MIXED-MODE AND CHAOTIC OSCILLATIONS VIA CANARD EXPLOSIONS IN LIGHT-EMITTING DIODES

Marzena Ciszak
CNR
Istituto Nazionale di Ottica
Italy
marzena.ciszak@inoa.it

Francesco Marino
Dipartimento di Fisica
Università di Firenze
Italy
marino@fi.infn.it

Sora F. Abdalah
CNR
Istituto Nazionale di Ottica
Italy
sora.abdalah@inoa.it

Kais Al-Naimee
CNR
Istituto Nazionale di Ottica
Italy
kais.al-naimee@inoa.it

Riccardo Meucci
CNR
Istituto Nazionale di Ottica
Italy
riccardo.meucci@inoa.it

Tito F. Arecchi
CNR
Istituto Nazionale di Ottica
Italy
tito.arecchi@inoa.it

Abstract
We demonstrate experimentally and theoretically the occurrence of complex sequences of periodic mixed-mode and chaotic oscillations in light emitting diodes with optoelectronic feedback. The experimental results have been qualitatively reproduced by a simple physical model of the system showing that the observed dynamics is the result of canard-phenomena in a 3D phase-space. We also investigate the transition between periodic and chaotic mixed-mode states and analyze the effects of noise on the chaotic attractors in coupled systems. In particular, we show that in the presence of white noise source, coupling enhances the coherence in the system response.

Key words
mixed-mode oscillation; chaotic spiking; light emitting diodes; optoelectronic feedback; coupled systems; noise;

1 Introduction

Oscillatory dynamics in chemical, biological and physical systems often takes the form of complex temporal sequences known as mixed-mode oscillations (MMOs) [Focus issue, 2008]. Typical time traces are characterized by a mixture of L large-amplitude relaxation spikes followed by S small-amplitude quasi-harmonic oscillations, while oscillations of intermediate amplitude do not occur. Sequences of this type are ubiquitous in nature and have been originally observed in chemical reactions more than 100 years ago [Ostwald, 1900], with the Belouzov-Zhabotinsky reaction discovered in the 70s being the most thoroughly studied example [Schmitz, Graziani and Hudson, 1977; Showalter, Noyes and Bar-Eli, 1978; Maselko and Swinney, 1987; Brons and Bar-Eli, 1991]. More recent studies involved surface chemical reactions [Bertram and Mikhailov, 2001; Bertram et al., 2003; Kim et al., 2001] electrochemical systems [Koper, 1995; Plenge, Rodin, Scholl and Krischer, 2001] neural and cardiac cells [Alonso and Llins, 1989; Medvedev and Cisternas, 2004], calcium dynamics [Kummer et al., 2000] and plasma physics [Mikikian, Cavarroc, Couedel, Tessier and Boufendi, 2008], to name just a few. As some bifurcation parameter is varied, MMOs can be ordered in periodic-chaotic sequences, in which intervals of periodic states are separated by chaotic states resembling random mixtures of the adjacent periodic patterns. In other cases, these mixtures can form periodic concatenations following the Farey arithmetic.

Several mechanisms can be at the origin of these phenomena [Focus issue, 2008], for instance the quasiperiodic route to chaos on an invariant 2-torus [Larter and Steinmetz, 1991] and the loss of stability of a Shilnikov homoclinic orbit [Arneodo, Argoul, Elezgaray and Richetti, 1993; Koper, 1995]. However, periodic-chaotic sequences and Farey sequences of MMOs do not necessarily involve a torus or an homoclinic orbit, but can occur also through the canard phenomenon [Benoit, Callot, Diener and Diener, 1981]. Here, a limit cycle born in a supercritical Hopf bifurcation experiences the abrupt transition from a small-amplitude quasi-harmonic cycle to large relaxation oscillations in a narrow parameter range (canard explosions). Although this sudden transition can be easily misinterpreted as a homoclinic bifurcation, here an exact homoclinic connection to a saddle-focus does not occur and therefore application of the Shilnikov theorem is not allowed. Such behavior is typical in 3D multiple time-scale dynamical systems, which can be described in terms of a fast 2D oscillatory subsystem, coupled to a slowly evolving variable acting as a quasistatic bifurcation parameter. The strong separation of time-scales may induce the switch between periods of small-amplitude and relaxation oscillations and makes the flow to pass very closely to the saddle-focus stationary state, thus ”simulating” trajectories close to the Shilnikov condition. For this reason, canard phenomena in 3D systems are often referred to as incomplete homoclinic scenarios [Koper, Gaspard and Sluyters, 1992]. Although most of the studies of this dynamics have been carried out in chemical systems, incomplete homoclinic scenarios have been recently predicted and observed also in semiconductor lasers with opto-electronic feedback [Al-Naimee, Marino, Ciszak, Meucci and Arecchi, 2009] and optical cavities with movable mirrors [Marino, Marin, Balle and Piro, 2007; Marino and Marin, 2011]. In these works, attention has been focused on the chaotic-spiking regime, a special kind of MMOs where large pulses are separated by an irregular number of quasi-harmonic oscillations, and on the excitable features of the small-amplitude periodic and chaotic attractors appearing just beyond the first supercritical Hopf bifurcation [K. Al-Naimee et al., 2010]. Here, we investigate experimentally the transition between periodic and chaotic mixed-mode states and analyze the effects of noise on chaotic attractors in two coupled systems.

