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NONLINEAR CONTROL DESIGN WITH INEQUALITY CONSTRAINTS: APPLICATION TO MAGNETIC LEVITATION SYSTEM

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Abstract
This paper deals with a generalization of classical backstepping nonlinear control technique, adjusted so that it may be used for a wider class of dynamical systems than the strict-feedback form. This extension of backstepping is interesting from our point of view, both in theory and in practice, since a large number of industrial applications may benefit from it. Theoretical aspects concerning design procedure (stability and optimality) are presented in a constructive manner, applied to a special type of magnetic levitation system. Robustness and performance issues with respect to other nonlinear control techniques already existing in known literature will be addressed, by plotting some compared simulation results.

Key words
Nonlinear control, backstepping, magnetic levitation system.

1 Introduction

The main tool used within this article is backstepping. This nonlinear control design procedure has become popular, due to its ability to address efficiently global stabilization and tracking problems, for nonlinear systems with unknown or uncertain parameters (see [Sepulchre, Jankovic and Kokotovic, 1997]). Theoretical aspects regarding robust and adaptive control of classical backstepping are systematically presented in [Krstic, Kanellakopoulos and Kokotovic, 1995].

Some examples of nowadays applications using backstepping controllers include flight control, robot joint manipulators [Krstic, Kanellakopoulos and Kokotovic, 1995]. These prove the increased interest towards nonlinear control techniques, in the same time with microcontroller technology advancements. Although nonlinear control solutions might not be considered as cost-appealing for a large number of industrial markets (mainly due to increased complexity, which translates itself into increased production costs), they address efficiently applications where uncertainty and accuracy are non-negligible.

Classical backstepping is a type of recursive design control that can be applied on linear systems that have *lower-triangular* structure, and on nonlinear systems of the *strict-feedback form* (see [Sepulchre, Jankovic and Kokotovic, 1997], [Khalil, 2002]).

In this paper we are concerned with a more general class of nonlinear systems, which will be called hereafter *non strict-feedback form*:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2^{\alpha_1} \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)x_3^{\alpha_2} \\ \quad \cdots \\ \dot{x}_{n-1} = f_{n-1}(x_1, .., x_{n-1}) + g_{n-1}(.., x_{n-1})x_n^{\alpha_{n-1}} \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) + g_n(x_1, x_2, .., x_n)u \end{cases} \tag{1}$$

with $n \in \mathbb{N}_0$ number of state variables; $f_i(\cdot)$ and $g_i(\cdot)$ are known nonlinear functions ($i = 1, .., n$). This system has input u and output $y = x_1$. The relative degree is *n*. The vector of state variables is defined as $x \triangleq [x_1 \; x_2 \; ... \; x_n]^T$.

The difference between class (1) and *strict-feedback form* systems is given by $\alpha_i \in \mathbb{R}_0$ power numbers of each x_{i+1} state variables, which are constant values in the case of *strict-feedback form*: $\alpha_i = 1$ ($i = 1, .., n-1$).

Our theoretical study has a certain number of practical applications that may benefit from it: the widely used solenoid electrical actuators found in automobile and aeronautics industry (for example fuel injectors [Pulkrabek, 2004]; magnetic bearings [Rodriguez et. al., 2000]; magnetic levitation systems [Brauer, 2006]). Further on, during this article, we focus our attention on magnetic levitation systems, by applying nonlinear control.

The classical approach of backstepping gives a continuous analytical form of the command. Depending on the type of system to be controlled, a discontinuous command might be appealing for better performance (as in the case of electrical actuators). We will present during this article a way to construct both continuous and discontinuous analytical commands, for a nonlinear system belonging to class (1).

Throughout the entire length of this article, we assume that all state variables x_i $(i = 1, .., n)$ are available, and use them for feedback. For *observer* backstepping design, one may see [Krstic, Kanellakopoulos and Kokotovic, 1995].

We will make use of Lyapunov theory in a constructive and systematical manner, highlighting its flexibility in spite of its perennial criticisms, as mentioned for example in [Sepulchre, Jankovic and Kokotovic, 1997]. This is possible due to system form (1), allowing for special candidate control Lyapunof functions, chosen according to preformance/robustness criteria, and continuous/discontinuous controller type.

