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FILTERING AND PARAMETERS ESTIMATION IN GYROS WITH AN ELASTIC SUSPENSION

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Abstract: In the paper the method and algorithm for increasing the accuracy of micromechanical gyroscopes is considered. This method is based on using Kalman filter for estimation of motion of sensing mass relatively output axis. This estimation is very effective because deflections of sensing mass relatively output axis are very small and measure by same angular minutes for micromechanical gyros (MMG) RR scheme. For this reason the level of the noises in the measurements are very significant. The paper presents a result of investigations of ability using Kalman filter for estimation parameters of MMG and angular rate of the mobile craft in which this gyroscope installed. Results of simulation are given and the questions of realization are considered.

Keywords: Kalman filter, micromechanical gyros, accuracy, estimation, elastic suspension.

1 Introduction

At the present time the great attention to improvement of micromechanical gyros (MMG) is given in [1, 2, 3, 4]. Fields of usage of the given type of gyros are determined first of all by accuracy rating of solved problems. Gears of low cost (less than 100 USD) which inaccuracy is in the range 10-1000 deg/hour, find wide application in automobile electronics, medical equipments, robotic technology, in consumer goods. There are prognoses that only for these directions, world production MMG can reach hundred million a year at a total cost of 4.5 billion dollars [5]. In difference from conventional types of the gyros, the accuracy of MMG improves in 10 times in each two years. Increase of accuracy of MMG to 0.01 deg/hour to 2020 is predicted. It creates backgrounds of making of miniature strapdown inertial navigation systems with correction from global navigation systems (GPS, GLONASS, etc.). The accuracy of such integrated systems will give ability to solve the problems of mission control of the wide type of the mobile craft. Improvement of the MMG goes in conventional directions for inertial sensors. With reference to MMG these directions are:

- Raise of exactitude of three-dimensional machining of silicon
- Raise of exactitude of microassembly operations
- Introduction of the embedded system for a temperature stabilization
- Maintenance of the given level of an evacuation of the sensor
- Increase of resolution, range and digit capacity of output converters of microdisplacements of the sensing mass.

In practice only these steps not enough for improve MMG to navigation accuracy rating. In [8] was consider the additional problems for increasing of MMG accuracy. In these papers the main attention is given to the circuitry analysis directed on:

- Compromise support between a conversion coefficient of measured angular rate and an instrument bandwidth
- Stabilizations of the characteristic which influence on the output signals of the gyro.

In this paper considers possibility to improve the accuracy of MMG with the help of Kalman filter.

2. Design concepts of MMG

A common element of all known designs of microgyros is the oscillating mass in elastic suspension. Angular velocity of the instrument relatively the inertial space results in the appearance of the Coriolis forces which determine the origin.

Excitation of primary oscillations in the microgyro is implemented with the help of a comb structured electrostatic driver of force. Measurement of secondary oscillations is carried out with the help of a capacitor transducer, one plate of which is mounted in the case of the device, and the other plate is assembled on the sensitive element. For creation of additional control efforts concerning a measuring axis the electrostatic driver of force on a design similar on capacitive an angle transmitter of turn of a countermeasure feeler is stipulated.

According to the type of movement of the sensing weight microgyros can be divided into four subclasses. In the scheme LL sensitive element during excitation of primary oscillations makes linear motion and the same linear motion in other plane at origin of Coriolis forces. In devices constructed by scheme RR, both vibration modes of a sensitive element are angular. Two vibration modes of a countermeasure feeler are realized in schemes LR and RL.

3. The main algorithm of estimation

At all variety of circuit designs of MMG, the simplified mathematical description of their dynamic is the same [8, 9]. As a first approximation movement of a sensitive element in RR-microgyroscope at influence of angular speed of the basis Ω_y can be described by the following equation

$$\ddot{\alpha} + 2\xi\omega\dot{\alpha} + \omega^2\alpha = 2\dot{\gamma}\Omega_y + kU(t) + g_w w(t) \quad (1)$$

where $\alpha(t)$ is the MMG angle rotation relatively of the sensitive axis, ω is a secondary oscillations resonant frequency, $\dot{\gamma}(t)$ is the instantaneous value of angular rate of MMG sensing mass relatively an axis of excitation of oscillations, γ_0 is an established amplitude of primary oscillations; $\Omega_y(t)$ is an angular rate of sensor frame or measured angular rate, U(t) is the control signal operating on the gyro, w(t) is a broad-band stochastic external disturbances.

