Passification-Based Adaptive Control with Implicit Reference Model ⋆

Alexander L. Fradkov and Boris Andrievsky *

* *Institute for Problems of Mechanical Engineering of Russian Academy of Sciences, 199178, 61, V.O. Bolshoy, Saint Petersburg, Russia*
e-mail: {fradkov_at_mail.ru, bandri_at_yandex.ru}

Abstract: A brief survey of the passification method in adaptive control based on applying the Yakubovich–Kalman–Popov Lemma to adaptive control systems is presented. The basics of the method were established in 1974 in the paper Fradkov, A. L. (1974). Design of an adaptive system of stabilization of a linear dynamic plant. *Autom. and Rem. Control,* (12), 1960–1966. Various types of the adaptive control systems with implicit reference model such as the systems of stabilization and tracking with the prescribed dynamics, systems with adaptive tuning of the low order control laws, and combined signal-parametric system are described. Description of the shunting method in the adaptive control problem is given. Some experimental adaptive control results for the "Helicopter" benchmark are described.

Keywords: method of passification, adaptive control

1. INTRODUCTION

The method of passification was born in 1974 (Fradkov, 1974) and since then was applied to a variety of design problems for nonlinear and adaptive control systems (see (Fomin *et al.*, 1981; Fradkov, 1990; Fradkov *et al.*, 1999; Andrievskii *et al.*, 1988; Andrievsky and Fradkov, 1994; Andrievskii *et al.*, 1996; Andrievskii and Fradkov, 1999)). Our briev survet shows that the method indeed results in simple adaptve control systems.

Consider systems affine in control

$$\dot{x} = f(x) + g(x)u, \quad y = h(x), \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^l$ are, respectively, the vectors of state, input, and output, $f(\cdot)$, $h(\cdot)$ are smooth vector functions of the argument x, and $g(\cdot)$ is the smooth matrix function. Let G be the given $m \times l$ matrix.

Definition 1. System (1) is called *G-passive* if there exists a nonnegative scalar function $V(x)$ (storage function) satisfying inequality

$$V(x) \le V(x_0) + \int_0^t u(t)^* G y(t)\, dt \tag{2}$$

for any solution $x(t)$ of system (1) with $x(0) = x_0$, $x(t) = x$. The system is called *strictly G-passive* if there exist a nonnegative scalar function $V(x)$ and a scalar function $\mu(x)$ such that $\mu(x) > 0$ for $x \ne 0$, (dissipation rate) satisfying inequality

$$V(x) \le V(x_0) + \int_0^t u(t)^* G y(t) - \mu(x(t))\, dt \tag{3}$$

In (2) and below the asterisk stands for matrix transposition and complex conjugation of its arguments (in the real case, simply transposition) . In what follows, we will discuss the strict G-passivity of linear systems

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{4}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^l$, and A, B, C are the matrices of appropriate dimensions. For linear systems, the storage function $V(x)$ can always be selected as quadratic form $V(x) = 0.5x^* Hx$ (or the Hermitian form in the complex case), and the dissipation rate can be chosen as the squared Euclidean norm $\mu(x) = \mu|x|^2$, $\mu > 0$.

Note that if $l = m$ and $G = I_m$ is the identity matrix, then G-passivity coincides with the ordinary passivity. In turn, passivity is very close to *hyperstability* introduced in 1964 by V.M. Popov for the linear systems (Popov, 1964) and is a special case of dissipativity (Willems, 1972) where integrand in (2) can be an arbitrary function of u, y (or u, x).

Passivity plays an important role in the problems of design of control systems because it is closely related to stability. One can readily see that if the storage function $V(x)$ is positive definite, then for $u = 0$ the passive system (1) is Lyapunov-stable, and for $u = -Ky$ it is asymptotically stable for any scalar or matrix $K > 0$. On the other hand, this property is rather restrictive. For example, for the strictly passive linear system (4) with transfer function $W(\lambda) = C^*(\lambda I - A)^{-1}B = \beta(\lambda)/\alpha(\lambda)$, the polynomials $\alpha(\lambda)$, $\beta(\lambda)$ must be Hurwitz for $m = l = 1$ and the difference of their degrees (so called relative degree) must be unity. That is why an interest arises to the possibility of making this system passive, that is, to passification by

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means of an output or state feedback. In what follows, we consider the following problems of passification by output feedback.

Problem A. Find an m-vector function $\alpha(y)$ $(m \times m)$-matrix function $\beta(y)$ such that system (1) with the output feedback

$$u = \alpha(y) + \beta(y)v, \tag{5}$$

where $v \in \mathbb{R}^m$ is the new input, is strictly G-passive.

Problem B. Find an m-vector function $\alpha(y)$ such that system (1) with the output feedback (5) is strictly G-passive with *fixed* $(m \times m)$-matrix function $\beta(y)$.

For linear systems, a passifying feedback is also sought in the class of linear laws, and the passification problems are formulated as follows.

Problem AL. Find an $m \times l$ matrix K and an $m \times m$ matrix L such that system (4) with the feedback

$$u = Ky + Lv, \tag{6}$$

where $v \in \mathbb{R}^m$ is the new input, $\det L \neq 0$ is strictly G-passive.