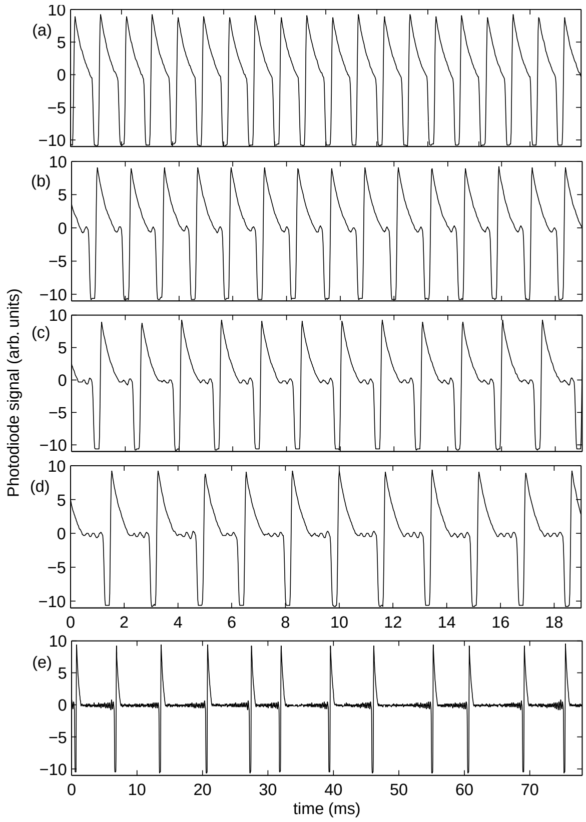

Figure 1. Experimental time-series of the optical intensity as V_n is decreased: a) V_n = 0.056, b) V_n = 0.039, c) V_n = 0.024, d) V_n = 0.016, e) V_n = 0.005.

2 Mixed mode oscillations: the experiment

The system here considered is a light-emitting diode (LED) with AC-coupled nonlinear optoelectronic feedback. The LED is driven by a constant voltage generated by a DC-power supply. The output light is sent to a photodetector producing a current proportional to the optical intensity. The corresponding signal is sent to a variable gain amplifier characterized by a nonlinear transfer function of the form $f(w) = Aw/(1 + sw)$, where A is the amplifier gain and s a saturation coefficient, and then fed back to the injection current of the LED. The feedback strength is determined by the amplifier gain, while its high-pass frequency cut-off can be varied (between 1 Hz and 100 KHz) by means of a tunable high-pass filter. A constant negative control bias from a low-voltage generator is added to the LED driving voltage through a mixer, allowing us the fine tuning of the control parameter. For zero control bias, the LED driving voltage and the feedback amplification are such that the system is in the relaxation oscillations regime (see Fig. 1a). In order to characterize the system dynamics, it is more meaningful to define the dimensionless control parameter $V_n = (V_0 - V)/V_0$, where V is the considered bias voltage and V_0 is the control voltage corresponding to the stationary state. In this way V_n represents the normalized distance between our operation point and the end of the transitions. We assign the MMO states the symbolic notation L^S where L gives the number of large amplitude oscillations, and S the number of small amplitude oscillations in a single periodic pattern. The aforementioned relaxation oscillation regime 1^0 is the dominant behavior of the system. The typical sequence of MMO states that is observed as V_n is decreased is shown in Fig. 1b-d, displaying a 1^1-, 1^2-, 1^3-periodic states respectively. At lower values of V_n, a chaotic spiking regime sets in, where large amplitude oscillations are separated by an irregular number of small-amplitude oscillations (see Fig. 1e). Decreasing V_n even further, the mean inter-spike interval decreases until the large amplitude spikes disappear and the system undergoes a sequence of small-amplitude chaotic and periodic attractors eventually reaching a stationary state via a supercritical Hopf bifurcation. The transition between these small-amplitude attractors and the chaotic spiking regime has been investigated in detail in Refs. [Al-Naimee, Marino, Ciszak, Meucci and Arecchi, 2009; Marino, Marin, Balle and Piro, 2007].

