Nonlinear Dynamics of the Spacecraft Gyromoment Control System with Plasma Thrusters at Initial Modes*

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Abstract: Recently, space science and engineering advanced new problem before theoretical mechanics and motion control theory: a spacecraft directed respinup by the weak restricted control internal forces. The paper presents some results on this problem, which is very actual for energy supply of the communication mini-satellites with plasma thrusters at initial mission modes.

Keywords: nonlinear dynamics, robust control, spacecraft, rotary oscillations

1. INTRODUCTION

On April 1, 2009 it is 50 years since the specialized department of *S. Korolev* Rocket-Space Design Bureau (Moscow), was set up in Krasnoyarsk-26 on the Yenissey, Siberia. In a short time period that small department had grown into the Nauchno - Proizvodstvennoe Obyedinenie (NPO) *Prikladnoy Mekaniki* (NPO PM). During these 50 years this company created over 1800 automatic spacecraft (SC) and became the respected Russian leader in development, manufacture and operations of navigation, coordinate-metric (geodetic) and telecommunication satellite systems. Acad. *M. Reshetnev*, colleague and follower of Acad. *S. Korolev*, was the General Director/Designer of the NPO PM from the very first days of the Company till his death on 26.01.1996. Today the NPO PM have name JSC "Acad. Reshetnev Information Satellite Systems" Company (ISS Reshetnev).

The priority dates of the ISS pioneering achievements:

- 18.08.1964 — first Soviet constellation of communication *Cosmos* series spacecraft had been created;
- 25.05.1965 — first Soviet communication satellite *Molniya-1* (Fig. 1), equipped with the world's first active 3-axial Attitude and Orbit Control System (AOCS), was put into high-elliptic orbit;
- 15.11.1967 — first Soviet navigation SC *Cosmos-192* was launched;
- 20.02.1968 — first Soviet geodetic satellite *Cosmos-203* was launched;
- 29.07.1974 — world-first retransmission satellite with an active 3-axial AOCS was put into geostationary orbit (GEO).

Fig. 1. The Russian communication SC *Molniya*

Starting from 1975, the ISS is actively conquering the GEO. The following spacecraft were launched:

- 22.12.1975 — first communication satellite of *Raduga* series,
- 26.10.1976 — first television satellite of *Ekran* series,

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Fig. 2. The Russian re-transmitter satellite *Lutch*

- 19.12.1978 — first communication satellite of *Gorizont* series,
- 25.10.1985 — first communication satellite of *Lutch* series with completely new precision digital AOCS, Fig. 2.

Geostationary spacecraft of *Gals* series (20.01.1994) and *Express* series (13.10.1994), Fig. 3, were put into nominal operations. After 30.09.1981 the ISS had started deployment of the first Soviet geodetic satellite system based on the spacecraft *GEO-IK*, and after 12.10.1982 — the well-known Russian space navigation system *Glonass*, Fig. 4.

The ISS is known all over the world as a designer and manufacturer of communication, navigation and geodetic spacecraft. It is less known that the ISS is a designer and manufacturer of various types of Attitude & Orbit Control Systems (AOCS), which are the most complex