Problem BL. Needed is to determine an $l \times m$ matrix K such that system (4) with the output feedback (6) is strictly G-passive with the fixed matrix L.

The problems of *G-passifiability*, that is, determination of the conditions for solvability of problems A, B, AL, and BL, are important for application of the method of passification. Solution of the problems of passification and passifiability AL, BL were formulated in (Fradkov, 2003) for the linear rectangle ($l \neq m$) systems. For the special case of quadratic ($l = m$) linear multidimensional systems, similar problems were considered in (Gu, 1990; Abdallah *et al.*, 1990; Weiss *et al.*, 1994; Huang *et al.*, 1999). Namely, theA special case of $L = K$ was studied in (Gu, 1990; Abdallah *et al.*, 1990), while the results of (Weiss *et al.*, 1994; Huang *et al.*, 1999) apply to the special case of $L = I$.

Introduce the following additional notation to formulate the solutions of the above problems:

$$\delta(\lambda) = \det(\lambda I_n - A), \quad W(\lambda) = C(\lambda I_n - A)^{-1}B,$$
$$A(K) = A + BKC, \quad \delta(\lambda, K) = \det\big(\lambda I_n - A(K)\big),$$
$$W(\lambda, K) = C\big(\lambda I_n - A(K)\big)^{-1}B,$$

where K is an $m \times l$ matrix. Obviously, $\delta(\lambda, K)$ and $W(\lambda, K)$ are, respectively, the characteristic polynomial and transfer matrix of the closed-loop system (4) with the feedback

$$u = Ky + v. \tag{7}$$

Let G be an $m \times l$ matrix. We determine $\varphi(\lambda) = \delta(\lambda)\det GW(\lambda)$, $\Gamma = \lim_{\lambda\to\infty} \lambda GW(\lambda)$. It is possible to show that $\varphi(\lambda)$ is a polynomial of the degree not greater than $n - m$ and invariant to feedback transformation (7). Since $\Gamma = GCB$, the $m \times m$ matrix Γ is also invariant to feedback transformation (7).

Definition 2. System (4) is called *G-minimum phase* if the polynomial $\varphi(\lambda)$ is Hurwitz (its zeros have negative real parts). System is called *strictly G-minimum phase* if it is minimum phase and Γ is nonsingular: $\det\Gamma \neq 0$. System is called *hyper-G-minimum phase* if it is minimum phase and Γ is symmetric and positive-definite: $\Gamma = \Gamma^* > 0$.

Now, it is possible to formulate the solvability conditions for the passification problems AL, BL.

Theorem 1. Let $\operatorname{rank} B = m$. System (4) is strictly G-passifiable by feedback (6) if and only if it is strictly G-minimum phase.

Theorem 2. Let $\operatorname{rank} B = m$. System (4) is strictly G-passifiable by feedback (6) with the fixed matrix L if and only if the system with the transfer matrix $W(\lambda)L$ is hyper-G-minimum phase.

The proofs of Theorems 1 and 2 can be found in (Fradkov *et al.*, 1999; Fradkov, 2003). They are based on solving the following algebraic problem posed and solved in (Fradkov, 1976). Given complex-valued matrices A, B, C, G, and R of respective dimensions $n\times n$, $n\times m$, $l\times n$, $m\times l$, and $n\times n$ ($m \leq n$, $l \leq n$), at that $R = R^* \geq 0$. Find the conditions for existence of the Hermitian $n \times n$ matrix $H = H^* > 0$ and complex-valued $m \times l$ matrix K such that

$$HA(K) + A(K)^*H + R < 0, \tag{8}$$
$$HB = (GC)^*, \tag{9}$$

where

$$A(K) = A + BKC. \tag{10}$$

Solution is provided by the following theorem.

Theorem 3. [(Fradkov, 1976)] For existence of the matrices $H = H^* > 0$, K that satisfy (8), (9), (10) and are real in the real case, it is sufficient and, if $\operatorname{rank}(B) = m$, then necessary, that the system with the transfer matrix $GW(\lambda)$ be hyper-minimum phase.

Note 1. It is possible to demonstrate (Efimov and Fradkov, 2006) that Theorem 3 retains its validity if the matrix $A(K)$ is defined instead of (10), by the relation $A(K) = A + BK$ or by the relation $A(K) = A + KC$.

Note 2. It follows from the proof of theorem that if the hyper-minimum phase condition is satisfied, then one can always select a matrix K satisfying (8), (9), (10) in the form $K = -\kappa G$, where κ is any sufficiently large scalar. At that, the lower boundary κ_0 for κ is as follows (Fradkov, 2003)

$$\kappa > \kappa_0 = \sup_{\omega\in\mathbb{R}^1} \lambda_{\max}\left(\operatorname{Re}\big(GW(i\omega)\big)^{-1}\right), \tag{11}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the matrix.

