Stability Analysis of a "Thyristor Voltage Controller – Induction Machine" Model

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Abstract–**Soft starters based on thyristorised voltage controller (TVC) are used as induction motor controllers in many industrial applications. Pulse-phase control system (PPCS) of the TVC can operate under two types of synchronization: line voltage synchronization and phase current synchronization. Oscillation processes can occur in stationary states of the PPCS with voltage synchronization as it has been shown in [1]–[3]. An averaging (linearizing) modeling approach does not allow to investigate this problem and to analyze the model stability adequately. Therefore, an analysis of the stationary state parametric stability is carried out by numerical calculation of a monodromy matrix in this paper.**

Index Terms–**Stability analysis, induction motors, soft start.**

I. INTRODUCTION

In order to mitigate the adverse effects of starting torque transients and high inrush currents in induction motors, a popular method is to use electronically controlled soft-starting voltages utilizing series-connected silicon-controlled rectifiers (SCRs).

At present soft start devices (hereafter softstarters) are developed based on the pulse-phase control system (PPCS) with thyristor voltage controller (TVC). TVC consists of the series-connected silicon-controlled rectifiers (SCRs), which are easy to produce, reliable and low-cost. In addition, SCRs allow switching of the currents with greater magnitude compared with transistors. It allows the TVC-based softstarters to compete with adjustable-speed AC drives based on transistor converter [1]–[4].

A complicated mathematical modeling and simulation is a disadvantage of the PPCS-TVC of induction machines (hereafter PPCS-TVC-IM). It's difficult to research the PPCS-TVC-IM behaviors in transient and stationary states. For example, stability of the stationary states has been researched with the help of linearized models of the PPCS-TVC-IM especially [2].

Pulse-phase control system (PPCS) of the TVC can operate under two types of synchronization: line voltage synchronization and phase current synchronization. Oscillation processes, characterized by deviation of angular velocity of IM exist in the stationary states of the PPCS-TVC-IM with voltage synchronization as it has been reported in [1]–[3]. An averaging (linearizing) modeling approach does not allow to investigate this problem and to analyze the model stability adequately. Therefore, both an analysis of the dynamic behaviors and control algorithm design must perform by non-linear model. In this paper, such complete (non-linear) model of the PPCS-TVC-IM is used for simulation and stability analysis.

The paper is organized as follows: an approach, based on the Kirchhoff and Ohm laws [5], of the non-linear PPCS-TVC-IM modeling is described in Section 2. Dynamic behaviors are compared by simulation of the PPCS-TVC-IM model with two types of synchronization. Before concluding, the parametric stability analysis of the stationary state, base on monodromy matrix numerical calculation is carried out for both types of the model synchronization in Section 3.

II. PPCS-TVC-IM. MODELING AND SIMULATION

Normally, modeling of the PPCS-TVC-IM is carried out as follows [2]–[4]: (i) the generalized two-phase IM model is represented; (ii) the supply conditions are obtained by operation mode analysis. But in this case the representation of non-symmetrical supply conditions is carried out by generalized IM model based on symmetrical supply condition assumption. Therefore this approach is not sufficiently valid.

In this paper, an approach, based on the Kirchhoff and Ohm laws, is applied to obtain a non-linear model under each operation mode, as has been proposed in [5]. At first, an analysis of the TVC-IM model operate is carried out to obtain the structure constancy intervals (SCI). The basic circuit is used for the control (Fig. 1.a). TVC-IM model is operated under following modes: "0" is a disconnected mode; "AB", "BC", "AC" are two-phase conduction modes; "ABC" is a three-phase conduction mode as shown in Fig. 1.b.

At second, a non-linear system of ordinary differential equations (ODEs) is corresponded to each SCI and is represented in the following form:

$$\frac{dx_i(t)}{dt} = f_i\left[x_1(t), x_2(t), ..., x_n(t), g(t)\right],\ x_i(t) \in \mathbf{R}^n, \quad t \in [t_j, t_{j+1}],\ i = 1,2,...,n;\ x_i(0) = x_i^0,\ i = 1,2,...,n, \tag{1}$$

where f_i, i=1…n are the non-linear functions; x_i, i=1...n are the state variables of the PPCS-TVC-IM model (angular velocity of the IM shaft; currents and/or flux linkages of both

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stator and rotor windings); $g(t)$ is a line voltage and n corresponds to a number of the state variables (n=4 in the case of the "0"-mode; n=5 in the case of the "AB"-, "BC"-, "AC"-mode; n=7 in the case of the "ABC"-mode).