For description of a moving craft often the following equations are used

$$\begin{aligned} \dot{X}_{cr} &= A_{cr}X_{cr} + B_{cr}u_{cr} + B_w w_{cr}, \\ y_{cr} &= C_{cr}X_{cr} + D_{cr}u_{cr} + v_{cr}. \end{aligned} \quad (2)$$

Equations (1) and (2) can be rewritten together in the following compact form

$$\begin{aligned} X(t) &= AX(t) + Bu(t) + w(t), \\ y(t) &= CX(t) + Du(t) + v(t), \end{aligned} \quad (3)$$

The elementary model, defining maneuverability of a moving craft on which the microgyro is installed, looks like

$$\dot{\Omega}_y(t) = -\frac{1}{T_\Omega}\Omega_y(t) + \sqrt{\frac{2}{T_\Omega}}\sigma_\Omega w_\Omega(t), \quad (4)$$

where T_Ω is a time constant of a moving craft; w_Ω is a random process of white noise type with unit intensity; σ_Ω is a root-mean-square error of angular rate of a moving craft .

This equation can be used for description of wide class of the moving craft. The same approach as will be conceded below can be used for more complex linear equations.

The equations (1) and 4) in the state space are recorded as follows:

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{\omega}_\alpha(t) \\ \dot{\Omega}_y(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & -2\xi\omega & 2\dot{\gamma}(t) \\ 0 & 0 & -\frac{1}{T_\Omega} \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \omega_\alpha(t) \\ \Omega_y(t) \end{bmatrix} + + \begin{bmatrix} 0 & 0 \\ g_w & 0 \\ 0 & \sqrt{\frac{2}{T_\Omega}}\sigma_\Omega \end{bmatrix} \begin{bmatrix} w(t) \\ w_\Omega(t) \end{bmatrix}. \quad (5)$$

The equation of measurements of an angle looks like

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha(t) \\ \omega_\alpha(t) \\ \Omega_y(t) \end{bmatrix} + v(t), \quad (6)$$

where $v(t)$ is error of angle $\alpha(t)$ measurement.

Generally nonstationary differential equations (5) and (6) can be recorded in the vector form (3).

The condition of observability for state vector $\mathbf{x}(t)$ on measurement $\mathbf{y}(t)$ looks like

$$rank\begin{bmatrix} C^T & A^T C^T & (A^T)^2 C^T \end{bmatrix} = n, \quad (7)$$

where n is an order of system of equations (3).

For system of equations (5) and the equations of measurements (6) condition of observability becomes

$$\text{rank}\begin{bmatrix} 1 & 0 & -\omega^2 \\ 0 & 1 & -2\xi\omega \\ 0 & 0 & 2\dot{\gamma}(t) \end{bmatrix} = 3.$$

This condition of observability is executed except for instants, for which $\dot{\gamma}(t) = 0$.

The equations of a nonstationary Kalman filter look like

$$\begin{bmatrix} \dot{\hat{\alpha}}(t) \\ \dot{\hat{\omega}}_\alpha(t) \\ \dot{\hat{\Omega}}_y(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & -2\xi\omega & 2\dot{\gamma}(t) \\ 0 & 0 & -\frac{1}{T_\Omega} \end{bmatrix} \begin{bmatrix} \hat{\alpha}(t) \\ \hat{\omega}_\alpha(t) \\ \hat{\Omega}_y(t) \end{bmatrix} + \begin{bmatrix} k_1(t) \\ k_2(t) \\ k_3(t) \end{bmatrix} \left(y(t) - \hat{\alpha}(t)\right). \quad (8)$$

or

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(t)\hat{\mathbf{x}}(t) + \mathbf{Bu}(t) + K(t)(y(t) - \hat{\alpha}(t)), \quad (9)$$

where

$$K(t) = P(t)C^T R^{-1}; \quad (10)$$

$$\dot{P}(t) = A(t)P(t) + P(t)A^T(t) + B_w Q B_w^T - \\ - P(t)C^T R^{-1} C P(t), \qquad (11)$$

where

Q is intensity of generating vector noise $\mathbf{w}(t)$, $Q = \begin{bmatrix} q_w & 0 \\ 0 & 1 \end{bmatrix}$;

R is intensity of noise of measurement $\mathbf{v}(t)$, $R = [r]$;

C is a matrix of measurements, $C = [1\ \ 0\ \ 0]$;

P_0 is initial conditions for the equation of Riccati (10).