Theorem 3 provides the solvability conditions for the matrix inequalities relating to the classical Kalman–Yakubovich–Popov Lemma (frequency theorem) for the case of special relations with the form $F(x, u) = y^{\mathrm{T}}u$ in the positive definite matrix H and the feedback matrix K. It may be called the feedback frequency theorem. Theorems 1, 2 and 3 may be called the passification theorems (Andrievskii *et al.*, 1996; Fradkov, 2003; Bobtsov and Nikolaev, 2005). There exist versions of the passification theorems for nonstrict matrix inequalities (weak passification) (Saberi *et al.*, 1990). Passification theorems were extended to the linear distributed systems (Bondarko *et al.*, 1979; Bondarko and Fradkov, 2003) and nonlinear systems (Byrnes *et al.*, 1991; Fradkov and Hill, 1998) and have numerous applications. Below applications to the design of implicit model reference adaptive systems (IMRAS) are considered.

2. APPLICATION OF THE PASSIFICATION METHOD TO THE PROBLEMS OF ADAPTIVE CONTROL

2.1 Adaptive Systems with Implicit Reference Model

Now we turn to the problem of control of the dynamic plants under essential *a priori* parametric uncertainty and the properties of external actions. Adaptation, that is, automatic tuning of the controller in the course of normal operation of the system, is one of the most universal and effective methods of its solution

Adaptive control can rely either on identification of the unknown parameters or on direct tuning of the controller coefficients according to the given performance index (objective functional). The latter approach which is called the *direct adaptive control* is usually based on defining the desired dynamics of the closed-loop system by means of some reference system, *reference model* (Petrov *et al.*, 1972; Landau, 1979). The passification theorem enabled design of adaptive controllers with *implicit reference model* having order much smaller than that of the control plant. The main results are presented below, more detailed presentation can be found in (Fomin *et al.*, 1981; Fradkov, 1990; Fradkov *et al.*, 1999; Andrievskii *et al.*, 1988; Andrievskii and Fradkov, 1999).

Let us consider the control plant (4) assuming for simplicity that $m = 1$. The following problem of adaptive stabilization was formulated in (Fradkov, 1974): find the adaptive output feedback law

$$u = \theta^{\mathrm{T}} y, \quad \dot{\theta} = \Theta(y), \tag{12}$$

allowing system (4), (12) to reach its objective

$$x(t) \to 0, \quad \theta(t) \to \text{const} \quad \text{for} \quad t \to \infty. \tag{13}$$

It is clear that objective (13) will be reached if the system has a quadratic Lyapunov function

$$V(x, \theta) = x^{\mathrm{T}} H x + 0.5(\theta - \theta_*)^{\mathrm{T}} \Gamma^{-1} (\theta - \theta_*) \tag{14}$$

with the properties

$$\begin{aligned} &V(x,\theta) > 0 \quad \text{for} \quad x \neq 0,\ \theta \neq \theta_*; \\ &\dot{V}(x,\theta) < 0 \quad \text{for} \quad x \neq 0. \end{aligned} \tag{15}$$

Existence in system (4), (12) of function (14) with the properties of (15) was shown (Fradkov, 1974; Fradkov, 1976) to be equivalent to the existence of the matrix $H = H^{\mathrm{T}} > 0$ and vector $\theta_* \in \mathbb{R}^l$ satisfying for some $G^{\mathrm{T}} \in \mathbb{R}^l$ the relations

$$HA(\theta_*) + A(\theta_*)^{\mathrm{T}} H < 0, \quad HB = C^{\mathrm{T}} G^{\mathrm{T}}, \tag{16}$$

where $A(\theta_*) = A + B\theta_* C$. In the case of solvable (16), the adaptive controller (12) providing the objective (13) takes on the form

$$u = \theta^{\mathrm{T}} y, \quad \dot{\theta} = -\Gamma(Gy)y. \tag{17}$$

Since (16) is nothing but a special case of relations (8), (9), (10) for $m = 1$, the property of *hyper-minimum phase* of the transfer function $GW(\lambda) = GC(\lambda I - A)^{-1} B$ meaning that $GW(\lambda)$ is minimum phase (its numerator is a Hurwitz polynomial), has unit relative degree (the difference of the degrees of the denominator and numerator), and positive high-frequency gain $GCB > 0$ is the necessary and sufficient condition for solvability of (16) relative to the pair (H, θ_*).

System (4), (17) and its extension to the tracking problems were named the *adaptive systems with implicit reference model (IMRAS)* because the variable $\delta(t) = Gy(t)$ can be shown to tend to zero for $l > 1$ faster than $y(t)$, that is, in the adaptive system $g^{\mathrm{T}} y(t) \cong 0)$ after the transient time. Stated differently, $\delta(t)$ can be interpreted as the generalized error of some implicit reference model. This approach was extended to the distributed (Bondarko *et al.*, 1979; Bondarko and Fradkov, 2003) and delay (Tsykunov, 1984) systems. Algorithm (17) was shown [Ch. 7](Fomin *et al.*, 1981) to reach objective (13) also for the nonlinear plants resulting from introduction into the right-hand sides of (4) of nonlinearities acting additively with control and satisfying the sector constraints,

The above-listed results allow one to formulate the procedural part of the method of passification as consisting of the following stages.

1. The new output $\widetilde{y}$ is determined as a linear combination of the outputs $\widetilde{y} = Gy$ so that the system becomes hyper-minimum phase with respect to the input u and output $\widetilde{y}$.