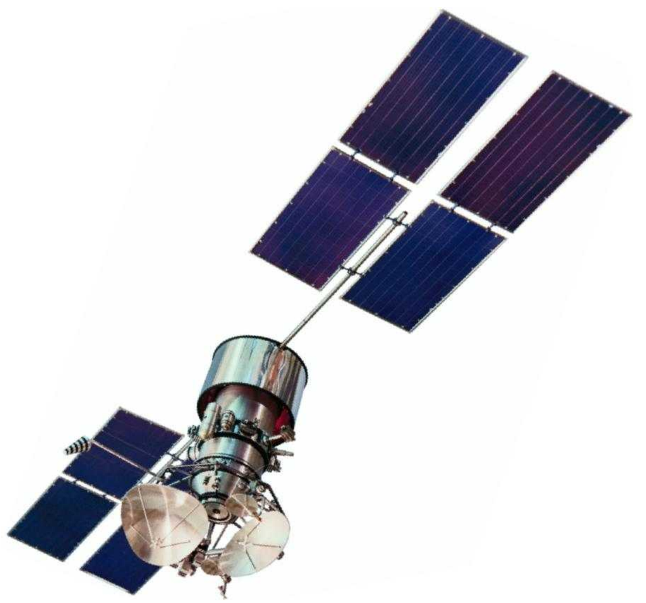

Fig. 3. The Russian geostationary satellite *Express*

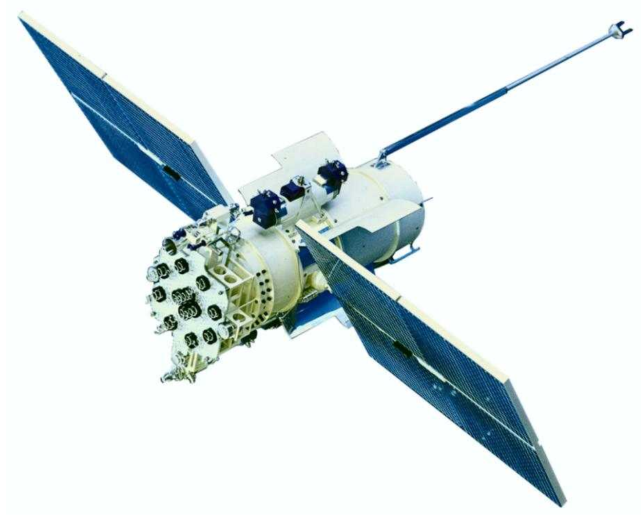

Fig. 4. The Russian navigation spacecraft *Glonass*

and expensive among the onboard subsystems. The ISS has developed a wide spectrum of spacecraft AOCS types:

- the *passive* systems, which use the Earth gravitation and magnetic fields for the SC attitude control;
- the *combined* systems with electromechanical drives used for autonomous pointing of receiving-transmitting antenna platforms with respect to the SC body, passively oriented in the orbit;
- the *active* systems, which use various types of electromechanical actuators, magnetic torques used for their desaturation on accumulated angular momentum (AM) as well as electromechanical systems of autonomous pointing Solar Array panels and antenna platforms.

Fig. 5. The geostationary spacecraft *Sesat*

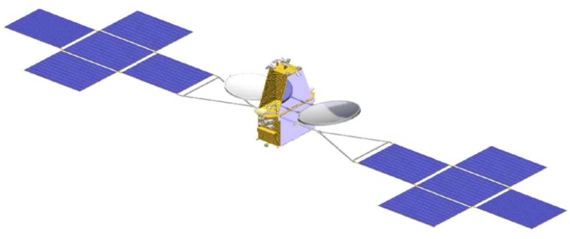

Fig. 6. The communication satellite *Express-AK*

For the purpose of orbit control (spacecraft orbit-keeping manoeuvres) the special types of propulsion units have been designed, using gas, thermal-catalytic and plasma thrusters. AOCS developed by ISS could be used for various spacecraft in circular, elliptic and geostationary orbits, see Fig. 5.

The ISS has accumulated substantial research and practical experience in solving problems of the algorithm synthesis for motion control of interconnected mechanical objects with complex dynamic structure under determinate and stochastic disturbances, as well as problems of configuring equipment and structure of such systems. Methodology and technology of their experimental development under the close-to-real-space flight environment have also been created. Substantial experience was also acquired in on-board software development of methods for on-ground and flight calibration, as well as spatial adjustment of the onboard sensors, antennas and actuators. Valuable information on real disturbing factors due to the spacecraft interaction with space environment has been obtained and systematized.

For over 30 years the ISS has successfully coordinated research and developments in orbit and attitude control of communication, navigation and geodetic spacecraft, which are conducted by the leading institutes of the RAS, by research institutes and design bureau of the FSA, and also by aerospace universities. ISS conducts joint research with the world-known scientific schools of Acad. B.N. Petrov and Prof. V.Yu. Rutkovsky (S.D. Zemlyakov, V.M. Sukhanov et al., Institute of Control Sciences, the RAS), Acad. V.M. Matrosov (The Stability and Nonlinear Dynamics Research Center, the RAS), Ye.I. Somov (Samara Scientific Center, the RAS), L.V. Dokutchaev, O.P. Klishev (TsNIIMash, the FSA), V.V. Malyshev, M.N. Krasilshikov (MAI), G.L. Degtyarev (KAI).