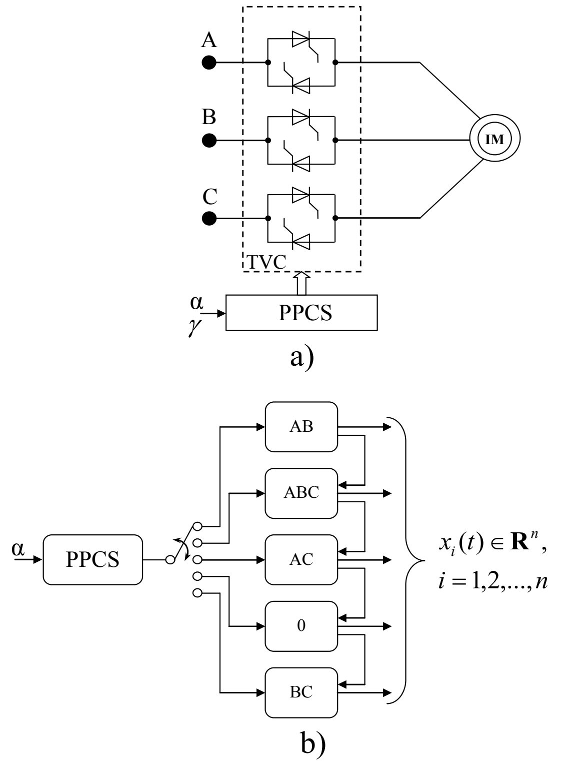

Fig. 1. (a) PPCS-TVC-IM model; (b) the operating scheme of the PPCS-TVC-IM

The mathematical models [6] of each SCI (1) are derived by using traditional assumptions of the electromechanical conversion energy theory (for example, the saturation neglecting of magnetic conductors, the plain air gap without stator and rotor slots, neglecting of the mechanical friction, etc.).

Two types of synchronization are used in PPCS-TVC [2], [3]:

(i) a current synchronization (CS); the control angle γ corresponds to the instant at which a thyristor is triggered on with the respect to the phase-current reaches zero (Fig. 2.a);

(ii) a line voltage synchronization (VS); the thyristor firing angle α from the point of zero crossing of each phase voltage (Fig. 2.b).

The dependence of the control angle α on γ is represented by this relation:

$$\alpha = \gamma + \delta, \tag{2}$$

where δ is an angle of current delay in inductive load circuits, depending on the equivalent circuit of the IM.

In this paper, all simulations are executed with the parameters of the IM "4A225M4Y3" (rated power P=55 kW); both the zero initial and mechanical load absence conditions are applied.

Simulation of the PPCS-TVC-IM model with both types of synchronization demands to estimate the angle δ under specified parameters of the IM.

For that purpose, let us draw the dependence $\alpha = f(\gamma)$ in the stationary state by means of numerical simulation of the PPCS-TVC-IM (the dots on the Fig. 3). The shape of the obtained curve allows approximating it by straight line (the line on the Fig. 3). The equation of the approximating line has the following form:

$$\alpha = \gamma + 64. \tag{3}$$

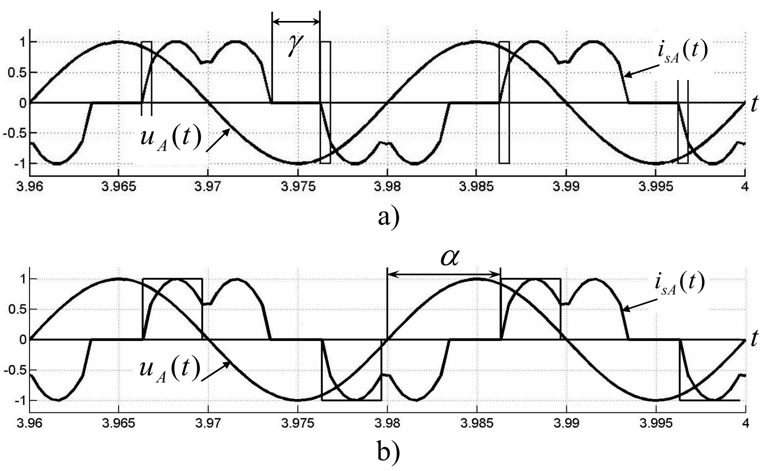

Fig. 2. Waveform of the stator current "A" ($i_{sA}(t)$,[p.u.]) and the line-voltage of the phase "A" ($u_A(t)$,[p.u.]) of the PPCS-TVC-IM with VS (a); also for the PPCS-TVC-IM with CS (b).

