# CHARACTERISTIC CONTROL ORIENTED WAVEFORM DESIGN AND RECONSTRUCTION IN A RADAR SYSTEM

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### Abstract

A dynamic system for waveform design and reconstruction based on the extraction and feedback of signal characteristic is proposed in this paper. When the radar waveform is regarded as the solution of a dynamic network, chaos and sparsity of the waveform can be applied to reduce the searching space and decouple parameters for the multi-objective optimization in traditional radar waveform design. Compared with compressive reconstruction, the reconstructed signal can be easily obtained from synchronization in the dynamic networks. The waveform design and reconstruction is explained in detail by a chaotic self-FM system.

### Keywords

Signal characteristics; Waveform design; Sparse signal; Dynamic system

#### **1** Introduction

Waveform design is being actively researched in radar systems. Capability of anti-interference and low intercept, target detection probability and some other performance of signal can be obtained through radar waveform design. Essentially, to meet various required indexes by waveform design or reconstruction need solve a complicated joint multi-objective optimization problem, and there are even some coupling and constraints in the various indexes. This problem is generally solved by multi-objective optimization algorithm[Cui, Li and Rangaswamy, 2014]; However, multiple group long duration signal cannot be generated or reconstructed by optimization methods efficiently.

Chaotic signal is widely used in radar waveform design because it has better ambiguity function, low probability of intercept, good anti-interference capability and orthogonal performance. Existing chaotic waveform is usually modulated[Jiang, Liu, Hu, and Bao, 2010] or converted[Willsey, Cuomo and Oppenheim, 2011] for good performance, such as lower peak-average power ratio and controllable spectrum shape. But the modulation process must obey some strict mathematical conditions, resulting in some limitations for engineering applications. In this paper, we innovatively propose a closed loop dynamic network controlled by the feedback of output signal, which output a chaotic signal with other excellent application features.

Willsey and Oppenheim from Lincoln Laboratory introduced a Lorentz-based chaotic system to generate MIMO radar waveform owning low PAPR(peak-average power ratio)[Willsey, 2011; Bradaric, 2012]. Through the structuring of the signal source based on classic chaotic system is easy to implement, it can't meet the requirements of radar development because of lacking radar background. Compared with Willsey's proposal, the dynamic



Figure 1. Radar signal characteristic oriented dynamic system and compressed reconstruction



Figure 2. Signal generation system

system presented in this paper using a closed loop structure constructed by sparse dictionary, sparse coefficient controller, observation matrix and signal synthesis can meet the demand of radar waveform.

#### 2 Radar Waveform Generator

The frame of waveform generator and reconstructor is shown in Figure 1. Firstly, we design sparse dictionary to meet the application features of radar waveform, such as low PAPR, controllable spectrum shape. Secondly we generate the output signal by observation matrix, sparse coefficient and signal synthesis. Compressive signal can be obtained from output signal and the predesigned observation matrix. To generate the sparse coefficient for outputting signal at the next time, we use the feedback function and time-delay to control the compressed signal. Because the waveform generation system is based on sparse coefficient and dictionary, the process of waveform reconstruction can be easily realized. The waveform design method is described in the following paragraph.

# 2.1 Design of feature-oriented dictionary optimization

Constructing a dictionary with enough capacity is

the fundamental step for sparse signal representation. Since the signal matrix is  $Y = [y_1, y_2, \dots, y_M]$ where  $y_i$  is *n* dimensional signal, and its corresponding sparse coefficient is  $X = [x_1, x_2, \dots, x_M]$ , the problem of creating an appropriate dictionary can be described as the following optimization equation (1) where  $\|\cdot\|_2$  is  $l^2$  norm,  $\|\cdot\|_0$  is  $l^0$  norm and  $k_0$  is the number of non-zero coefficients. Since the form of *Y* is unknown, the optimization equation (1) is hard to solve directly.

$$\min_{\boldsymbol{\psi},\boldsymbol{X}} \sum_{i=1}^{M} \|\boldsymbol{Y} - \boldsymbol{\psi}\boldsymbol{X}\|_{2} \\
subject to \|\boldsymbol{x}_{i}\|_{0} \leq k_{0}, 1 \leq i \leq M$$
(1)

