ADAPTIVE ROBUST DAMPING OF THE INERTIAL-SENSOR ERRORS DURING INTEGRATED PRIMARY AND SECONDARY PROCESSING OF SIGNALS

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Abstract

This paper is devoted to the problem of damping of random residual drifts and random residual biases of the output signals of sensors of strapdown inertial navigation systems. (SINSs). The proposed solutions of the above problem rely on the implementation of closed-loop damping schemes. The mathematical support of such systems is based on joint procedures for the estimation of SINS errors and for subsequent compensation for such errors. The accomplishment of the purpose of this paper is based on the bringing of the problem of digital signal processing to a Kalman construction and on the tuning of such a construction to the functioning under a priori uncertainty and possible discordant measurements. The results of experimental studies are given, which corroborate the effectiveness of applying the proposed approach in practice.

Keywords

Nonlinear system, stochastic system, applications

1 Introduction

Accuracy and reliability of strapdown inertial navigation systems (SINSs) are dependent on the technical condition of sensors, i.e., of gyros and accelerometers.

The present-day level of the development of onboard electronics permits one to estimate the inertial-sensor errors and to compensate for such errors in the course of primary signal processing in real time. To do this, use can be made of mathematical and software tools that were formerly employed only for a second information processing in the integration of navigation systems (NSs). Such tools rely on the models of NS errors and on the Kalman filtering of the noise of observations. An analysis of studies in the field of integrated primary processing of inertial-sensor signals indicates that the possibility exists of a practical implementation of analytical approaches to the improvement of the sensors accuracy characteristics.

The purpose of this paper is an increase in the accuracy characteristics of SINSs, based on the estimation of inertial sensors instrumental drifts and on compensation for such drifts at the level of primary and secondary signal processing.

2 Structure of a System for the Primary Processing of Inertial-Sensor Signals

The accomplishment of the purpose formulated in the present paper is based on the bringing of the problem of digital signal processing to a Kalman construction and on the tuning of such a construction to the functioning under a priori uncertainty and possible discordant measurements. As in known [Schmidt, 2004], estimation systems that are Kalman ones in structure include loops intended for parameters prediction and for their updating on the basis of observation processing. When implementing the prediction loop, provision should be made for models that reflect variations in sensor output signals between the sessions where observations are formed. We propose that such models should be constructed, on a real-time basis, from the moving sample of readings of sensor signals by the use of the Chebyshev orthogonal polynomials. In view of the smoothing properties of the Chebyshev polynomials, it is apparently also possible to perform preliminary restoration of the valid signal at the prediction stage. Updating of the predicted signal and estimation of the instrumental drifts of sensors are realized from the processing of observations. As observations, we propose that the residual between the predicted and actual sensor signals should be used, along with the appropriate invariants. The invariants can be “a priori” known physical quantities, such as a change in the rotation angle of an inertial measurement unit, a change in the sensor output signal of the appropriate order, etc.

The block diagram of the proposed system for primary signal processing, together with the loop intended to estimate and compensate sensor errors of a laser inertial measurement unit (LIMU) is shown in Figure.1, where the following notation is introduced: Θ_i are observed readings of the ring laser gyro (RLG) output signal; $\hat{\Theta}_{i/i-1} = \sum_{k=0}^{m} \hat{q}_k P_k(t_i)$ is the predicted value of the RLG output signal; $P_k(t_i)$ are the Chebyshev normalized orthogonal polynomials; P_m is the module for adaptive polynomial smoothing. Adaptation proceeds by use of recurrence smoothing of a retrospective sample of RLG signal readings from the observed results; $\hat{q}_k$ are weight coefficients;