4. Simulation of filtering and parameters estimation in gyros

Equations (5), (8), (10) and (11) are used for simulation and investigation of the properties of MMG with KF.

In Fig. 1, 2, 3 the functions of three component of matrix $K(t)$, which calculated with using of formula (10) are given. All these coefficients are functions of the time. The frequency of oscillations corresponds to ν and 2ν, where ν is the frequency of primarily oscillations of sensing mass. For testing example this frequency is ν =3000Hz.

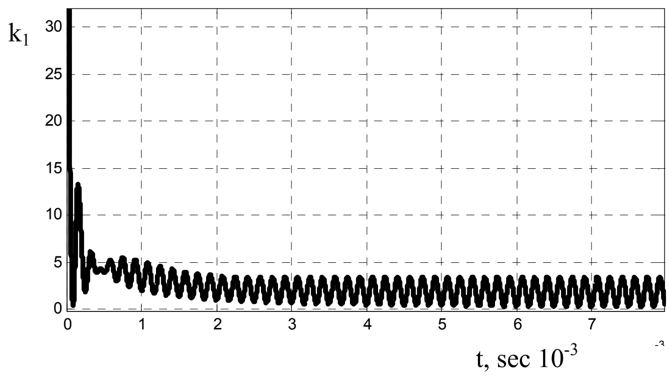

Fig.1 Dependence of the first factor of a matrix K from a time.

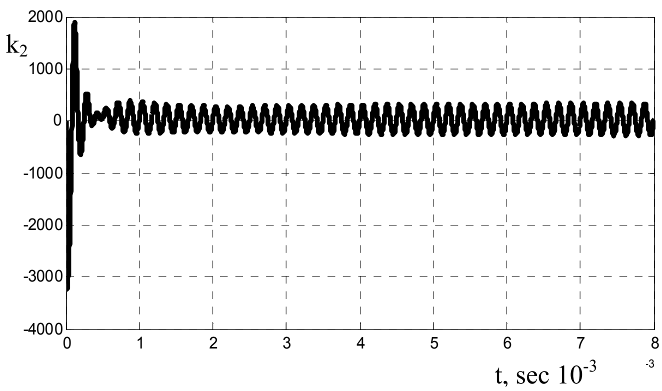

Fig.2 Dependence of the second factor of a matrix K from a time.

This is rather big frequency and for realization of considered filtration algorithm in numerical controller. Methods of practical simplification of this algorithm will be explained in

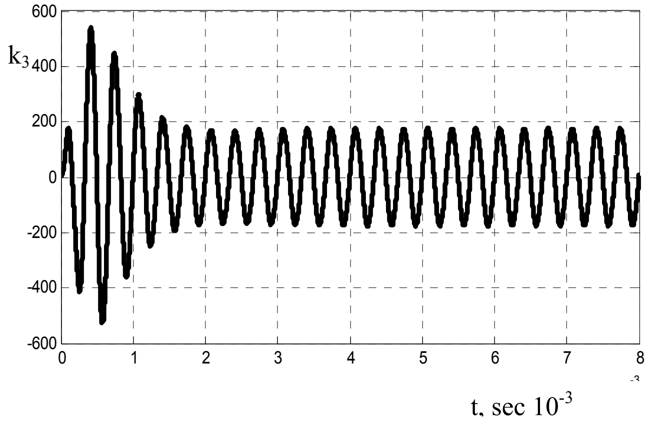

Fig.3 Dependence of the third factor of a matrix K from a time.

The Fig. 1-3 show that for realization of considered filtration algorithm in numerical controller, it is necessary to use the step of integration smaller then $\frac{1}{20-40}\frac{1}{2\nu}$ or to use special methods of simplification.. It is not possible for simplification to use constant average values of coefficients k2 and k3 because the functions k2(t) and k3(t) oscillate relatively zero with high frequency. For this reason it is convenient to generate these functions on the bases of measured primarily oscillations of sensing mass. In Fig. 4 the time response of estimation of the mobile craft velocity