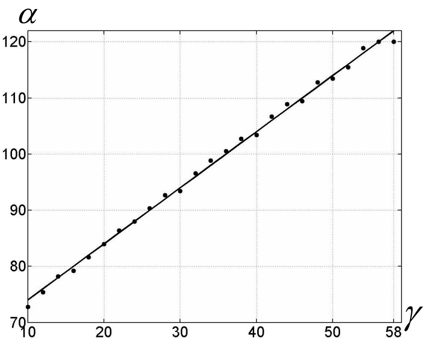

Fig. 3. The dependence of the control angle α on γ.

As shown by several papers (for example, [2], [3]), the linearized PPCS-TVC-IM model with VS is not stable in the range of subsynchronous angular velocity. To validate this result, let us simulate the stationary state of the PPCS-TVC-IM with equal initial conditions for each kind of synchronization.

The time dependences of the IM shaft angular velocity $\omega_r(t)$ and both the stator current and current envelope $\hat{I}_{sA}(t)$ of the winding "A" are shown in the Fig. 4.a. The control angle α is set to 89 degree. A moment of inertia (J) on the IM shaft is equal to the rated IM inertia moment (J=J_{rated}).

The relative deviation of the angular velocity of the IM shaft around stationary value is calculated as follows:

$$\Delta\omega = \frac{\omega_\sigma}{\overline{\omega}_r} \cdot 100\%, \qquad (4)$$

where $\omega_\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\omega_{r,i} - \overline{\omega}_r\right)^2}$ is the mean square deviation of $\omega_r(t)$ around mean angular velocity $\overline{\omega}_r = \frac{1}{n}\sum \omega_{r,i}$ in the stationary state.

Plotting of the current envelope $\hat{I}_{sA}(t)$ (Fig. 4.a) allows to determine the period of oscillation $T\big|_{\alpha=89} = 2.14$ sec. The dependence of the harmonic current magnitudes on current frequency (f) (Fig. 4.b) shows that the quasi-particle components (f=3·f_{base},5·f_{base},7·f_{base}...) exist around fundamental harmonic f_{base}=50 Hz. It is necessary to emphasize the considerable peak-to-peak value of the IM angular velocity. The relative deviation of the angular velocity reaches $|\Delta\omega| = 3\%$.

Let us draw the dependences showed above for the PPCS-TVC-IM with CS. Expressing (3) in term of γ and substituting $\alpha = 89$, we get: $\gamma = 35$ degree. The time dependences of the $i_{sA}(t)$ and $\omega_r(t)$ are showed in the Fig. 5.a for the PPCS-TVC-IM with CS. Dependence of the harmonic components of $i_{sA}(t)$ on the frequency f is showed in the Fig. 5.b.

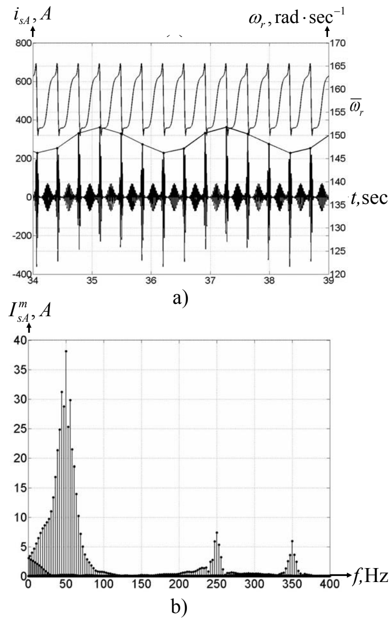

Fig. 4. (a) Electromechanical processes of the PPCS-TVC-IM with VS in the stationary state; (b) the dependence of the amplitude I^m_{sA} on frequency f of the current harmonic components.

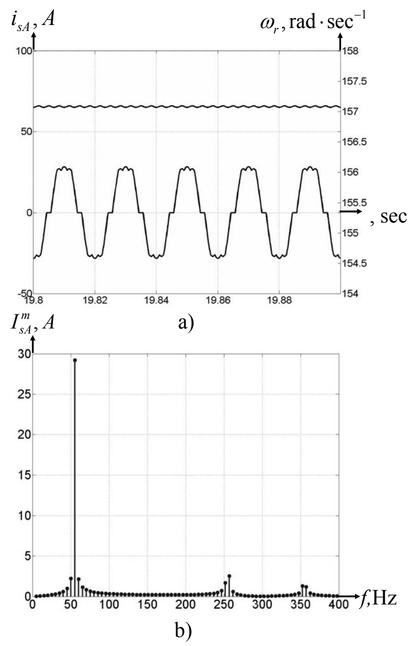

Fig. 5. (a) Electromechanical processes of the PPCS-TVC-IM with CS in the stationary state; (b) the dependence of the amplitude I^m_{sA} on frequency f of the current harmonic components.