Dictionary can be obtained with some basic functions, such as the Fourier set or the DOG set, which have no correlation with the required signal. We can also train some signal with obvious characteristics by some dictionary training algorithms to create the dictionary, such as the least mean square recursive



Figure 4. Nonlinear self-FM system diagram

algorithm. Although the dictionary generated by training method has better sparsity, other characteristics are not ensured. Therefore, here we construct hierarchical tree type dictionary by parametric waveform sets with multi-objective optimization algorithm. This method can not only reduce the time complexity and improve the correlation with the signal needed, but also make joint optimization of multiple application features easy to achieve. We can use the waveforms in Ref. [Liu, 2013] which are optimized as our dictionary to generate longer complex waveforms by the dynamical system described in Figure 1.

# 2.2 The dynamic-network based signal synthesis and analysis

The signal generation system is shown in Figure 2. We synthesize the output signal by the predesigned sparse dictionary and the controlled sparse coefficients from the compressive signal and feedback controller.

As shown in Figure 2, the synthetic signal is  $Y_n = \psi X_n$  where  $\psi$  is the dictionary designed in paragraph 2.1 and  $X_n$  is the given sparse coefficient. We can get the compressive sampling of signal  $Y_n$  by the pre-constructed observation matrix  $\Phi$ , and obtain the sparse coefficient  $X_{n+1}$  for next time by the feedback controller  $h(\Phi Y_n)$ . To keep the dynamic system chaotic lasting long time and ensure the sparsity, controllable spectrum shape and low PAPR of the output signal, we need design apposite  $h(\cdot)$  by analysis rich dynamical behavior of the waveform generator.

# 2.3 The compressive-sampling based signal reconstruction

In compressed sensing theory, signal reconstruction is achieved by solving complex optimization problems. In our presented method, the signal reconstruction can be realized by the sparse coefficient and dictionary. The coefficient is the result of compressed signal after the transformation  $h(\cdot)$ , and the dictionary is designed by some optimization rules. In the system presented here, the transformation function  $h(\cdot)$  and dictionary  $\psi$  are known, reconstructed signal  $Y_n$  can be got from compressed data  $\Phi Y_n$  directly as shown in Figure 3.

### 3 Example of a Waveform Generator

In order to verify the feasibility of the waveform design and waveform reconstruction introduced in Section 2 and describe this system clearly, we present a simple case here. A closed loop self-feedback frequency modulation (FM) system[Hu, Li, Zhang, Liu and Zhao, 2012] is shown in Figure 4. In this waveform generator, we use Fourier set as the

dictionary, and limit the number of non-zero coefficient to 1 and quantify the amplitude of the parse coefficient X to 0 and 1. The Fourier set and the sparse coefficient correspond to the VCO in Figure 4, compressed sampling of the signal is achieved by the under-sampling of signal output signal x, feedback controller function  $h(\cdot)$  is the polynomial about under-sampling data  $x_n$  shown in  $f(x_n)$ . We control the system chaotic and ensure the waveform with long time complexity, ideal ambiguity function and controllable spectral shape by designing and optimizing function  $h(\cdot)$ . The characteristics of the waveform is shown in Figure 5, and the PAPR of output signal is 2.0001 which is close to singlefrequency sinusoidal signal.





(b) Frequency spectrum with one notch





Since the function  $h(\cdot)$  and the dictionary  $\psi$ are known, as shown in Figure 6, we reconstruct waveform by the system based on minimizing the synchronization error.





The system in Figure 6 can reconstruct waveform  $x_1(t)$  from compressive sampling data. The results

of simulation analysis and experimental verification are shown in Figure 7. Figure 7(a) shows that  $x_2(t)$ is tend to  $x_1(t)$  after a short transient process. The waveform in Figure 7(c) and the synchronization profile in Figure 7(d) both verify the effectiveness of the reconstruction system.



(a) Simulation result





(c) Waveform observed by oscilloscope







## 4 Conclusion

A characteristic control oriented waveform

design and reconstruction for radar application is proposed in this paper. We design the dynamic system using a closed loop structure from signal synthesis, observation matrix, sparse dictionary and sparse coefficient controller. Constructing suitable dictionary and feedback function by analysis the dynamic behavior of the system can achieve multiple application characteristic and rapid reconstruction of the radar waveforms.

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