2. The control law is selected in the output feedback form. For the nonadaptive case, it is given by

$$u = -\kappa \widetilde{y} = -\kappa G y, \tag{18}$$

and for the adaptive one by (17).

3. If the original control plant cannot be made passive by selecting the output—for example, the plant transfer function has the relative degree greater than unity, the number of measurable variables is insufficient, and so on, then its model is simplified so as to satisfy the passification condition. For example, if there are stable multipliers with small time constants in the denominator of the plant transfer function, then one may try to drop them and carry out design by the *reduced model* (Fradkov, 1990; Popov and Fradkov, 1983; Ioannou and Kokotović, 1983; Fradkov, 1987). Other possible tricks are introducing a *parallel feedforward compensator* (shunt, see Sec. 2.5), observer, and so on.

We note that although the system G-passivity coincides with passivity in output $\widetilde{y} = Gy$, for these two cases the problems of passification do not coincide. Indeed, in the first case the passifying feedback is sought in the form $u = -Ky + Lv$, and in the second case, as $u = \kappa G y + Lv$, the sizes of the matrices K and κ being distinct. In particular, if the output $\widetilde{y} = Gy$ is scalar, then κ also will be scalar. Then, the algorithm of adaptive stabilization will be as follows:

$$u = \kappa \widetilde{y}, \quad \frac{d\kappa}{dt} = -\gamma \widetilde{y}^2 \tag{19}$$

(the so-called "universal controller" (Ilchmann, 1991)). This algorithm was proposed in various publications beginning from (Byrnes and Willems, 1984). Despite its apparent simplicity (it has only one adjustable parameter) and the same asymptotic properties as algorithm (17), it is less flexible. In particular, algorithm (19) does not allow one to realize the principle of implicit reference model. Let us consider some special cases.

2.2 Adaptive Stabilization and Tracking for the Input–Output Systems

Adaptive stabilization. Let the linear time-invariant system (4) with scalar input and output be represented by the input–output equation

$$A(p)y(t) = B(p)u(t), \quad t \geq 0, \tag{20}$$

where u, y are scalar variables, $A(p) = p^n + a_{n-1}p^{n-1} + \ldots + a_0$ and $B(p) = b_m p^m + b_{m-1}p^{m-1} + \ldots + b_0$ are the polynomials of the operator of time differentiation $p \equiv d/dt$. We denote by k the *relative degree* of system (20), $k = n - m > 0$. In compliance with the formulation of the problem of adaptive control, we hold that the coefficients a_i, b_j $(i = 0, \ldots, n-1,\ j = 1, \ldots, m)$ are the unknown *a priori* parameters of the control plant model (20).

We first consider the problem of stabilization of plant model (20)—reduction of $y(t)$ to zero from the nonzero initial state. The desired dynamics of the process of stabilization can be defined by some differential equation to which the output of the plant $y(t)$ must obey. In the classical *Model Reference Adaptive Systems* (MRAS) this equation is realized explicitly as the dynamic unit, that is, the reference model incorporated in the adaptive controller (Petrov *et al.*, 1972; Landau, 1979). A somewhat different scheme of solution is realized in the adaptive systems with the *implicit reference model* (IMRAS) that are described here.

We introduce the signal of *adaptation mismatch* (*error*) $\sigma(t)$ as

$$\sigma(t) = G(p)y(t), \tag{21}$$

where $G(p) = p^l + g_{l-1}p^{l-1} + \ldots + g_1 p + g_0$ is some given Hurwitz polynomial of the operator $p \equiv d/dt$. The coefficients g_i of the polynomial $G(p)$ are defined by the control system designer starting from the desired dynamics of the stabilization process with regard for the mentioned below requirement on the value of its degree l. The adaptation algorithm must drive the mismatch $\sigma(t)$ to zero. By assuming that $\sigma \equiv 0$, we obtain that $y(t)$ satisfies the equation

$$G(p)y(t) = 0. \tag{22}$$

Therefore, (22) defines the reference model which is not explicitly realized in the adaptive controller (as a dynamic unit) and is expressed implicitly through the coefficients g_i $(i = 0,\ 1, \ldots, l-1)$. Therefore, (22) can be called the IRM.

We take the following control law in the main loop:

$$u(t) = \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \tag{23}$$

where $k_i(t)$ $(i = 0, \ldots, l)$ are the adjustable parameters of the controller. In our problem, the following structure of the adaptation algorithm stems from the requirement of passifiability:

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t), \quad k_i^0 = k_i(0), \tag{24}$$

where $\gamma > 0$ is the *gain* of the adaptation algorithm and k_i^0 are the initial values of the adjusted parameters, $i = 0, \ldots, l$.

We make use of Theorem 3 in order to check operability of the closed-loop system with the plant model (20) and adaptive controller (21), (23), (24). With this aim in view, we introduce the vector G consisting of the coefficients g_i of the polynomial $G(p)$ and the transfer function $W(\lambda)$ of the plant model (20) from the input u to the vector $[y, \dot{y}, \ldots y^{(l)}]^{\mathrm{T}} \in \mathbb{R}^{l+1}$ as

$$G = [g_0, g_1, \ldots, 1], \quad W(\lambda) = \frac{B(\lambda)}{A(\lambda)} \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^l \end{bmatrix}, \quad \lambda \in \mathbb{C}.$$

By applying Theorem 3 to the system with the transfer function $GW(\lambda)$ we obtain the following conditions for operability of the adaptive controller (23), (24) (Fradkov, 1974; Fomin *et al.*, 1981; Fradkov, 1990):

1. the polynomial $B(p)$ is Hurwitz and $b_0 > 0$;

2. $l = k - 1$, where $k = n - m$ is the relative degree of the equation of the plant model (20).