In the current practice, geostationary satellites enjoying 15-year life and high-accurate station-keeping maneuvers are equipped with thruster unit based on plasma reaction thrusters (RTs) having high specific pulse and large power consumption. While designing mini-satellite weighted of 400 to 800 kg (as Russian satellite *Express-AK*, Fig. 6) it is very attractive to employ plasma RTs only for all modes. The constrains at the problem are as follows (Titov et al., 2003):

• On separating from a launcher, a spacecraft (SC) obtains an initial angular rate up to 20°/s. During that SC rotation an electric power required for the on-board equipment is generated by solar arrays panels (SAPs) or by chemical batteries. An energy generated by the SAPs depends on an angle between their normal and direction towards the Sun.

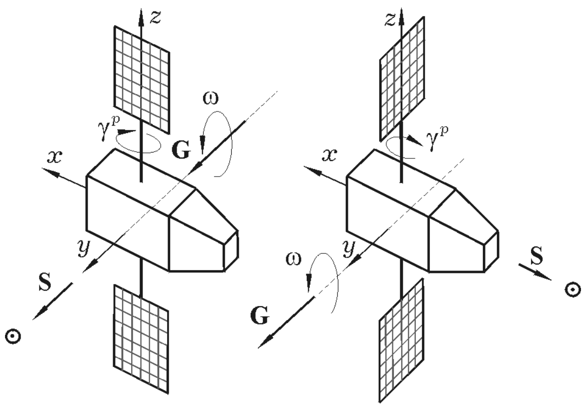

Fig. 7. The rotating SC attitude over the Sun

• Plasma RT enjoy small thrust values (about several grams) and large power consumption (magnitude of 1 to 1.5 kW). Small thrusts and therefore small control torques are the cause of a long time period required to damp initial SC rate. The plasma RTs can be activated a specified time period T_a from several *hours* to several *days* after the separation.

• Severe requirements applied to the mass of the attitude & orbit control system (AOCS) installed on a mini-satellite result in the fact that the angular momentum (AM) of a gyro moment cluster (GMC) based on the reaction wheels (RWs) or on the single-gimbal control moment gyroscopes — gyrodines (GDs), is significantly lower then the SC's AM obtained after its separation.

The engineering problem is to ensure such motion of a SC separated with *no plasma RTs used*, under which the energetic conditions are met, and then after the specified period T_a to complete a SC orientation towards the Sun by plasma RTs. The approach applied is based on two main assumptions:

- the plasma RTs are applied to perform two tasks: (i) satellite attitude control and unloading of an accumulated AM, and (ii) satellite orbit control;
- a small-mass GMC having a small AM is applied at initial mode without joining-up the RTs.

At a separation time moment t_0, a satellite body AM vector $\mathbf{K}_0 \equiv \mathbf{J}\boldsymbol{\omega}(t_0) = \mathbf{G}_0$ has an *arbitrary* direction, therefore the principle problem is to *coincide* this satellite vector with the maximum inertia satellite body axis Oy using *only* the GMC having small resources for the AM and control torque variation domains. Essentially nonlinear dynamical processes are arising from a moving the *total* AM vector $\mathbf{G}(t)$ of mechanical system with respect to the satellite body reference frame (BRF) Oxyz. Moreover, a Sun sensor is switched on, the Sun position is determined within the BRF and, if required, the SAPs are turned by an angle γ^p, $0 \leq \gamma^p \leq 270°$. In result, the SC angular rate is set along the axis Oy which is perpendicular to the SAPs rotation axis. Depending on the initial vector $\mathbf{G}$ angular position and direction $\mathbf{S}$ towards the Sun, the SAPs will be illuminated either continuously when the vectors $\mathbf{G}$ and $\mathbf{S}$ have coincided, or periodically if $\mathbf{G}\perp\mathbf{S}$, see Fig. 7. At this phase of the SC mission, the GMC is applied to generate control torques and plasma RTs are *not activated*. At next phase of the AOCS initial modes the RTs are turned on and generate the control torques to damp a SC angular rate.