2 Nonlinear Modeling. Applications

The purpose of this section is to give examples of applications that may benefit from the proposed design control. The common key-element to the class of solenoid actuators mentioned during "Introduction" section, making all these systems belong to class (1) is magnetic force, most of the times modeled by:

$$F_{mag}(x_1) = -\frac{\mu_0 N^2 S x_3^2}{8 x_1^2} \qquad (2)$$

with μ_0 permeability of free space (vacuum) or air; N number of coil turns; S transversal surface of *armature* in region of *air gap*.

Formula (2) is essential for our study: due to x_3^2 appearing on numerator, we will be able to construct a model belonging to the class (1) and apply the control technique earlier presented. It can be shown that most *solenoid actuators* can be modeled so that they belong to general class (1), due to $F_{mag} \triangleq -\frac{\partial W}{d x_1}$ (see [Brauer, 2006]), with W magnetic energy stored within *air gap*. As presented during "Introduction" section, there is a wide range of industrial solenoid applications. This aspect is an important result of this paper and of our present study.

Next, we will introduce a nonlinear system belonging to class (1), modeled using formula (2). Experimental platforms presented in [Rodriguez et. al., 2000] and [Barie et. al., 1996] were the starting point in our quest for models of magnetic applications.

2.1 Magnetic Suspension System

Magnetic suspension, as it appears in the literature, is a particular class of *magnetic levitation* systems. Sometimes, the two terms are confounded each other.

Experimental apparatus consists of metallic *plunger*, positioned vertically, with its longitudinal axis perpendicular to the ground; it moves upwards-downwards due to joint effect of gravity and electromagnetic force. Bearing friction is neglected and thus not appearing in system (3). For more structural details, we guide the reader to [Brauer, 2006].

We give a simplified version of model with parameter values:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g + \dfrac{C}{m_3}\left(\dfrac{x_3}{c - x_1}\right)^2 \\ \dot{x}_3 = \dfrac{1}{L}\Big[- R_3 x_3 + U \Big] \end{cases} \qquad (3)$$

with x_1, x_2, x_3 position, velocity of metallic plunger and current in the coil of electromagnet respectively; $g = 9.81 m/s^2$ gravitational acceleration; $C = 0.005 Nm^2/A^2$ magnetic force constant; $m_3 = 0.0844 kg$ mass of metallic plunger; $R_3 = 10\Omega$ coil resistance; $L = 10^{-2} H$ self-inductance is assumed to be constant value, according to an simplification hypothesis; $c = 0.011 m$ maximum allowed position for metallic plunger; U voltage across actuator coil; Ox_1 axis has been chosen so that magnetic force $F_{mag}(x_1)$ is always positive; $x_1 = c$ corresponds to closed *air gap*.

During next section, we calculate analytically, global stabilizing control U.

2.1.1 Linear-tangent model

During "Simulation" section, we compare results of our proposed control technique with other known controllers, some of which are linear. For this reason, the following linear-tangent model will be used.

The simplest stationary linearized i.e. linear time-invariant (LTI) model one can get from nonlinear system (3), can be realized by applying linearization method (around an equilibrium point) accordingly [Bolton, 2002], [Anderson, 1971]. Let us define:

$$\left| \begin{array}{l} x = x_e + \tilde{x} \\ U = U_e + \tilde{U} \end{array} \right. \qquad (4)$$

with $x = [x_1 \; x_2 \; x_3]^T$ vector of *state variables*; $x_e = [x_{1e} \; x_{2e} \; x_{3e}]^T$ and U_e equilibrium point; $\tilde{x}$ and $\tilde{U}$ "small" variations around equilibrium point.

From system (3), by setting up all functions on the right side of *equal* sign to zero, one gets equilibrium points class. The choice for actual equilibrium point to be used for linear control is as follows: x_{1e} is at the middle of $D_{x_1} = [0, c]$ *admissible range* for x_1 i.e. $x_{1e} = c/2$; $x_{3e} = (c - x_{1e})\sqrt{\frac{m_3 g}{C}}$; $U_e = R_3 x_{3e}$. Numerically, the results are: $x_{1e} = 0.0055$; $x_{2e} = 0$; $x_{3e} = 0.07078$; $U_e = 0.7078$.