These conditions imply that the plant must be minimum phase and a sufficient number l of the derivatives of its output must be used in the control law. The value of l is defined by the relative degree of the plant transfer function. Therefore, the order of the reference model may be small even if the control plant obeys a high-order equation. Additionally, the order of the plant model may be unknown when designing the control algorithm, which is a specialty of the systems with IRM as compared with the traditional MRAS. Another specialty of these systems lies in the possibility of using the IRM not only in the problems of tracking the reference signal, but also in the problems of stabilization. The model output, that is, its response to the reference signal, is used in the systems with explicit RM. In the problems of stabilization such RM "grows blind." Finally, on choosing the main loop of the MRAS, an important part is usually played by the *matching condition* (Petrov *et al.*, 1972; Fomin *et al.*, 1981; Landau, 1979), which means that there must be controller coefficients providing coincidence of the closed-loop system equations with the RM equations. In many cases, this condition is extremely restrictive. We note that the above Condition 1 $b_0 > 0$ implies that the common sign of the coefficients in the right-hand side of the control plant equations (20) must be known on designing the algorithm. If it is negative, that is, $b_0 < 0$, then one has just to change the sign of the coefficient γ in the adaptation algorithm (24).

With algorithm (24), it is important that in the course of adaptation $\sigma(t)$ usually decays much faster than the system transients. As a result, the coefficients of controller (23) reach steady state values, and the output $y(t)$ of the plant model (20) follows the IRM equation (22).

Robustification of the adaptation algorithms. The adaptation algorithm (24) is rarely used in practice in the form in which it is set down, which is due to the fact that the coefficients of controller (23) can grow indefinitely under the action of external disturbances on the control plant model (20) or in the presence of errors of the sensitive elements. To avoid this, various *methods of robustification* (“regularizing”) algorithm (24) (Fomin *et al.*, 1981; Fradkov, 1979) were developed among which introduction of the *parametric feedback* and introduction of the *dead zone* are the basic ones.

The adaptation algorithm regularized by a parametric feedback is as follows:

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \quad (25)$$

where the coefficient $\alpha \geq 0$ of the algorithm *parametric feedback* was introduced. This coefficient is chosen by the designer of the control algorithm. One must bear in mind that the robustification by feedback allows one just to make the system trajectories to hit some bounded neighborhood of the origin, rather than to make the plant output to tend asymptotically to zero (Fomin *et al.*, 1981; Fradkov, 1990; Fradkov, 1979). With such a method of regularization, the adaptation error signal $\sigma(t)$ also does not necessarily tend to zero.

The above method of regularization is applicable also if in the control loop there are some nonlinearities (such as signal quantization and time sampling in the digital control systems) and dynamic perturbations (small additional inertiality in the control loop) (Fomin *et al.*, 1981).

The passification-based implicit reference model method was extended also to the problems of tracking the reference signal (Fomin *et al.*, 1981; Fradkov, 1990; Andrievsky and Fradkov, 1994; Andrievskii and Fradkov, 1999; Andrievskii, 1979). Now we dwell in more detail on the results obtained.

The aforementioned requirement on the relation between the number l of measured derivatives and the relative degree k of the control plant transfer function proves to be too rigid for may practical problems. Various kinds of structures of the main loop of the adaptive control systems—with *control by an intermediate variable*, with *adjustable-dynamics controller*, and with *parallel compensator* (“shunt”)—were obtained to soften this condition. These structures rely on the conditions of Theorem 3.

Adaptive tracking systems [p. 391](Fomin *et al.*, 1981), (Andrievskii, 1979). Let us consider the problem of tracking the reference signal $r(t)$ with the prescribed dynamics by the plant model (20).

The adaptation error signal $\sigma(t)$ is defined as

$$\sigma(t) = G(p)y(t) - D(p)r(t), \quad (26)$$

where the polynomial $G(p)$ was defined above and $D(p)$ is the operator polynomial like $D(p) = d_q p^q + d_{q-1}p^{q-1} + \ldots + d_1 p + d_0$. The adaptation algorithm must provide the tendency of the mismatch $\sigma(t)$ to zero: asymptotically or with some error $\Delta > 0$

$$|\sigma(t)| \leq \Delta \quad \text{for} \quad t \geq t_*, \quad (27)$$

where t_* is some *adaptation time*.

The signal $\sigma(t)$ may be interpreted as the error of satisfying the relation

$$G(p)y(t) = D(p)r(t), \quad (28)$$

(28) being like (22) an IRM, but for the problem of tracking.

