comes smaller and with similar probability.

If the applied noise has a low amplitude, probability of jumping to the other state is low, but increasing the noise intensity the double-well potential is tilted symmetrically up and down, in this way periodically raising and lowering the potential barrier.

Figure 3 shows the (x, y) phase space of the corresponding temporal series presented in Figure 2. Here it is possible to observe the initial attractor phase space, for a noise $N = 0$, and its evolution when noise is increased. The changes in the dynamical response presents a noisy attractor that jumps between the commutation surfaces of the 9-scrolls defined dynamic, from which the system is generated.

One statistical tool used to evaluate fluctuations of systems is the Detrended Fluctuation Analysis (DFA) [Peng et al., 1994]. The DFA allows to measure a simple quantitative parameter, the scaling exponent β_ν which characterizes a signal correlation properties. The main ad-

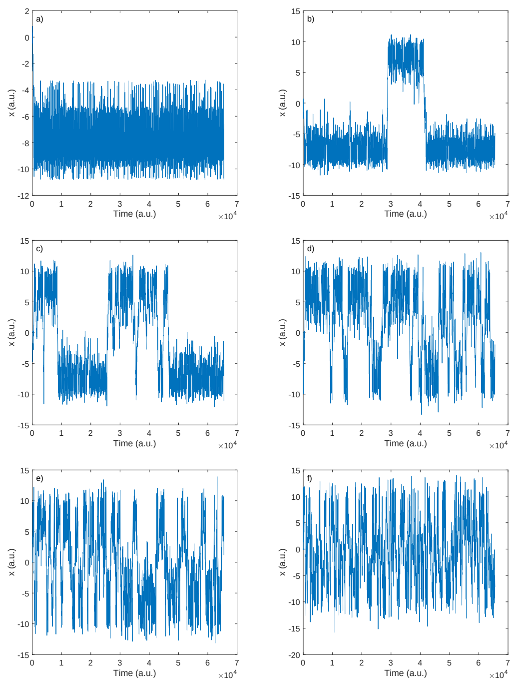

Figure 2. Temporal series obtained from the *eq. (9)* for different noise amplitude: (a) $N = 0$, (b) $N = 1$, (c) $N = 1.5$, (d) $N = 2.5$, (e) $N = 3$, and (f) $N = 4$. Noise amplitude values in arbitrary units.

vantage of the DFA, over many other methods, is that allows the detection of long-range correlations of a signal embedded in seemingly nonstationary time series, and also avoids the spurious detection of apparent long-range correlations that are an artifact of nonstationarity. Fluctuation function $F(\nu; s)$ obeys the following power law scaling relation:

$$F(\nu; s) \sim s^{\beta_\nu}, \qquad (10)$$

for which the time series is segmented in s pieces with length ν. When the scaling exponent $\beta_\nu > 0.5$, three distinct regimes can be defined as follows:

1. If $\beta_\nu \sim 1$, DFA defines $1/f$ noise.
2. If $\beta_\nu > 1$, DFA defines a non stationary or uncorrelated behavior.
3. If $\beta_\nu \sim 1.5$, DFA defines Brownian motion or Gaussian noise.

In order to analyze the noise effects over the dynamical changes of the bistable studied system, a noise amplitude variation $0 \leqslant N \leqslant 4$ in steps of $\Delta_N = 0.001$, was applied. Figure 4 is made by considering the average of 50 temporal series for each noise amplitude. Figure 4 shows the evolution of the slope obtained by the DFA method, where β_ν remains unchanged until a value $N = 0.245$. It can be observed, from the mean line, that a change in the slope occurs after $N > 0.245$, and then it continues increasing until a laminar region for $\beta_\nu = 1.34$, with $1.23 \leqslant N \leqslant 2$, showing a perturbations robustness region, then the slope value decreases slowly with the noise amplitude increment.

From Figure 2((b)-(f)), it is possible to see an intermittent behavior evolution with the noise increase. In Figure 3 it can be seen that the obtained intermittency not only visits the commutation surfaces defined for the bistable attractor, but also visit and lives in all the switching surfaces that creates the natural attractor. As can be seen in Figure 5(b), for $\zeta = 0.056$, the obtained behavior corresponds to a 9-scrolls monostable attractor, for which each scroll have a equilibrium point, and in the case of $\zeta = 0.0585$, the behavior corresponds to the bistable attractor which has been studied. In the case of the bistable attractor, each scroll have oscillations around two fixed points, depending on the initial conditions. In Table 1, the equilibrium location for the 9-scrolls attractor and the bistable analyzed attractor is displayed.

As mention in section 2, the equilibrium points of the analyzed system (when $N = 0$), behaves according to the equation $\chi^* = -A^{-1}[0, 0, \alpha\zeta f(x; k, h, p)]'$, and have the property that whenever the state of the system start at χ_i^*, it will remain at χ_i^* for all future time, being the system autonomous. When $N > 0$, the system equilibrium respond to the following equation, $\chi^* = -A^{-1}[0, 0, \alpha\zeta f(x; k, h, p)]' - A^{-1}[0, 0, N\eta]$, where η is a time dependent function which varies in every iteration, becoming the system into a non-autonomous one,