Linearizing around (x_e, U_e), by using Jacobian matrix, one gets new matrices A and B, making up linear system:

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{U} \qquad (5)$$

We are now faced with linear control problem of finding $\tilde{U}$. Once $\tilde{U}$ is analytically expressed, we test the controller directly on nonlinear system (3) by replacing $U = U_e + \tilde{U}$ from (4).

3 Controller Design

The proposed feedback controller will be presented in a step-by-step manner, hereafter. We divide this control design procedure into two subparts: stability analysis and optimality.

For simplification purposes, some structural constraints (of mechanical or electrical nature) will be disregarded and are left unmodeled (see for example, subsection 3.3).

3.1 Stability analysis

The procedure for finding stabilizing control U is pretty much the same with respect to classical backstepping [Sepulchre, Jankovic and Kokotovic, 1997], only the Control Lyapunov Function (CLF) is different. Also, we keep the same terminology definitions like *virtual control*, but add some new ones like *adjustment parameters* k_i ($i \in \{1, 2, 3\}$), used for *tuning* final command $U = U(k_i)$.

The final command U will be calculated in n steps, with n being the number of state variables (in our case $n = 3$).

For the first step, let us consider only first equation of (3): we define *virtual control* ϕ_{x_2} and replace it instead of x_2; $\varepsilon_1 = x_1 - x_{1ref}$ with $x_{1ref}(t)$ desired reference trajectory for $x_1(t)$; chosen CLF is $V_1(x_1) = \frac{1}{2}\varepsilon_1^2$. We impose $\dot{V}_1(x_1) = -k_1\varepsilon_1^2 < 0$, $k_1 \in \mathbb{R}_0^+$, in order to ensure global asymptotic convergence (GAS) of $\varepsilon_1(t) \to 0$. All explicit calculations done, we get $\phi_{x_2} = \dot{x}_{1ref} - k_1\varepsilon_1$ i.e.:

$$\phi_{x_2} = \dot{x}_{1ref} - k_1(x_1 - x_{1ref}) \tag{6}$$

For the second step, let us consider the subsystem consisting of first two equations of (3): we define *virtual control* $\phi_{x_3^2}$ and replace it instead of x_3^2 ; $\varepsilon_2 = x_2 - \phi_{x_2}$ and use the CLF: $V_2(x_1, x_2) = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2$. We impose

$$\dot{V}_2(x_1, x_2) = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2 < 0, k_2 \in \mathbb{R}_0^+ \tag{7}$$

and get $\phi_{x_3^2}$.

For the purpose of this paper, we will skip tedious calculations and just present the result:

$$\phi_{x_3^2} = -\frac{m_3(c - x_1)^2}{C}\Big[(k_1 + k_2)(x_2 - \dot{x}_{1ref}) + (1 + k_1k_2)(x_1 - x_{1ref}) - g - \ddot{x}_{1ref}\Big] \tag{8}$$

At the third step of design procedure, let us consider the whole system (3): we define $\varepsilon_3 = x_3^2 - \phi_{x_3^2}$ and use the CLF:

$$V_3(x_1, x_2, x_3) = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2 + \frac{1}{2}\varepsilon_3^2$$

At this point, we have (at least) two choices for $\dot{V}_3$:

3.1.1 Continuous control

By imposing:

$$\dot{V}_3(x_1, x_2, x_3) = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2 - k_3\varepsilon_3^2 < 0, k_3 \in \mathbb{R}_0^+$$

one may calculate explicitly final command U_{EBC} given in relationship (9).

3.1.2 Discontinuous control

By imposing:

$$\dot{V}_3(x_1, x_2, x_3) = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2 - k_3|\varepsilon_3| < 0, k_3 \in \mathbb{R}_0^+$$

and using relationship $|\varepsilon_3| = \varepsilon_3 \cdot sign(\varepsilon_3)$ [Khalil, 2002], we are able to calculate explicitly final command U_{EBD}, given in (9).

Remark1: In (9), by simply omitting to write $sign$ in U_{EBD}, one gets U_{EBC}.