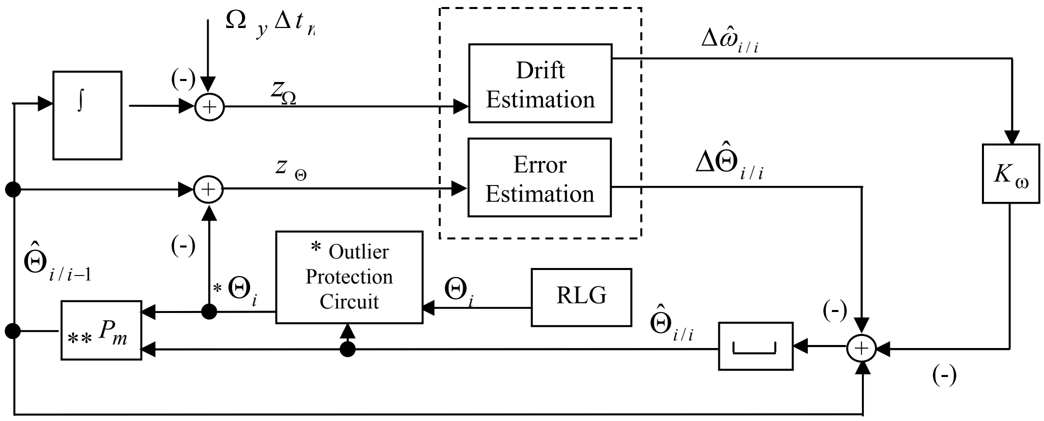

Figure. 1 Block diagram of the system for primary signal processing

$v_j = z_j - \hat{z}_j$ is the residual between the actual value z_j and the predicted value $\hat{z}_j = H_j m_j$ of observations; m_j, $\hat{x}_{i/i}$ are the estimates of the error vector $x_i = [\Delta\Theta_i \ \Delta\omega_i]^T$ at the *i*-th step after the *j*-th component and the whole vector z_i of observations are processed, respectively; H_j is the row vector of coupling coefficients; $\Delta\Theta_i$, $\Delta\omega_i$ are the angular error of the RLG and its instrumental drift at the *i*-th instant of time t_i, respectively; α_j is a scaling parameter; $\sqcup$ is the delay by one bit; $\Omega_y \Delta t_n$ is the rotation angle (an invariant) of the LIMU *oy*-axis in the inertial space over the time $\Delta t_n = t_i - t_{i-n}$ when the base has no motion with reference to the Earth; $K\omega$ is a damping coefficient.

3 Analysis of Schemes for the Damping of INS Errors

The damping of SINS errors includes the procedures of their estimation and subsequent compensation for such errors. In its turn, compensation procedures provide for the updating of SINS parameters from estimates, with due regard for the predicted dynamic behavior of errors. The evolution of methods intended for the damping of SINS errors is dependent on the feasibility of hardware support of computational procedures. Modern onboard computers perform the estimation of SINS errors by application of an extended Kalman filter (EKF) [Schmidt, 2004] and its modifications [Chernodarov, Patrikeev, Budkin, Golikov, Larionov, 2004]; moreover, such computers implement different compensation procedures for the estimates obtained. At the same time, studies [Rogers, 2003, Titterton, 2004], conducted in the field of updatable INSs point to the pressing necessity of making improvements in compensation schemes for SINS errors, which operate effectively under the conditions of “a priori” uncertainty and dynamic noise environment.

A traditional scheme is an open-loop one intended to compensate for the estimates of SINS errors; this scheme is shown in Figure 2, where the following notation is introduced: Y_{LIMU} is the vector of parameters that are reckoned by the LIMU; Y_{GPS} is the vector of parameters that are reckoned by the GPS; *z* is the vector of observations; $\hat{x}$ is the vector of estimates of SINS errors.

In such a scheme, SINS updated parameters are not used in algorithm for inertial reckoning.

Furthermore, accumulated estimates maintain EKF smoothing properties with regard to short-time discordant observations. The effectiveness of error damping according to the open-loop scheme is limited by high-accuracy SINSs with precision sensors, which are functioning under “a priori” known noise conditions. In such SINSs, sensor precision is maintained at the hardware level and also by means of factory calibration and preliminary calibration.