The frequency spectrum of $i_{sA}(t)$ (Fig. 5.b) shows that the value of quasi-particle components are small. The relative deviation of the angular velocity does not exceed the $|\Delta\omega| = 4.8 \cdot 10^{-3}\%$.

Therefore, previous analysis shows that the magnitude of the state variable oscillation processes can reach significant values in the stationary states of the PPCS-TVC-IM with voltage synchronization. In this case, an application of the averaging modeling approach shows instability as it has been reported in [2]. To show the necessary of the non-linear PPCS-TVC-IM model using, stability analysis this one will carry out by numerical calculation of the monodromy matrix in the next section.

III. Stability research of the PPCS-TVC-IM models

Theoretical stability analysis of the non-linear switched models is a very complicate problem, but it can be executed by numerical methods. One of them is a wide-spread approach, based on monodromy matrix numerical calculation [7], [8]. The main idea of this technique is at first to derive the monodromy matrix of the steady-state model. After that, the matrix eigenvalues calculation allows to make the conclusion about stability behavior of the investigated system [7], [8].

For the monodromy matrix evaluation purpose, let us give the map of the shift as follows:

$$\mathbf{X}_k=\mathbf{F}(\mathbf{X}_{k-1}), \tag{5}$$

where $\mathbf{X}_k$ is the augmented vector of the state variables at the instant $t=kT$; $\mathbf{F}(\mathbf{X}_{k-1})$ is the vector-function set of the system (1), transformed to the autonomous form.

Sequence of points $\mathbf{X}_{ck}$, $k=1,2,..,m$ of the map (5) corresponds to the periodical process with period T_m which is multiple of period T of the line-voltage ($T_m=mT$, $m=1,2,\ldots$). Therefore we have:

$$\mathbf{X}_{ck} = \mathbf{F}^{(m)}(\mathbf{X}_{ck}), \tag{6}$$

where $\mathbf{F}^{(m)}(\mathbf{X}_{ck})$ are the m-consecutive iterations of the map (5).

Coordinates of the fixed points are determined as follows: the observable model is integrated on the period T with maximum distance between elements of the vectors $\mathbf{X}_{k-1}$ and $\mathbf{X}_k$ calculation until the distance is not less than the required accuracy.

Local stability of the m-cycle $\mathbf{X}_{ck}$, $k=1,2,..,m$ is researched through incremental equation as follows. Let $\Delta_k=\mathbf{X}_k-\mathbf{X}_{ck}$, $k=1,2,..,m$ be infinitesimal perturbance of the state vector $\mathbf{X}_k$ from coordinates of the m-cycle $\mathbf{X}_{ck}$ then disturbance on $k+1$ step in linear approximation has the following form:

$$\Delta_{k+1}=\mathbf{J}_k\Delta_k, \tag{7}$$

where $\mathbf{J}_k = \left.\dfrac{d\mathbf{F}}{d\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}_{ck}}$ is the Jacobian matrix calculated at $\mathbf{X}_{ck}$, $k=1,2,.., m$.

Evolution of the disturbance Δ_k throughout m iterations of the map (3) is calculated as follows:

$$\Delta_{m+1}=\mathbf{M}\Delta_1, \tag{8}$$

where $\mathbf{M} = \prod_{k=1}^{m} \mathbf{J}_k$ is the monodromy matrix corresponding to the m-cycle.

Thereby, if the following condition is satisfied than we can say that the m-cycle of the system is asymptotically stable [7]:

$$|\rho_i| < 1, i = 1,2,...,n, \tag{9}$$

where $\rho_i, i = 1,2,...,n$ are the eigenvalues of the matrix $\mathbf{M}$.

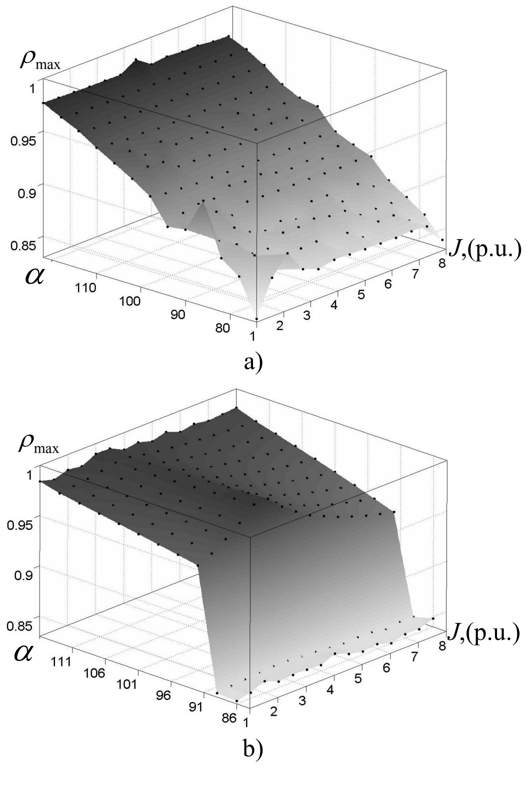

Fig. 6. $\rho_{\max} = f(\alpha, J)$: (a) – the PPCS-TVC-IM with CS; (b) – the PPCS-TVC-IM with VS.