Remark2: In equation (9) one may notice two singularity points: $x_1 = c$, which is particular to this type of system (3) and $x_3 = 0$, issue arising for all class (1).

At this phase of our proposed design procedure, we have a class of stabilizing control $U = U(k_i)$ ($i \in \{1, 2, 3\}$). Optimal k_i parameter choice will be discussed next.

3.2 Optimality

Since we need reasonable "a priori" tracking performance imposed on magnetic suspension behaviour, we appeal classical optimisation theory [Griva, Nash and Sofer, 2009].

Since optimal solution for the final command $U = U(k_i)$, $k_i \in \mathbb{R}_0^+$ ($i = 1, .., n$), might be difficult to infer, we propose to use numeric computation search algorithms (see [Griva, Nash and Sofer, 2009]), capable of solving numerically minimization problems (for example, using adequate software like MATLAB®/-Optimization Toolbox; LabVIEW™/Optimization VIs Palette, or Scilab™/Optimization Tools).

Thus, optimal *adjustment parameters* k_i will be adjusted numerically, with respect to following multivariable minimization problem, integrating a nonconvex *objective function*:

$$\min_{k_i} \int_{H_c} \left(\varepsilon_1(t)^2 + \rho_1 \cdot max \begin{bmatrix} 0 \\ |U(t)| - U_{max} \end{bmatrix} \right) dt \tag{10}$$

$$
\begin{aligned}
U_{EBC} = R_3x_3 - \frac{L}{2}(k_1+k_2+k_3)x_3 + \frac{L}{2x_3}\Bigg[- \frac{C}{m_3(c-x_1)^2}\Big[x_2 - \dot{x}_{1ref} + k_1(x_1 - x_{1ref})\Big] \\
+\frac{m_3(c-x_1)}{C}\Big[-(1+k_1k_2)(x_1-x_{1ref}) - (k_1+k_2)(x_2-\dot{x}_{1ref}) + g + \ddot{x}_{1ref}\Big]\Big[k_3(c-x_1) - 2x_2\Big] \\
+\frac{m_3(c-x_1)^2}{C}\Big[-(1+k_1k_2)(x_2-\dot{x}_{1ref}) - (k_1+k_2)(-g-\ddot{x}_{1ref}) + \dddot{x}_{1ref}\Big]\Bigg] \\
U_{EBD} = R_3x_3 - \frac{L}{2}(k_1+k_2)x_3 + \frac{L}{2x_3}\Bigg[- \frac{C}{m_3(c-x_1)^2}\Big(x_2 - \dot{x}_{1ref} + k_1(x_1 - x_{1ref})\Big) \\
-k_3\, sign\Bigg(x_3^2 - \frac{m_3(c-x_1)^2}{C}\Big[-(1+k_1k_2)(x_1-x_{1ref}) - (k_1+k_2)(x_2-\dot{x}_{1ref}) + g + \ddot{x}_{1ref}\Big]\Bigg) \\
-2\frac{m_3}{C}(c-x_1)x_2\Big[-(1+k_1k_2)(x_1-x_{1ref}) - (k_1+k_2)(x_2-\dot{x}_{1ref}) + g + \ddot{x}_{1ref}\Big] \\
+\frac{m_3(c-x_1)^2}{C}\Big[-(1+k_1k_2)(x_2-\dot{x}_{1ref}) - (k_1+k_2)(-g-\ddot{x}_{1ref}) + \dddot{x}_{1ref}\Big]\Bigg]
\end{aligned}
\tag{9}
$$

with $\varepsilon_1 = x_1 - x_{1ref}$ $(i = 1,..,n)$; $\rho_1 \gg 0$ ponderation-parameter used to adjust the importance of control $U(t)$ penalty with respect to $\varepsilon_1(t)$; $U_{max} \in \mathbb{R}_0^+$ is maximum admissible voltage across actuator coil, value imposed by manufacturer.

We got very good results in terms of tracking performance ($\varepsilon_1(t) \to 0$) and numerical convergence towards some local solution $k_{i\ opt}$, by using Nelder & Mead downhill simplex search algorithm (see [Griva, Nash and Sofer, 2009]). Other numerical algorithms like "steepest descent" and "gradient methods" proove poorer results in terms of convergence, mainly due to non-convex nature of *objective function* in (10).