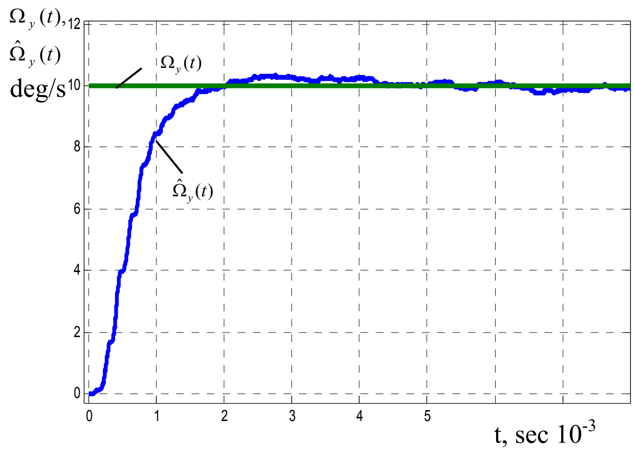

Fig.4 Angular velocity of the mobile craft $\Omega_y(t)$ and estimation of this velocity $\hat{\Omega}_y(t)$

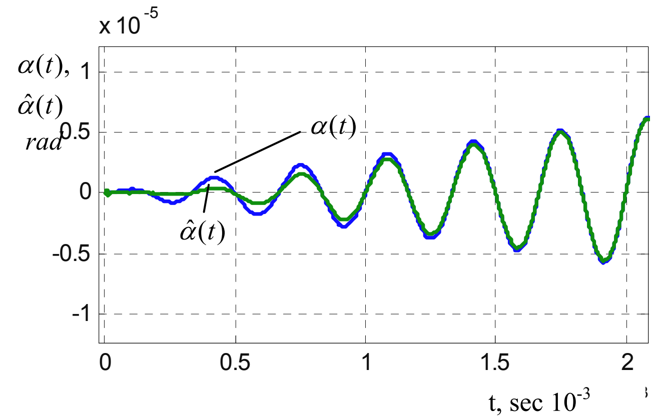

Fig.5 Angular deflection of the sensing mass $\alpha(t)$ and it estimation $\hat{\alpha}(t)$ for the first 2 msec

In Fig. 5 an angular deflection of the sensing mass $\alpha(t)$ and it estimation $\hat{\alpha}(t)$ for the first 2 msec after appearing constant angular rate $\hat{\Omega}_y(0)$ =0 is shown.

In Fig. 6 the same curves for the first 0.2 sec after appearing constant angular rate $\hat{\Omega}_y(0)$ =0 is shown.

Presented simulations show the effectiveness of the considered algorithm for estimation output oscillation $\alpha(t)$ and angular velocity of the mobile craft $\Omega_y(t)$. Response time for the estimation of angular velocity of the mobile craft and for the estimation output oscillation are not more 2-3 msec for considered model of microgyro.

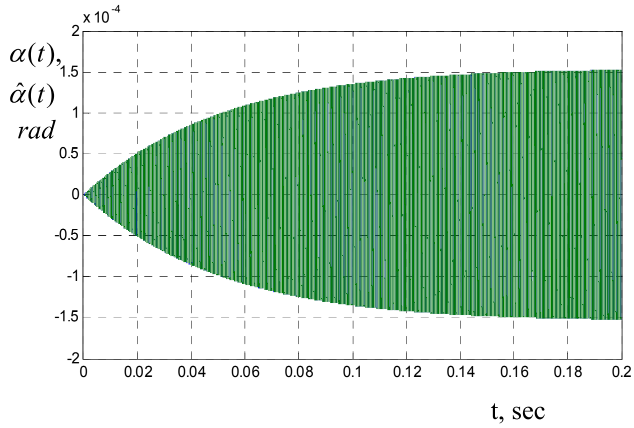

Fig.6 Angular deflection of the sensing mass $\alpha(t)$ and it estimation $\hat{\alpha}(t)$ for the first 0.2 sec

Because of symmetry of a matrix $P(t)$ in equation Riccati (11) can be rewritten in the following scalar form:

$$
\begin{aligned}
\dot{p}_{11} &= 2p_{12};\\
\dot{p}_{12} &= p_{22} - \omega^2 p_{11} - 2\xi\omega p_{21} + 2\dot{\gamma}(t)p_{31};\\
\dot{p}_{13} &= p_{23} - \frac{p_{23}}{T};\\
\dot{p}_{22} &= -\omega^2 p_{12} - 2\xi\omega p_{22} + 2\dot{\gamma}(t)p_{32} + g^2 q_w;\\
\dot{p}_{23} &= -\omega^2 p_{13} - 2\xi\omega p_{23} + 2\dot{\gamma}(t)p_{33} - \frac{p_{23}}{T};\\
\dot{p}_{33} &= -\frac{2p_{33}}{T} + \frac{2}{T_\Omega}\sigma^2{}_\Omega.
\end{aligned}
\tag{11}
$$