3 Mixed mode oscillations: theoretical model and numerical results

The dynamics of LEDs is determined by two coupled variables (intensity and carrier density) evolving with very different characteristic time-scales. The introduction of a third degree of freedom (and a third much slower time scale) describing the AC-feedback loop, leads to a 3D system, displaying the multiple time-scale competition between optical intensity, carriers and the feedback nonlinear filter function. Since the dynamics is mainly governed by the AC-feedback loop, the coherence proper of a laser does not play any role since the fast dynamics of the matter-field interaction is adiabatically eliminated on such slow time scale. Therefore the system dependence on the phase of the optical field can be neglected and what remains is just the dependence on the output light intensity. In these conditions, experiments in LEDs are equivalent to experiments in semiconductor lasers, but much more easily controllable. The evolution of the photon density S and carrier density N is described by the usual rate equations appropriately modified in order to include the AC-coupled feedback loop

$$\begin{aligned}\dot{S} &= [g(N-N_t)-\gamma_0]S+\gamma_c N \\ \dot{N} &= \frac{I_0+f_F(I)}{eV}-\gamma_c N-g(N-N_t)S \\ \dot{I} &= -\gamma_f I + k\dot{S}\end{aligned} \qquad (1)$$

where I is the high-pass filtered feedback current (before the nonlinear amplifier), $f_F(I) \equiv AI/(1+s'I)$ is the feedback amplifier function, I_0 is the bias current, e the electron charge, V is the active layer volume, g is the differential gain, N_t is the carrier density at transparency, γ_0 and γ_c are the photon damping and population relaxation rate, respectively, γ_f is the cutoff frequency of the high-pass filter and k is a coefficient proportional to the photodetector responsivity. For numerical purposes, it is useful to rewrite Eqs. (1) in dimensionless form. To this end, we introduce the new variables $x=\frac{g}{\gamma_c}S$, $y=\frac{g}{\gamma_0}(N-N_t)$, $w=\frac{g}{k\gamma_c}I-x$ and the time scale $t'=\gamma_0 t$. The rate equations then become

$$\dot{x} = x(y-1)+\gamma y \qquad (2)$$

$$\dot{y} = \gamma(\delta_0 - y + f(w+x) - xy) \qquad (3)$$

$$\dot{w} = -\varepsilon(w+x) \qquad (4)$$

where $f(w+x) \equiv \alpha\frac{w+x}{1+s(w+x)}$, $\delta_0=(I_0-I_t)/(I_c-I_t)$, $I_c = eV\gamma_c(\frac{\gamma_0}{g}+N_t)$, $\gamma=\gamma_c/\gamma_0$, $\varepsilon=\omega_0/\gamma_0$, $\alpha = Ak/(eV\gamma_0)$ and $s=\gamma_c s'k/g$. Figure 2 shows some of these patterns, obtained by numerical integration of Eqs. (2,4). As the parameter δ_0 is decreased, we observe the complete sequence of transitions going from the 1^0-state to the chaotic spiking regime, thus reproducing qualitatively the experimental results.