The choice for optimal *adjustment parameters* in (9) and after rounding to nearest integer values, is as follows:

- for U_{EBC}: $k_1 = 40$; $k_2 = 40$; $k_3 = 1000$
- for U_{EBD}: $k_1 = 40$; $k_2 = 40$; $k_3 = 20$

To conclude upon this subsection, we resume by saying that, numerical optimization proves to be a powerful tool, reducing both prototyping time and costs. Nowadays industrial applications, using online and/or offline optimization tools include aeroplane fuel consumption and automotive pollution reduction (see [Pulkrabek, 2004]).

3.3 PWM Power Converter

Power conversion is the interface between control laws and the plant. In electronic systems, the power is often delivered through pulse modulation, a family of near-linear methods that switch transistors at high frequency. Seen from the frequency domain, modulation inserts a short delay in the control loop. Modulation also injects nonlinear effects, such as ripple, which are usually ignored in the frequency domain (see [Ellis, 2004]).

In the case of our magnetic suspension application, a particular type of Pulse Width Modulation (PWM) *power converter* is being used. For the purpose of our simulations, we include a special block in-between *controller* and *plant* for simulating PWM, using center-aligned *triangular waveform* for the carrier signal [Neacsu, 2006]. Also, we assume an ideal *H-bridge* to be used as DC-to-DC power converter, thus allowing for both positive and negative values, for current x_3 and voltage U (see [Neacsu, 2006], [Bolton, 2002]).

4 Simulations

To test the efficiency of proposed extended backstepping presented in section 3, we plot some comparative simulation results, in terms of robustness and performance (regulation, tracking). For this reason, we introduce following controller acronyms: EBC (extended backstepping continuous control, according to 3.1.1); EBD (extended backstepping discontinuous control, according to 3.1.2); SMD (classical sliding mode discontinuous control [Utkin, Guldner and Shi, 1999]); PID (classical proportional-integral-derivative controller); LQR (classical linear-quadratic regulator [Anderson, 1971]); PoP (pole placement [Anderson, 1971]); EFL (exact feedback linearization [Khalil, 2002]).

Simulations were run in MATLAB®/Simulink® on a workstation powered by Linux®.

Adjustment parameters $k_{i\ opt}$ $(i = 1,..,n)$ will be fixed throughout all simulation scenarios related to extended backstepping; for sliding mode, we keep the same sliding surface.

When PWM *power converter* block is interposed in-between *controller* and *plant*, simulation results (in terms of stability/instability), depend highly on simulation *sample time* and frequency of PWM triangular signal carrier. For signal carrier of frequency $\nu_c \geq 2\ kHz$, and low values for simulation *sample time* ($T_e \leq 10^{-5} second$), we get virtually same results, with, or without PWM *power converter* block.

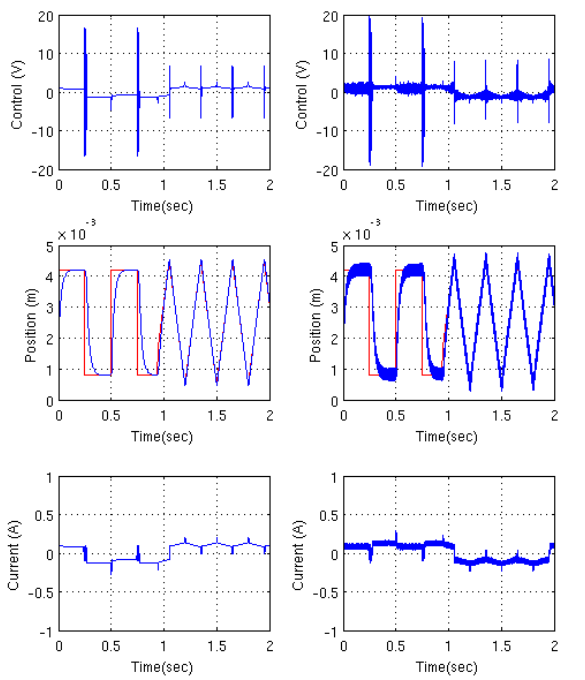

Figure 1. Magnetic suspension simulation results for EBC control: without perturbation (left column) and with perturbations (right column). From top to bottom: (a) command; (b) plunger position x_1 (blue) and given reference x_{1ref} (red); (c) current x_3.