It is assumed that, in this case, the linearity of LIMU errors with respect to the reference phase path and hence the correctness of EKF inclusion in an SINS configuration must be ensured.

One can essentially reduce requirements that are imposed for sensors and for their service conditions when use is made of a closed-loop scheme for the damping of SINS errors.

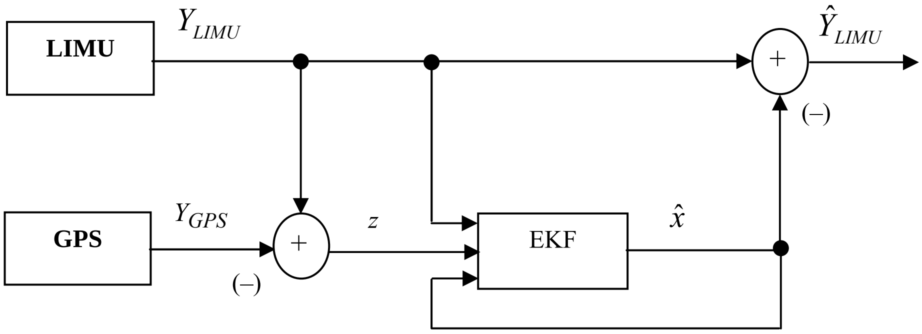

Figure 2. Open-loop scheme for the damping of LIMU errors

Such a scheme is generally implemented as shown in Figure 3. In addition to the foregoing notation, in figures 3 the following designations are used: K_x is a damping coefficient; RKF is a robust Kalman filter. With analytical compensation for the estimates of SINS errors by a closed-loop scheme, the updated parameters are used in algorithm for inertial reckoning. Here estimate compensation may be either complete compensation ($K_x = 1$) or a partial one ($0 < K_x < 1$). Under complete compensation [Maybeck, 1982], the estimates are not accumulated and the EKF may lose its smoothing properties with regard to discordant observations. That is why, the need arises for robust protection [Chernodarov, Patrikeev, Budkin, Golikov, Larionov, 2004] of the EKF from its divergence. At the same time, when a scheme with estimate partial compensation is implemented, it is essential that estimate damping coefficients should be formed with due regard for the dynamic behavior of SINS errors between observation sessions. By analogy with approaches to hardware integration of navigation systems [Schmidt, 2004], we can form loosely-coupled and tightly-coupled closed-loop schemes for the damping of LIMU errors.

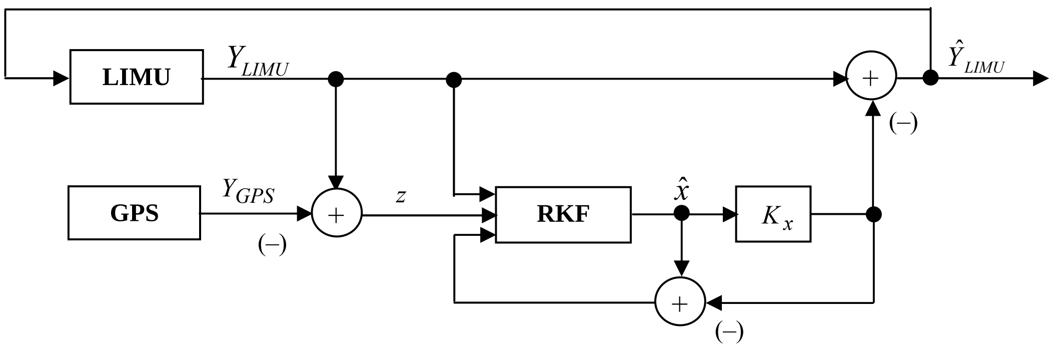

Figure 3 Closed-loop scheme for the damping of LIMU errors

4 Some Peculiarities of the Elaboration of Algorithms for Analytical Compensation for Sensor Errors

In the implementation of a loop meant for analytical compensation for sensor errors, the problem of determining the coefficients K_x of estimate damping still remains topical.