4 Noise effects on coupled chaotic systems

Let us consider two bidirectionally coupled LED systems:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x},\xi_{y_1})+\mathcal{K}(\mathbf{y}-\mathbf{x}) \\ \dot{\mathbf{y}} &= \mathbf{f}(\mathbf{y},\xi_{y_2})+\mathcal{K}(\mathbf{x}-\mathbf{y})\end{aligned} \qquad (5)$$

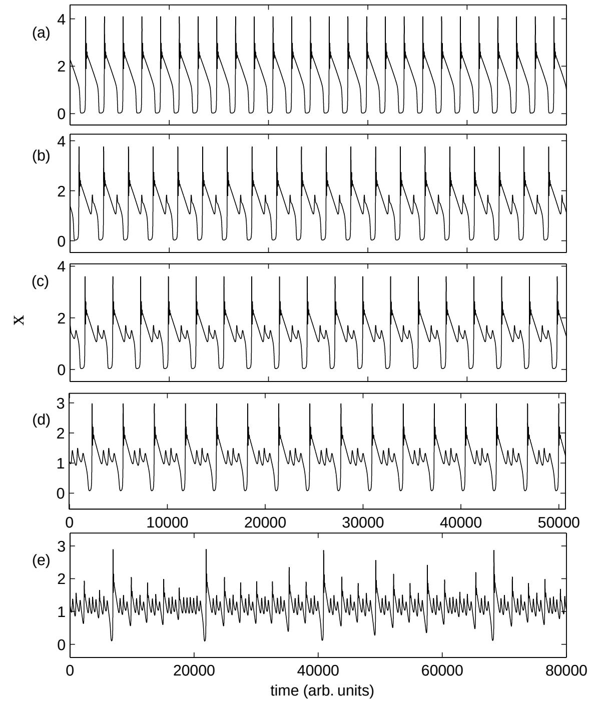

Figure 2. Time-series of the normalized optical intensity x as obtained by numerical integration of Eqs. (2,3,4): a) $\delta_0 = 2.45$, b) $\delta_0 = 2.35$, c) $\delta_0 = 2.3$ d) $\delta_0 = 2.13$, e) $\delta_0 = 2.11$. Fixed parameters: $\alpha = 1.007, \gamma = 0.01, \varepsilon = 5\times10^{-4}, s = 0.2$.

where $\mathbf{x} = \{x_1, y_1, w_1\}$ and $\mathbf{y} = \{x_2, y_2, w_2\}$ are vectors containing the system variables and $\mathcal{K}$ is a coupling matrix. The systems, both in the chaotic spiking regime, are coupled bidirectionally through the bias current, so we take the matrix $\mathcal{K}$ containing only one non-zero and positive value K. The coupling has been chosen sufficiently weak in order to avoid chaotic synchronization. Therefore each system follows a different chaotic spiking temporal evolution. Independent noise signals ξ_{y_1} and ξ_{y_2} are added to the bias currents of the two systems. Experimental measurements and numerical simulations, show that the dependence of the coefficient of variation (CV) on noise amplitude D changes its shape. In particular, it appears that the minimum in CV, corresponding to the maximum coherence, decrease depending on the coupling strength K. The same occurs for stochastic incoherence, which corresponds to maximum values in CV. In Fig. 3 we plot CV versus noise intensity obtained experimentally (Fig. 3 (a-b)) and numerically for two coupled systems defined(Fig. 3 (c-d)). All this indicate that the coherence of each unit is enhanced by the coupling. On the other side, the incoherence is worsened.

5 Conclusion

In conclusion, we reported experimental evidence of complex periodic and chaotic mixed-mode oscillations in a light-emitting diode with optoelectronic feedback. The experimental results have been qualitatively repro-

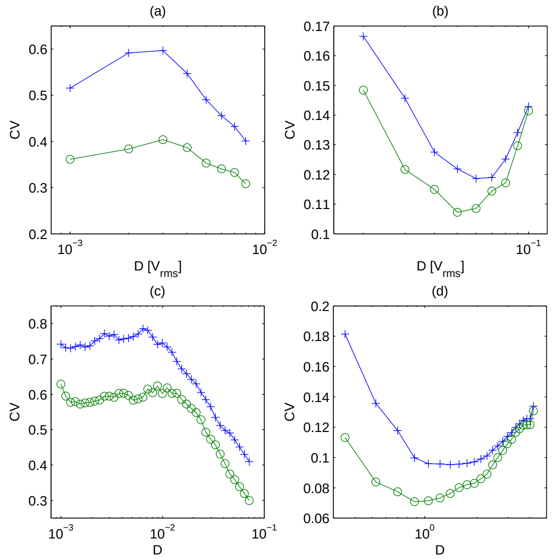

Figure 3. Experimental measurements of CV versus noise intensity for two light emitting diodes with optoelectronic feedback coupled with: (a) $K = 0$ (+) and (b) $K = 0.09$ (o). Numerical calculations of CV versus noise intensity for the coupling strengths: (c) $K = 10^{-5}$ (+) and (d) $K = 10^{-3}$ (o).