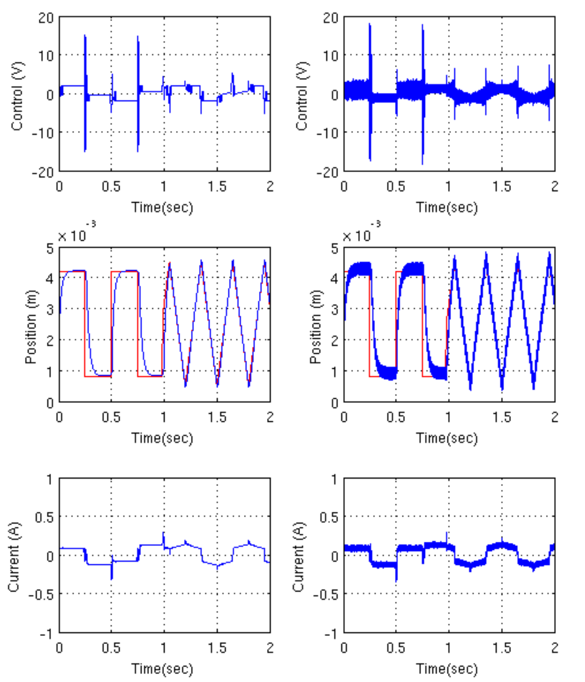

Figure 2. Magnetic suspension simulation results for EBD control: without perturbation (left column) and with perturbations (right column). From top to bottom: (a) command; (b) plunger position x_1 (blue) and given reference x_{1ref} (red); (c) current x_3.

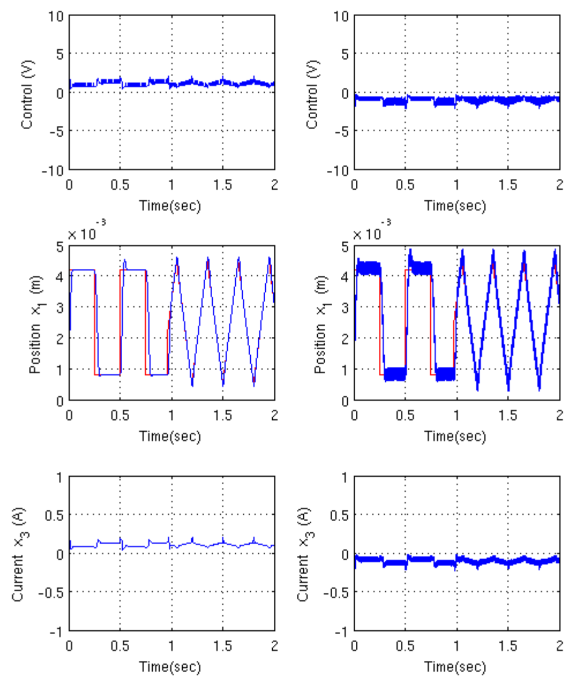

Figure 3. Magnetic suspension simulation results for SMD control: without perturbation (left column) and with perturbations (right column). From top to bottom: (a) SMD control; (b) plunger position x_1 (blue) and given reference x_{1ref} (red); (c) current x_3.

We address robustness and tracking issues during first two sets of simulations (see Fig. 1-2), where we apply proposed nonlinear control technique (EBC on Fig. 1, EBD on Fig. 2), and compare results with SMD on Fig. 3. Simulation scenarios include *square waveform* reference signal for the first half of simulation time and center-aligned *triangular waveform* signal for the other half. Figures have two columns: on the left we plot results without perturbation, and on the right column we apply additive *uniform white noise* $\nu_i(t)$ for each *state variable* x_i $(i \in \{1,2,3\})$: $E\{\nu_i(t)\} = 0$ and standard deviation $\sigma_i = 5\% \cdot [max(D_{x_i}) - min(D_{x_i})]$, with D_{x_i} *admissible range* of x_i. D_{x_i} is according to physical constraints: $D_{x_1} = [0, 11]$ millimeters; $D_{x_3} = [-1, 1]$ Amper. Since there is no constraint for plunger velocity, let us choose $D_{x_2} = [-1, 1]$ m/sec.