The above coefficients are required to set the level of confidence in the estimates being formed, with due regard for the fact that observations are stochastic in character and for the predicted dynamic behavior of errors. With the proper choice of damping coefficients, the linearity of SINS errors with respect to the reference phase path is maintained. Therefore, nominal conditions of EKF functioning are ensured. However, with complete compensation for the estimates accumulated, the module of error prediction is excluded from the EKF configuration; this module maintains EKF smoothing properties. This is connected with the fact that after such a compensation, the estimates of errors are set equal to zero. That is why, the need arises for the formation of compensation procedures in the form of the control actions $u(t)$, intended to have control over the estimates of SINS errors between instants of time, at which observations come.

Such procedures may be constructed according to the following scheme:

$$d\hat{Y}\big/dt = F(\hat{Y},t) + u(t)\,,$$
$$d\hat{x}\big/dt = A(t)\hat{x}(t) + u(t)\,, \qquad (1)$$

where $u(t) = -K_x(t)\hat{x}(t)$; $\hat{Y} = \hat{Y}(t)$ is the vector of SINS parameters; $\hat{x} = \hat{x}(t)$ is the vector of the estimates of SINS errors; $F(\hat{Y},t)$ is a function that represents, in the general form, the right-hand sides of SINS equations and sensor error equations; $A(t) = \left.\frac{\partial F(Y,t)}{\partial Y}\right|_{Y=\hat{Y}}$ is the matrix of partial derivatives.

To form the matrix of the compensation coefficients $K_x = K_x(t)$, one can use the technique for exponential reduction of the errors of a nonlinear dynamical system [Bezbogov, 1984]. In this case, $K_x = A + \Lambda$, where $\Lambda = diag(\Lambda_1 \ldots \Lambda_n)$ is the diagonal matrix of error damping coefficients $(\Lambda_i > 0)$, and Eqs. (1) take the form:

$$d\hat{Y}\big/dt = F(\hat{Y},t) - (A + \Lambda)\hat{x}\,;\, d\hat{x}\big/dt = -\Lambda\hat{x}\,. \qquad (2)$$

In a discrete case, Eqs. (2) can be shown to have the following form:

$$\hat{Y}_{i/i-1} = \tilde{F}(\hat{Y}_{i-1/i-1}, t_i) - (\Phi_i - \Lambda_{i/i-1})\hat{x}_{i-1/i-1}\,; \quad (3)$$

$$\hat{x}_{i/i-1} = \Lambda_{i/i-1}\hat{x}_{i-1/i-1}\,, \qquad (4)$$

where $\hat{x}_{i/i-1}; \hat{x}_{i-1/i-1}$ are the "a priori" (predicted) and "a posteriori" (after processing the observations) estimates of the vector of errors, respectively; Φ_i is the transition matrix for the vector x_i of errors.

In this paper, a two-level procedure for optimization of the damping loop is proposed. At the first level, robust (with respect to outliers) estimates of SINS errors are formed, and at the second level, coefficients of error damping are formed. The basis for an algorithm for robust estimation is the generalized parameter $\beta_j = \nu_j \big/ \alpha_j$, where $\nu_j = z_j - \hat{z}_j$ is the residual between the actual value z_j and the predicted value $\hat{z}_j = H_j m_j$ of observations; m_j, $\hat{x}_j$ are the estimates of the vector of SINS errors after processingthe j-th component and the whole vector of observations; H_j is the row vector of coupling factors; α_j is a scaling parameter.

The algorithm for robust estimation results from the solution of the following problem:

$$\hat{x}_i = \underset{x_i}{\operatorname{argmin}} \sum_{i=i_0}^{i_f} \rho(\beta_i)\,, \qquad (5)$$

where $\rho(\beta_j) = -\ln f(\beta_j)$ is a likelihood function; $f(\beta_j)$ is a probability density function.

The solution of the problem (5), in detailed form, was presented in [Chernodarov, Patrikeev, Budkin, Golikov, Larionov, 2004]. The novelty of this part of the present paper lies in the algorithm for the optimal damping of SINS errors, which is combined with the robust estimation filter into a single structure.