In the paper, only principle aspects of strongly nonlinear dynamics related to the robust controlled coincidence of the SC body Oy axis with the SC's AM vector $\mathbf{G}$ are presented. Results early obtained, see Fig. 3 in Somov et al. (2004), are direct proofs for large efficiency of the GDs as compared with the RWs. The solution achieved is based on the methods for synthesis of nonlinear robust control (Somov, 2002; Somov and Rayevsky, 2002) and on rigorous analytical proof for the required SC rotation stability (Somov et al., 2003c, 2004). These results were verified by computer simulation (Somov et al., 1999a) of strongly nonlinear oscillatory processes at respinuping a flexible SC.

2. THE PROBLEM BACKGROUND

Most satellites contain a GMC to provide gyroscopic stability of a desired attitude of the SC body, problems of gyrostat optimal control (Krementulo, 1977; Chernousko et al., 1980; Somov and Fatkhullin, 1975; Junkins and Turner, 1986) and synthesis of control laws (Zubov, 1975, 1982, 1983) had been studied. V.I. Zubov's results were essentially developed by Ye.Ya. Smirnov (1981) and his successors (Smirnov et al., 1985; Smirnov and Yurkov, 1989). Here a Lyapunov function is applied with small parameter for its crossed term. This idea for mechanical systems rises to G.I. Chetayev (1955). Instead that A.V. Yurkov (1999) applied a large parameter for a position term at the Lyapunov function.

The SC spinup problems have been investigated by numerous authors, see Hubert (1981b); Huges (1986); Guelman (1989); Hall (1995a,b) et al. C.D. Hall (1995a) have been obtained a bifurcation diagram for all gyrostat spinup equilibria *manifolds*. Different approaches were applied to *convert* the intermediate axis spin equilibrium to those of major axis spin (to *respinup* the SC body) by variation of the RWs AM (Hubert, 1981a,b; Huges, 1986; Salvatore, 1991). If *enough* AM is added, the desired spin is globally stable in the presence of energy dissipation (Huges, 1986). However, no literature was found suggesting the SC respinup *feedback* control by the GMC having *small* resources, when the SC body AM vector have a *large* value and an *arbitrary* direction.

3. MATHEMATICAL MODELS

3.1 Spacecraft rigid model

Let we have a *free* rigid body (RB) with one fixed point O and any GMC. An inertia tensor $\mathbf{J}$ of the RB with a GMC is a arbitrary diagonal one, i.e. $\mathbf{J} = \lceil J_x, J_y, J_z \rfloor \equiv \mathrm{diag}\{J_i, i = x, y, z \equiv 1\div 3\}$ within the BRF Oxyz. Model of the RB motion is presented at well-known vector form

$$\dot{\mathbf{K}} + \boldsymbol{\omega} \times \mathbf{G} = \mathbf{M} \equiv -\dot{\boldsymbol{\mathcal{H}}}, \qquad (1)$$

where $\boldsymbol{\omega} = \{\omega_i\}$ is an absolute angular rate vector of the RB; $\mathbf{K} = \mathbf{J}\,\boldsymbol{\omega}$ is an AM vector of the RB equipped with a GMC; $\mathbf{G} = \mathbf{K} + \boldsymbol{\mathcal{H}}$ is a total AM for mechanical system in the whole; $\boldsymbol{\mathcal{H}}$ is a *column-vector* of a GMC *total* AM determined in the BRF.

3.2 Spacecraft flexible model

Simplest model of a *free* flexible body (FB) motion is presented also at the vector-matrix form with standard notations

$$\mathbf{A}^{\rm o}\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}\} = \{\mathbf{F}^{\omega}, \mathbf{F}^{q}\}, \qquad (2)$$

where $\mathbf{G} = \mathbf{G}^{\rm o} + \mathbf{D}_q\dot{\mathbf{q}}$; $\mathbf{G}^{\rm o} = \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta})$;

$$\mathbf{A}^{\rm o} = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q \\ \mathbf{D}_q^{\rm t} & \mathbf{A}^q \end{bmatrix}; \qquad \begin{aligned} \mathbf{A}^q &= \lceil a_j^q, j = 1\div n^q \rfloor \\ \mathbf{q} &= \{q_j, j = 1\div n^q\}; \end{aligned}$$

$$\mathbf{F}^{\omega} = \mathbf{M} - \boldsymbol{\omega}\times\mathbf{G};\ \mathbf{F}^{q} = -\{a_j^q((\delta^q/\pi)\,\Omega_j^q\,\dot{q}_j + (\Omega_j^q)^2\,q_j)\}.$$