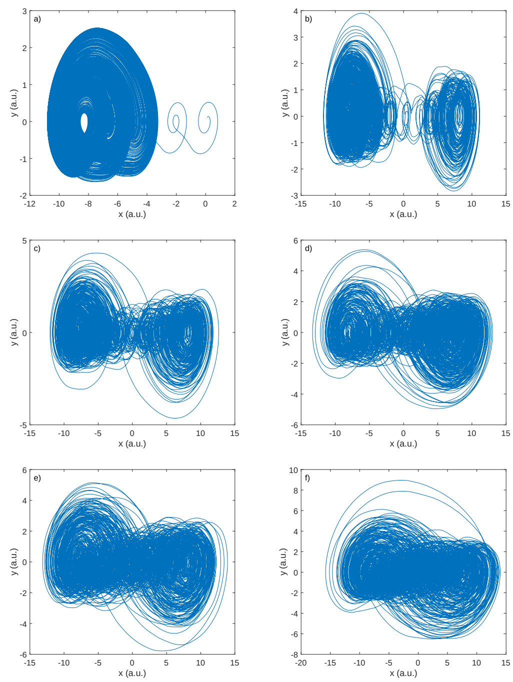

Figure 3. Phase spaces (x, y), corresponding to the temporal series in Figure 2, obtained from the *eq. (9)* for different noise amplitude: (a) $N = 0$, (b) $N = 1$, (c) $N = 1.5$, (d) $N = 2.5$, (e) N= $N = 3$, and (f) $N = 4$. Noise amplitude values in arbitrary units.

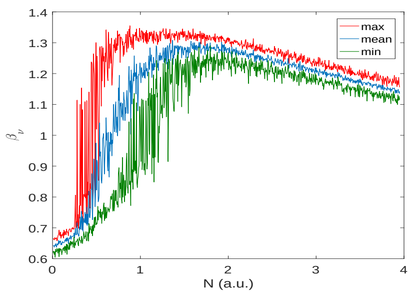

Figure 4. With the increasing of the applied noise amplitude it can be seen the evolution of the slope β_ν with the maximum, minimum and mean values.

thus the system does not have equilibrium points [Khalil, 2002]. An important point to remark is that the vector field associated to linear operator A oscillates in the range of x state, that depends of noise amplitude.

After the applied noise, the system jumps between all the commutation surfaces unachievable for $N = 0$. As seen in Figure 5, the noise amplitude $1.23 \leqslant N \leqslant 2$ can offer the possibility to distinguish the natural attractor structure. Moreover, the induced stochastic dynamics between the nine domains in the system can not be observed in Figure 3(d), (e) and (f), because of the noise amplitude ($N > 2$).

In Figure 5, a comparison between the monostable attractor (1(c), dark blue color), with added noise $N = 1.5$, and the 9-scrolls natural attractor is shown. As mention before the noise induce a visit of the nine domains in the system, for $N \backsimeq 1.5$, shown in Figure 5(a).

Table 1. Equilibrium points location for the analyzed attractors, $(\chi_i^*, 0, 0)$, $N = 0$.

Equilibrium points for the 9-scrolls attractor, $\zeta = 0.056$								
χ_1^*	χ_2^*	χ_3^*	χ_4^*	χ_5^*	χ_6^*	χ_7^*	χ_8^*	χ_9^*
-8	-6	-4	-2	0	2	4	6	8
Equilibrium points for the bistable attractor, $\zeta = 0.0585$								
χ_1^*	χ_2^*	χ_3^*	χ_4^*	χ_5^*	χ_6^*	χ_7^*	χ_8^*	χ_9^*
-8.3	-6.2	-4.1	-2.1	0	2.1	4.1	6.2	8.3

4 Conclusion

In this work numerical simulations over a bistable attractor generated by means of a multiscroll system with added noise are presented. Results are obtained over a bistable region which is very close of the natural attractor of the system, modulated by means of Gaussian noise. It was observed that an intermittent attractor appears between the initial states, but with the noise amplitude increase, stochasticity is higher and domains are visited with the absence of fixed points. A DFA analysis reveals that for noise amplitude $N = 0.245$, the intermittent attractor appears, for $1.23 \leqslant N \leqslant 2$, the fluctuation analysis remains with a constant slope $\beta_\nu = 1.34$, and for $N > 2$ the behavior turns to uncorrelated, showing an indistinguishable attractor in the phase space. In the case of noise amplitude $1.23 \leqslant N \leqslant 2$, the system shows a constant behavior between Brownian motion and uncorrelated fluctuations. There are several works that presents multiscrolls attractors useful to the design of systems with memory applications, in this case, the added noise could be applied to dynamically change the memory properties of the system, or in modeling systems immune to perturbations.

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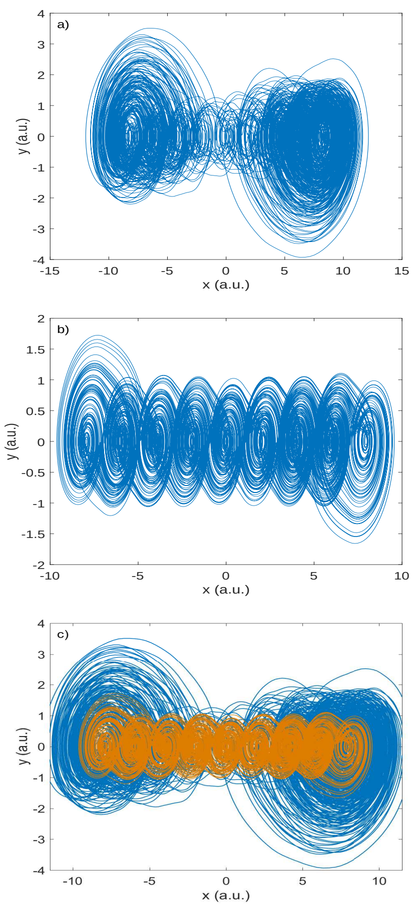

Figure 5. Phase spaces (x, y), corresponding to temporal series, (a) shows intermittent behavior for a noise amplitude $N = 1.5$, (b) shows the natural attractor of 9-scrolls, and (c) shows that the intermittent attractor visits the same commutation surfaces defined for the natural attractor.

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