Uniform white noise stands for electrical noise found on faulty measurement equipement (for exemple the sensor itself, signal conditioning equipement, acquisition board electronics etc.).

Magnetic suspension can be controlled using other known control techniques, like SMD, EFL and PID, LQR, PoP. The choice of controller should be done according to different criteria like cost, performance and robustness. From industrialist point of view, PID, LQR or PoP might be the "right" choice offering reduced costs for good performance. When more advanced control techniques might be needed, nonlinear control (like EBC, EBD, SMD) should have priority.

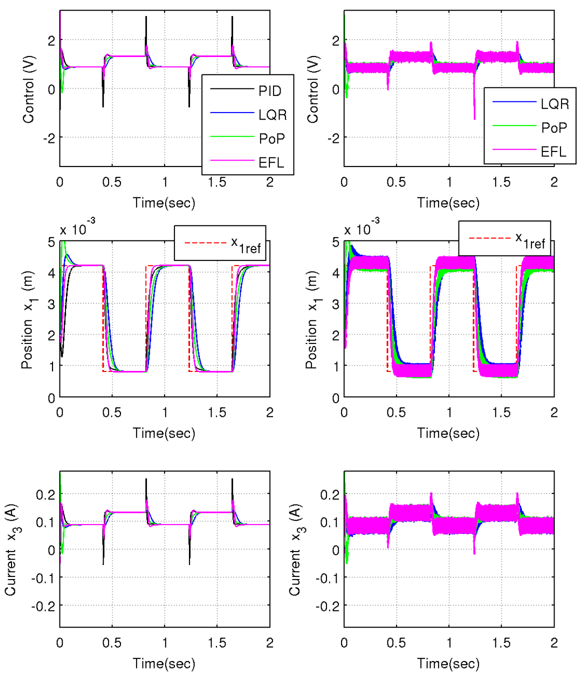

Figure 4. Magnetic suspension simulation results for linear control: without perturbation (left column) and with perturbations (right column). From top to bottom: (a) Controller type; (b) plunger position x_1 and given reference x_{1ref} (red curve); (c) current x_3.

Next, we compare simulation results, in terms of command $U = U(t)$ evolution, between EBC and EBD with SMD and PID, LQR, PoP, EFL. “Optimal” controller parameters have been calculated numerically using offline search algorithms (see [Griva, Nash and Sofer, 2009]). In Fig. 3 (SMD controller) as well as in Fig. 4 (linear controllers), one may notice less *peaking* values for the command, in vecinity of points defined by fast-changing reference x_{1ref}, with respect to EBC and EBD. This is one drawback of our proposed controllers, namely there is a risk of running saturated. Solutions will be proposed in future works.

Remark1: Since we got instable simulation results for PID controller applied on *perturbed* model, we avoided plotting them in Fig. 4 (right column).

Remark2: Open-loop control has not been an issue of present paper. However, flatness-based open-loop control (see [Fliess et. al., 1999]) might be tried due to do flatness property of class (1).

Simulation results are satisfactory and according to expectations. Command U and state-space variables x_i rest within *admissible range*. Some *peaking* phenomena may be observed each time reference value changes rapidly. By filtering input reference, for example using some first-order filter, *peaking* reduces, *robustness* increases, and consequently performance, in terms of *response time*, reduces a little bit.

5 Conclusions and Perspectives

In this paper we deal with theoretical and numerical issues regarding a proposed controller design for a special class of nonlinear dynamical systems. This control technique is based on an extension of classical backstepping, applicable to a wider range of nonlinear models than the so-called *strict-feedback form* systems. Then we apply it to a magnetic levitation system, describing systematically the step-by-step procedure, dividing it into two parts: stability and optimality. The paper ends with some relevant simulations, comparing this approach to other known control techniques, like sliding mode and linear optimal control.

As perspectives, we will primarily focus on correcting some undesired controller issues, like *peaking*. Afterwards, some experimental validation of control laws will be undergone.

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