By analogy with (23), we take the main-loop control law

$$u(t) = k_r(t)\big(D(p)r(t)\big) + \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \quad (29)$$

where $k_r(t)$, $k_i(t)$ ($i = 0, \ldots, l$) are the adjustable parameters. We make use of the following regularized adaptation algorithm:

$$\begin{aligned} \dot{k}_r(t) &= \gamma\sigma(t)D(p)r(t) - \alpha\big(k_r(t) - k_r^0\big), \\ \dot{k}_i(t) &= -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big) \end{aligned} \quad (30)$$

where $\gamma > 0$ and $\alpha \geq 0$ are the algorithm parameters and k_r^0 and k_i^0 are the initial estimates of the suitable values of the adjusted parameters, $i = 0, \ldots, l$. As was shown in (Fomin *et al.*, 1981; Fradkov, 1990; Andrievsky and Fradkov, 1994; Andrievskii, 1979), the dissipativity of the closed-loop system (20), (28)–(30) is provided if the aforementioned Conditions 1 and 2 and the conditions for boundedness of the perturbations and the rate of variation of the reference signal are satisfied.

We note that the above conditions do include neither the degree q of the polynomial $D(p)$ nor its coefficients. The polynomial degree $D(p)$ is limited by the possibility of differentiating the command signal $r(t)$ and is selected by the control system designer. Further development of the method is concerned both with extension of the class of plants under consideration and development on its basis of practical schemes of adaptive control. Some results are set forth below.

2.3 Adaptive Tuning of the Low Order Controllers

The systems with IRM may be used for adaptive tuning of the standard controllers in the course of system operation (Andrievsky and Fradkov, 1994; Andrievskii and Fradkov, 1999). Let us consider, for example, the following proportional integral control law in the main loop:

$$u(t) = k_P(t)e(t) + k_I(t)\int_0^t e(\tau)d\tau, \quad (31)$$

where $e(t) = r(t) - y(t)$ is the tracking error and $k_P(t)$ and $k_I(t)$ are the adjusted coefficients of the controller. Let us take the second-order IRM

$$T^2p^2y(t) + 2\xi Tpy(t) + y(t) = r(t), \quad (32)$$

where $p = d/dt$ is the operator of time differentiation and T and ξ are the parameters selected at IRM design and defining the desired behavior of the closed-loop system. By applying the operations of integration and filtering, we represent the error of adaptation σ as

$$\sigma(t) = T^2y(t)\omega_f + (2\xi - T\omega_f)Ty_f(t) - \int_0^t e_f(\tau)d\tau, \quad (33)$$

where $y_f(t)$ and $e_f(t)$ are the outputs of the low-frequency filters to whose inputs the respective signals $y(t)$ and $e(t)$ are fed,

In this case, the adaptation algorithm (30) is as follows:

$$\begin{aligned} \dot{k}_P(t) &= \gamma\sigma(t)e(t) - \alpha\big(k_P(t) - k_P^0\big), \\ \dot{k}_I(t) &= \gamma\sigma(t)\int_0^t e(\tau)d\tau - \alpha\big(k_I(t) - k_I^0\big). \end{aligned} \quad (34)$$

We note that algorithm (34) is designed using filtration of the mismatch signal $\sigma(t)$. Admissibility of such transformation of the signal from the standpoint of stability of the closed-loop adaptive system was substantiated in [Sec. 7.1.3](Fomin *et al.*, 1981).

2.4 Combined Signal-Parametric Control Algorithms with Implicit Reference Model

Let us consider now the application of the passification theorem to the design of the controllers of the variable-structure systems (VSS) (Andrievskii and Fradkov, 1999; Utkin, 1992) and the signal-parametric adaptive controllers (SPAC) (Fradkov, 1990; Andrievskii *et al.*, 1988; Andrievskii *et al.*, 1996; Andrievskii and Fradkov, 1999; Stotsky, 1994). We again consider the linear system (4) whose control objective is $\lim_{t\to\infty} x(t) = 0$. Let ensuring the *sliding mode* motion over the surface $\sigma = 0$, where $\sigma = Gy$ and G is the given $l \times n$ matrix, be chosen as the auxiliary objective. We use the following control algorithm:

$$u = -\gamma \operatorname{sign} \sigma, \quad \sigma = Gy, \tag{35}$$

where $\gamma > 0$ is some chosen parameter. As was shown in (Fomin *et al.*, 1981; Andrievskii *et al.*, 1988), this objective is reached for system (4), (35) if there exist a matrix $P = P^{\mathrm{T}} > 0$ and vector K_* such that $PA_* + A_*^{\mathrm{T}}P < 0$, $PB = GC$, $A_* = A + BK_*^{\mathrm{T}}C$. As follows from Theorem 3, these conditions are satisfied only if the transfer function $GW(\lambda)$, where $W(\lambda) = C\bigl(\lambda I_n - A)^{-1}B$, is hyper-minimum phase and the sign of the high-frequency transfer coefficient, that is, the sign of GCB which is assumed to be positive at the algorithm design, is known. If these conditions are satisfied, then for a sufficiently great coefficient γ we get $\lim_{t\to\infty} x(t) = 0$. To eliminate dependence of system stability on the initial conditions and the plant parameters, a "signal-parametric" (or "combined") adaptive control algorithm was proposed (Andrievskii *et al.*, 1988; Andrievskii and Fradkov, 1999) instead of (35):

$$\begin{aligned} u &= K^{\mathrm{T}}(t)y(t) - \gamma \operatorname{sign} \sigma, \quad \sigma(y) = G\,y \\ \dot{K}(t) &= -\sigma(y)\Gamma y(t), \end{aligned} \tag{36}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$ and $\gamma > 0$ are the matrix and scalar gains of the algorithm.