These equations compose nonstationary system of ordinary differential equation with sinusoidal high frequency parameter $\dot{\gamma}(t)$. This is instantaneous value of the primarily oscillations of the sensing mass, which are measured in the conventional MMG. The function $\dot{\gamma}(t)$ synchronize the components of the matrix *P(t)*. Through the equation (10), which in scalar form can be written in the following form:

$$K(t)=[p_{11}(t)/r \quad p_{12}(t)/r \quad p_{13}(t)/r], \tag{12}$$

where coefficients $k_1(t)$, $k_2(t)$, $k_3(t)$ will be synchronized with primarily oscillations. This is very significant fact because the frequency of primarily oscillations can changed during the gyroscope maintenance.

In Fig. 7 is show the configuration of the system for estimation of the craft angular rate $\Omega_y(t)$ and parameters of the microgyro $\hat{\alpha}(t)$ and $\hat{\dot{\alpha}}(t)$. Here $\dot{\gamma}_m(t)$ is the measured value of $\dot{\gamma}(t)$ and $\Delta\dot{\gamma}(t)$ is error of this measurement. In the unit Riccati Equation, the equations (12) are integrated. Equation (12) is used for calculation of time depended matrix K(t).

For investigation of the dynamical properties of considered algorithm was used the Simulink program, which structure in the Fig. 8 is presented. The program allows changing properties of moving craft and MMG, level of noises and errors.

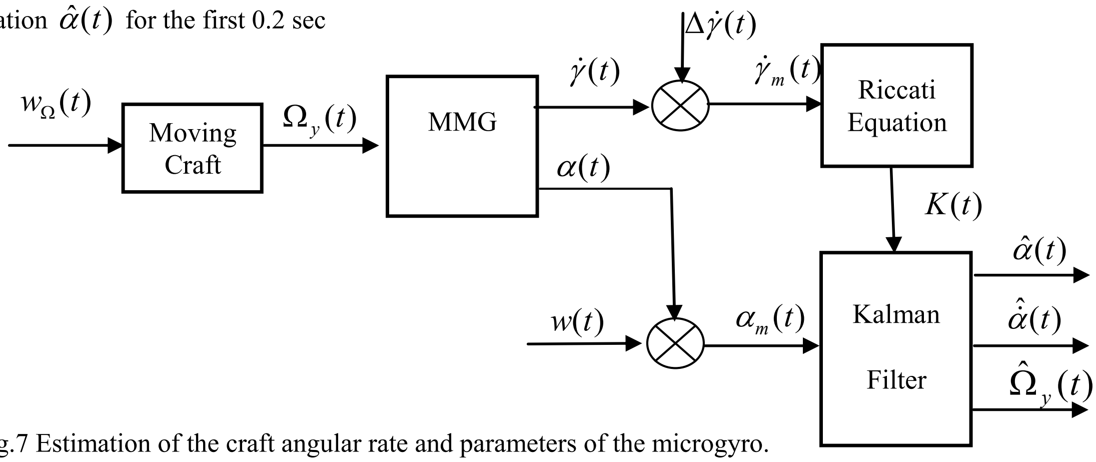

Fig.7 Estimation of the craft angular rate and parameters of the microgyro.

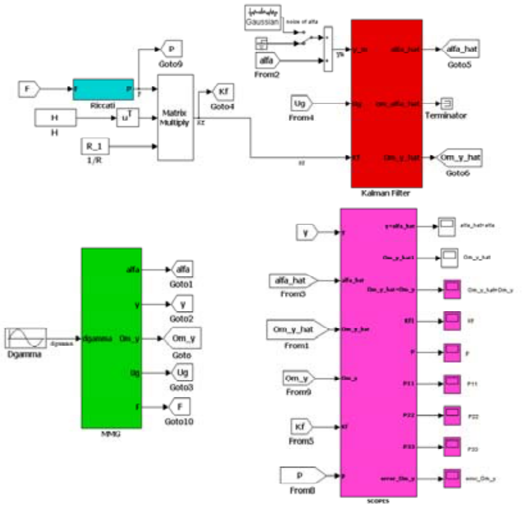

Fig.8 The Simulink structure of the program

Conclusion

1. The algorithm of parameters estimation of MMG is presented.

2. It is shown, that all components of a state vector are observed at any maneuvers of a moving craft.

3. Modeling has confirmed serviceability and efficiency of the offered algorithm at a modification of parameters of the gyro during its operation.

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