In the realization of Eqs. (3) and (4), the problem of determining the vector of the coefficients $\hat{\bar{\Lambda}}_i = \left[\hat{\Lambda}_{1(i)}, \ldots, \hat{\Lambda}_{n(i)}\right]^{\mathrm{T}}$, which are optimal ones according to the appropriate criterion still remains topical. We propose that the optimality criterion should be constructed on a basis of the residuals η_i between the "a priori" estimates $\hat{x}_{i/i-1}$ and the "a posteriori" estimates $\hat{x}_{i/i}$ of the vector of errors, i.e., $\hat{\bar{\Lambda}}_i = \arg\min\limits_{\bar{\Lambda}_i} 0.5 \sum_{i=i_0}^{i_f} \eta_i^{\mathrm{T}} P_{i/i}^{-1} \eta_i$, (6)

Where $\eta_i = \hat{\tilde{x}}_{i/i-1}\hat{\bar{\Lambda}}_{i/i-1} - \hat{\tilde{x}}_{i/i}$;

$$\hat{\tilde{x}}_{i/i} = diag\{\hat{\tilde{x}}_{1(i/i)}, \ldots, \hat{\tilde{x}}_{n(i/i)}\}\,; \qquad (7)$$

$\hat{x}_{i/i}$; $P_{i/i}$ are respectively the estimate of the vector of SINS errors and its covariance matrix, which were obtained at the i- th instant of time from i observations.

The solution of the problem (6) on condition that $\hat{\bar{\Lambda}}_{i/i-1} = \hat{\bar{\Lambda}}_{i-1/i-1}$ can be obtained using the Bellman-Shridhar technique [Sage, Melse, 1972] of the form:

$$\hat{\bar{\Lambda}}_{i/i-1} = \hat{\bar{\Lambda}}_{i-1/i-1}\,; \qquad (8)$$

$$S_{i/i-1} = S_{i-1/i-1} + M_i\,; \qquad (9)$$

$$S_{i/i} = S_{i/i-1} - S_{i/i-1}\hat{\tilde{x}}_{i/i-1}^{\mathrm{T}}(\hat{\tilde{x}}_{i/i-1}S_{i/i-1}\hat{\tilde{x}}_{i/i-1}^{\mathrm{T}} + P_{i/i})^{-1}S_{i/i-1} \qquad (10)$$

$$\hat{\bar{\Lambda}}_{i/i} = \hat{\bar{\Lambda}}_{i/i-1} + S_{i/i}\hat{\tilde{x}}^{\mathrm{T}} P_{i/i}^{-1}\eta_i\,, \qquad (11)$$

where $\hat{\bar{\Lambda}}_{i/i-1}$, $\hat{\bar{\Lambda}}_{i-1/i-1}$ are respectively the predicted and updated (by the residual η_i) estimates of the vector of coefficients that characterize the values of uncompensated errors; $S_{i/i}$ is the covariance matrix for the errors of forming the compensation signals, which are caused by dynamic noise environment; M_i is the matrix for coefficients of the intensity of disturbances in the compensation channel.

When executing the algorithm (8) – (11), the need arises for the inversion of the matrix in Eq. (10). The following solutions of the above problem are feasible:

- employment of the U-D technology for the realization of covariance equations [Chernodarov, Enyutin, Patrikeev, 2000];
- setting a limit on the number of parameters that have variable damping coefficients;
- approximate solution of Eq. (10) on the basis of sequential processing of the components of the vector of residuals (7).

Thus, the algorithm (8) – (11) reflects the technology intended for adaptive damping of INS errors from their estimates that are formed by the RKF.