It is worth to note that convergence to zero in a finite time is an important property of the VSS with forced sliding modes. It is possible to prove (see, for example, (Fradkov, 1990)) that this property is satisfied for any bounded domain of the initial states of system (4), (36).

2.5 Shunting Method for Adaptive Systems

The problem of reducing the number of plant state variables used in the adaptive control algorithm is all-important. An appreciable number of recent publications was devoted to the development of the adaptive control system design methods intended to weaken the requirements on the current information about the plant state variables which manifests itself also in the desire to reduce the number of the derivatives of the plant output used in the control algorithm (Druzhinina *et al.*, 1996; Nikiforov and Fradkov, 1994). Complexity (high order) of the proposed algorithms which hinders their realization and reduces their noise immunity is the disadvantage of the existing methods. One of the approaches to this problem, the *shunting method* is based on using a parallel compensator ("shunting unit" or "shunt") (Bar-Kana, 1987; Iwai and Mizumoto, 1994; Kaufman *et al.*, 1994; Andrievsky and Fradkov, 1994; Andrievskii and Fradkov, 1999; Fradkov, 1994; Andrievsky *et al.*, 1996). Its essence lies in making the *extended plant* (comprising the control plant itself and the compensator) hyper-minimum phase. The plant and shunt outputs constitute the output used to generate the control action. Therefore, the design of the adaptation algorithm is based on the so-called extended plant whose transfer function is equal to the sum of the transfer functions of the plant itself and the shunt. The requirement on system operability lies in Hurwitz stability of the numerator of the transfer function of the extended plant, which must be provided by certain choice of the transfer function and the shunt parameters. In particular, equality to unity of the relative degree of the extended plant, which is involved in the strictly minimum phase condition, is satisfied mechanically if the shunt transfer function has the unit relative degree and the degree of the shunt denominator is one less than the relative degree of the plant transfer function.

Let us consider the following structure. We feed the control signal $u(t)$ both to the plant input and some additional unit ("*parallel compensator*", or "*shunt*") whose output is added to that of the control plant when generating the control signal. The basic concept of this approach lies in providing a strictly minimum phase property of the extended plant comprising both the control plant itself and the compensator. Let as before the control plant be defined by (20). We introduce additional unit (shunt) with the transfer function $W_c(\lambda) = \dfrac{B_c(\lambda)}{A_c(\lambda)}$, where $A_c(\lambda)$, $B_c(\lambda)$ are polynomials of the degrees n_c and m_c, respectively, $n_c = m_c + 1$, and $A_c(\lambda)$ is a Hurwitz polynomial. The output of the extended plant $\overline{y}(t)$ is the sum of the control plant output and the output of the shunt to whose input the signal $u(t)$ is fed:

$$\begin{aligned} &A_c(p)y_c(t) = B_c(p)u(t), \\ &\overline{y}(t) = y(t) + y_c(t), \quad p \equiv d/dt. \end{aligned} \tag{37}$$

The extended plant has the following transfer function from the input $u(t)$ to the output $\overline{y}(t)$:

$$\begin{aligned} \overline{W}(\lambda) &= \frac{B_c(\lambda)A(\lambda) + B(\lambda)A_c(\lambda)}{A(\lambda)A_c(\lambda)} \\ &= \frac{\overline{B}(\lambda)}{A(\lambda)A_c(\lambda)}. \end{aligned} \tag{38}$$

One can readily see that the relative degree k of the extended plant (38) is $k = n + n_1 - \max(m_1 + n, m + n_1) = 1$. Consequently, the condition for hyper-minimum phase will be met if $\overline{B}(\lambda)$ is a Hurwitz polynomial. We note that in this structure it is assumed that only the plant output is measured and not its derivatives, which substantially simplifies realization of the control algorithm and improves its noise immunity.

The shunting unit may be selected differently. It was suggested in (Andrievsky and Fradkov, 1994; Andrievskii and Fradkov, 1999; Fradkov, 1994) to use as shunt a system with the transfer function

$$W_c(\lambda) = \frac{\kappa\varepsilon\big(\varepsilon\lambda + 1\big)^{k-2}}{\big(\lambda + \alpha\big)^{k-1}}, \quad \alpha > 0. \tag{39}$$

The following Theorems 4 and 2.5.1 set forth the properties of the extended plant (38) with shunt (39).

Theorem 4. [(Andrievsky and Fradkov, 1994; Fradkov, 1994)] Let the function $W(\lambda)$ (20) be minimum phase ($B(\lambda)$ is the Hurwitz polynomial) and have relative degree $k > 1$ and $B(0) > 0$. Then, there exist number $\kappa_0 > 0$ and function $\varepsilon_0(\kappa) > 0$ such that the transfer function $\overline{W}(\lambda) = W(\lambda) + W_c(\lambda)$ is hyper-minimum phase for all $\kappa > \kappa_0$ and $0 < \varepsilon < \varepsilon_0(\kappa_0)$.