Let us draw the surfaces evolution of the maximum eigenvalue $\rho_{\max} = \max_{1\le i\le n}|\rho_i|$ as the function of the control angle and IM inertia moment. The surfaces $\rho_{\max}$ for PPCS-TVC-IM with both CS and VS are shown in the Fig. 6.a and 6.b, respectively. The control angle α is calculated by (3) to plotting of Fig. 6.a and Fig. 7.a.

The analysis of the surfaces showed above allows to conclude: the periodical processes of the observable models are stable (in the sense suggested by Lyapunov) because the condition (9) is fulfilled in the turndown of parameters.

However, the angular velocity oscillation of the IM, showed in the previous section, may have unacceptable amplitude for some applications. Let us draw the evolution surfaces of relative deviation of the angular velocity $\Delta\omega$ as function of the control angle and IM inertia moment. The surfaces $\Delta\omega$ for PPCS-TVC-IM with both CS and VS are shown on the Fig. 7.a and 7.b, respectively.

The surface analysis showed in the Fig. 7 allows to emphasize that the relative deviation of the angular velocity $\Delta\omega$ of the PPCS-TVC-IM with VS reaches the value 6%; the relative deviation of the angular velocity $\Delta\omega$ of the PPCS-TVC-IM with CS is much less and approximately equal to $\Delta\omega \approx 0.08\%$ in comparison to the system with VS. Thereby, the PPCS-TVC-IM with CS can have wider application than similar system with VS.

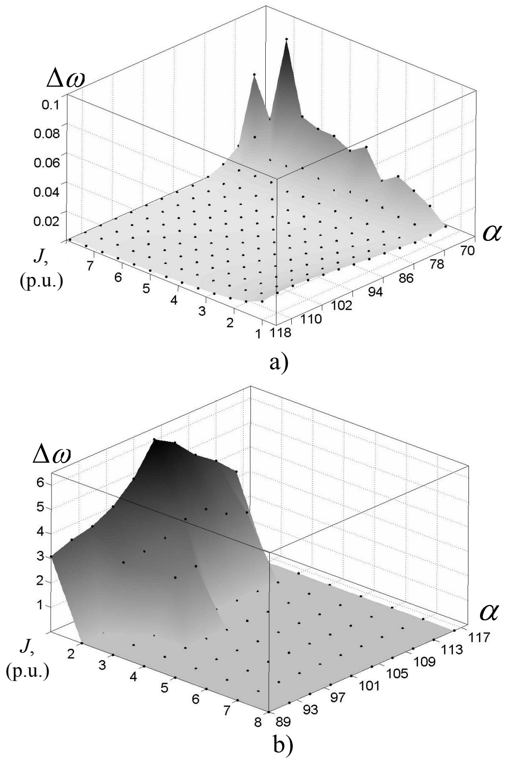

Fig. 7. $\Delta\omega = f(\alpha, J)$: (a) – the PPCS-TVC-IM with CS; (b) – the PPCS-TVC-IM with VS.

IV. Conclusion

The non-linear PPCS-TVC-IM model with two types of synchronization was described and simulated. The parametric stability analysis of the stationary state, base on monodromy matrix numerical calculation was carried out for both types of the model synchronization. Periodical processes of the stationary state are stable in both the turndown of the control angle and range of the IM inertia moment $J \in [J_{rated}; J_{rated} \cdot 8]$. However, the oscillation processes exist in the stationary states of the PPCS-TVC-IM with voltage synchronization in the IM inertia moment range $J \in [J_{rated}; J_{rated} \cdot 3]$. In this case, the use of an averaging (linearizing) modeling approach does not allow to analyze the model stability adequately. Therefore, the dynamic behavior analysis of the softstarter models with VS must execute by non-linear PPCS-TVC-IM model application.

As shown in this paper, CS of the PPCS-TVC-IM decreases the oscillations in the stationary state. It makes CS of the PPCS-TVC-IM more attractive for implementation against the VS. Especially, it can use in both the soft start devices and energy-saving drive systems.

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