5 Analysis of the Results of Studies

A three-axis inertial measurement unit based on the ZLK-16 Zeeman-type ring laser gyros [Azarova, Golyaev, Dmitriev, et. al. 1999] has been the object of our studies. Random angular drifts of uncalibrated RLGs of such a type are of the order of $0.1 \div 0.5$ deg/h. Hardware that was formed according to the "effectiveness-cost" criterion gives ground to classify the SINS version presented here among the systems of intermediate accuracy. Such systems are just the ones in which it is apparently expedient to implement analytical compensation for sensor residual drifts.

The results of a comparison analysis of SINS operation when using open-loop and closed-loop schemes for the damping of sensor errors were obtained on a basis of the reckoning of motion parameters from the recorded signals of sensors.

Figure 4 shows the following: an output signal (the light-colored graph, arc sec) of the LIMU "vertical" gyro; a signal smoothed through the use of an adaptive robust polynomial procedure (the dark-colored graph) of the same gyro. The smoothing was performed with a frequency of picking the signal of the RLG equal to 1 kHz. In Fig. 5 is depicted an actual instrumental RLG drift , which was determined as a mean value of zero bias on time intervals equal to 60 s. The estimates of the instrumental drift of the above-mentioned gyro (of its autocorrelated component) was obtained when processing the observations z_Ω (see Fig.1) with a frequency of 0.5 Hz. In Figs. 6 and 7, the errors $\delta\hat{\omega}$ of estimating the RLG drift $\Delta\omega$ are shown. Fig. 6 reflects the dynamic behavior of error when RLG drift is damped, and Fig. 7 reflects the above behavior when the RLG drift are not damped.

An analysis of the results obtained has shown that the polynomial smoothing with a loop for estimation and damping of random residual drifts of the output signals of sensors permits one to reduce the RLG angular error by no less than an order of magnitude.

Θ, *arc sec*

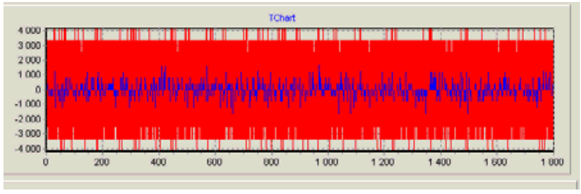

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Figure 4 Output signal of the LIMU

$\Delta\omega$, *deg/h*

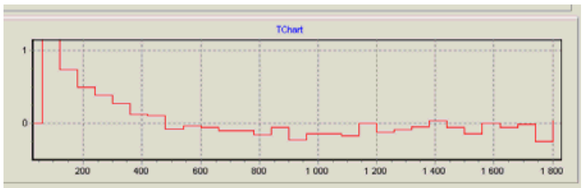

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Figure 5 Actual instrumental RLG drift

$\delta\hat{\omega}$, *deg/h*

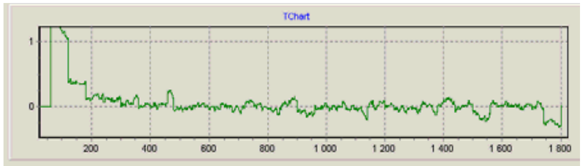

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Figure 6 Error of estimating the RLG drift

$\delta\hat{\omega}$, *deg/h*

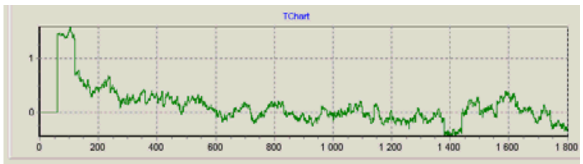

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Figure 7 Error of estimating the RLG drift

Conclusion

In the paper presented here, the authors draw your attention to the importance of systems approach to the improvement of the operational characteristics of integrated navigation systems. Such approaches enable us to combine the capabilities of algorithmic and hardware means intended to improve the accuracy and reliability of SINSs. Moreover, algorithmic means may be of preferable importance in SINSs built around inexpensive sensors of poor and intermediate accuracy. The implementation of closed-loop schemes intended for the damping of sensor drifts is based on joint procedures for estimation of and for analytical compensation for SINS errors. The continuous evolution of such schemes meant for SINS construction is closely connected with problems of the optimal control of stochastic systems

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