Theorem 2.5.1 [(Andrievsky *et al.*, 1996)] Let the function $W(\lambda)$ be stable ($A(\lambda)$ is the Hurwitz polynomial) and have the relative degree $k > 1$ and $W(0) > 0$. Then, for any $\varepsilon > 0$ there exists a sufficiently great κ_0 such that $\overline{W}(\lambda) = W(\lambda) + W_c(\lambda)$ is hyper-minimum phase for all $\kappa \geq \kappa_0$.

It follows from Theorem 4 that introduced can be the shunt (39) of the order $\deg(A_s(\lambda)) = k - 1 = n - m - 1$ which for a sufficiently large κ and sufficiently small ε satisfies the condition for hyper-minimum phase of the extended plant (38) for any minimum phase control plant and an arbitrary bounded domain of parameters. It follows from Theorem 2.5.1 that for another way of selecting the parameters of the shunt (39) the condition for hyper-minimum phase is satisfied for the stable (and, possibly, non-minimum phase) plants. In this case, the shunt equation can be simplified by taking $W_c(\lambda) = \kappa/(\lambda + \alpha)$ instead of (39).

We note that the above statements guarantee satisfaction of the condition for hyper-minimum phase either for the minimum phase or stable control plants, but in some narrower domain of feasible values of the plant parameters it is possible to make the shunted system hyper-minimum phase simultaneously for unstable and non-minimum phase plants. Another advantage of this way of shunting is the possibility of selecting a small value of the static shunt transfer coefficient, which in the problems of tracking leads to small error caused by using in the control law the output $\overline{y}(t)$ of the extended plant (38) instead of the output $y(t)$ of the control plant itself. The shunting method underlies new combined structures of the adaptive control systems uniting the methods of passification, shunting, and identification on the sliding modes and design of the robust controllers for solution of the applied control problems (Andrievskii and Fradkov, 1999; Andrievsky *et al.*, 1996; Andrievsky and Fradkov, 2002; Andrievsky and Fradkov, 2003*a*; Andrievsky and Fradkov, 2003*b*; Fradkov and Andrievsky, 2004; Fradkov and Andrievsky, 2005).

2.6 Applications

Many recent technical and scientific publications are devoted to the design of laboratory stands controlled by personal computers. In this connection, the problems of control of various kinds of the helicopter laboratory setup are of great interest. One of the impressive devices of this kind is represented by the "Helicopter" benchmark (Apkarian, 1999) intended for testing the flight control laws under varying conditions. A photograph of the "Helicopter" is shown in Fig. 1. Experimental results for adaptive control of the "Helicopter are presented in (Andrievskii *et al.*, 2005; Andrievsky *et al.*, 2005; Andrievskii and Fradkov, 2006; Andrievsky *et al.*, 2007; Fradkov *et al.*, 2007*a*).

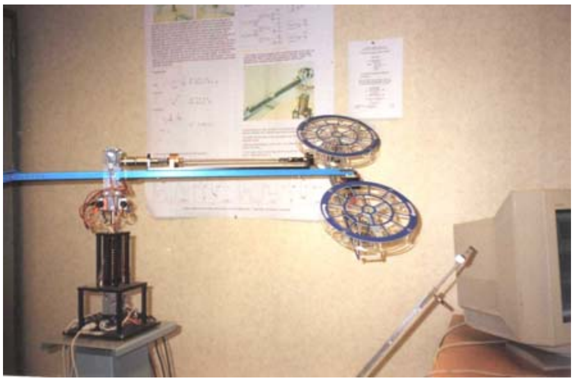

Fig. 1. Photograph of the "Helicopter" benchmark.

Among other applications are adaptive synchronization of chaotic systems (Fradkov and Markov, 1997; Fradkov *et al.*, 2000; Andrievsky and Fradkov, 2000; Andrievsky, 2002; Fradkov *et al.*, 2006; Fradkov *et al.*, 2007*b*) and irrigation systems (Tsykunov, 1984).

3. CONCLUSIONS

The algorithms of the passification method that were born by the frequency theorem have inherited its procedural simplicity and clarity of application. Over more than in thirty years of its history the method was considerably developed both in theoretical and practical terms. Quite real application fields came into existence in the flight control and the message transmission by modulated chaotic signals. Finally, new experimental confirmations of adaptive system operability appeared. They indicate to practicality of the approach whose theoretical fundamentals were laid in the works of V.A. Yakubovich.

At the same time, a number of problems still remain unsolved. Also there are actively explored fields of research among which the recent works on the necessary and sufficient conditions for robust passification under parametric norm-bounded indefiniteness (Peaucelle *et al.*, 2005) and on the necessary and sufficient conditions for adaptive passification (Peaucelle *et al.*, 2006) deserve mentioning.

A practically important question of selecting the matrix G providing passivity or minimum phase of the system with the transfer function $GW(\lambda)$ still remains unsolved (in the existing publications it was solved only for the zero relative degree $W(\lambda)$ (Sun *et al.*, 1994)). Finally, a new path of research concerned with the adaptive control under limited throughput of the communication channels deserves mentioning (Fradkov *et al.*, 2006).

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