duced by a simple physical model of the system showing that the observed dynamics is the result of canard-phenomena in a 3D phase-space. We have shown that the bidirectional coupling of the systems in the chaotic regime and in the presence of white noise changes the global system dynamics causing more regular firing rate. In particular, we have shown that stochastic coherence can be enhanced by coupling, i.e. the minimum value of CV becomes smaller than that in the case of uncoupled systems. On the other hand, the stochastic incoherence is worsened, giving the smaller maximum of CV. These results reveal that noise amplitude can be considered as a parameter which controls the collective dynamics of the systems. The same phenomenology has been found numerically in a model of our experimental setup.

Acknowledgements

MC acknowledges Marie-Curie European Reintegration Grant (within 7th EC Programme) and SFA acknowledges ICTP-TRIL for financial support. Work partly supported by the contract ”Dinamiche cerebrali caotiche” of Ente Cassa di Risparmio di Firenze.

References

For a review: Focus issue on ”Mixed-mode oscillations: experiment, computation and analysis” Chaos, **18**, 015101-015115 (2008).

W. Ostwald, Z. Phys. Chem., Stoechiom. Verwandtschaftsl. **35**, 33 (1900).

R. A. Schmitz, K. R. Graziani, and J. L. Hudson, J. Chem. Phys. **67**, 3040 (1977).

K. Showalter, R. M. Noyes, and K. Bar-Eli, J. Chem. Phys. **69**, 25142524 (1978).

J. Maselko and H. L. Swinney, Phys. Lett. A **119**, 403 (1987).

M. Brons and K. Bar-Eli, J. Phys. Chem. **95**, 8706 (1991).

M. Bertram and A. S. Mikhailov, Phys. Rev. E **63**, 066102 (2001).

M. Bertram et al. Phys. Rev. E **67**, 036208 (2003).

M. Kim et al. Science **292**, 1357 (2001).

M. T. M. Koper, Physica D **80**, 72 (1995).

F. Plenge, P. Rodin, E. Scholl, and K. Krischer, Phys. Rev. E **64**, 056229 (2001).

A. A. Alonso and R. R. Llins, Nature (London) **342**, 175 (1989).

G. S. Medvedev and J. E. Cisternas, Physica D **194**, 333 (2004).

U. Kummer et al. Biophys. J. **79**, 1188 (2000).

M. Mikikian, M. Cavarroc, L. Couedel, Y. Tessier, and L. Boufendi, Phys. Rev. Lett. **100**, 225005 (2008).

R. Larter and C. G. Steinmetz, Philos. Trans. R. Soc. London, Ser. A **337**, 291 (1991).

A. Arneodo, F. Argoul, J. Elezgaray, and P. Richetti, Physica D **72**, 134 (1993).

M. T. M. Koper, Physica D **80**, 72 (1995).

E. Benoit, J. L. Callot, F. Diener, and M. Diener, Collect. Math. **32**, 37 (1981).

M. T. M. Koper, P. Gaspard, and J. H. Sluyters, J. Chem. Phys. **97**, 11 (1992).

K. Al-Naimee, F. Marino, M. Ciszak, R. Meucci, and F. T. Arecchi, New J. Phys. **11**, 073022 (2009).

F. Marino, F. Marin, S. Balle, and O. Piro, Phys. Rev. Lett. **98**, 074104 (2007).

F. Marino, and F. Marin, Phys. Rev E **83**, 015202 (2011).

K. Al-Naimee, F. Marino, M. Ciszak, S. F. Abdalah, R. Meucci, and F. T. Arecchi, Eur. Phys. J. D **58**, 187 (2010).