3.3 The GMC based on the GDs

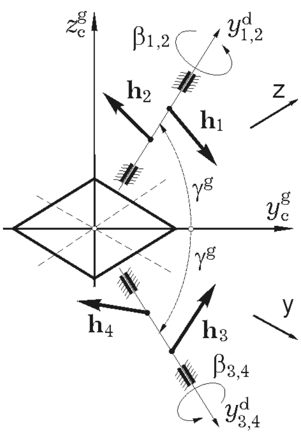

Fig. 8. The *2-SPE* scheme

It is suitable to present any GMC type using a canonical reference frame (CRF) $\mathbf{E}_{\rm c}^{\rm g}(x_{\rm c}^{\rm g}, y_{\rm c}^{\rm g}, z_{\rm c}^{\rm g})$. The necessary location of the required domain $\boldsymbol{\mathcal{S}}$ of the GMC AM $\boldsymbol{\mathcal{H}}$ variation within the BRF is achieved by the CRF orientation versus the BRF. Applied *2-SPE* (*2 Scissored Pair Ensemble*) scheme on 4 GDs with *own* AM h_g is presented in Fig. 8. Within *precession theory* of control moment gyros the AM vector $\boldsymbol{\mathcal{H}}$ by this scheme have the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = h_g\mathbf{A}_{\gamma}\mathbf{h}$ with constant non-singular matrix $\mathbf{A}_{\gamma}$, where a normed vector $\mathbf{h} = \sum \mathbf{h}_p(\beta_p)$ made up from units $\mathbf{h}_p(\beta_p)$, column $\boldsymbol{\beta} = \{\beta_p\}$ presents the GD's precession angles, column $\mathbf{h} \equiv \{\mathsf{x}, \mathsf{y}, \mathsf{z}\}$, where $\mathsf{x} = \mathsf{x}_{12} + \mathsf{x}_{34}$; $\mathsf{x}_{12} = \mathsf{x}_1 + \mathsf{x}_2$; $\mathsf{x}_{34} = \mathsf{x}_3 + \mathsf{x}_4$; $\mathsf{y} = \mathsf{y}_1 + \mathsf{y}_2$; $\mathsf{z} = -(\mathsf{z}_3 + \mathsf{z}_4)$; $\mathsf{x}_p = C_{\beta_p}$; $\mathsf{y}_p = S_{\beta_p}$; $\mathsf{z}_p = S_{\beta_p}$ with $S_\alpha \equiv \sin\alpha$ and $C_\alpha \equiv \cos\alpha$. At the command column $\mathbf{u} = \{\mathrm{u}_p\}$ the vector of the GMC output control torque have the form

$$\mathbf{M}^{\rm g} = -\dot{\boldsymbol{\mathcal{H}}} = -h_g\mathbf{A}_{\rm h}(\boldsymbol{\beta})\mathbf{u}; \quad \dot{\boldsymbol{\beta}} = \mathbf{u}, \qquad (3)$$

where matrix $\mathbf{A}_{\rm h}(\boldsymbol{\beta}) = \mathbf{A}_{\gamma}\mathbf{A}_h(\boldsymbol{\beta})$ and matrix

$$\mathbf{A}_h(\boldsymbol{\beta}) \equiv \partial\mathbf{h}(\boldsymbol{\beta})/\partial\boldsymbol{\beta} = \begin{bmatrix} -\mathsf{y}_1 & -\mathsf{y}_2 & -\mathsf{z}_3 & -\mathsf{z}_4 \\ \mathsf{x}_1 & \mathsf{x}_2 & 0 & 0 \\ 0 & 0 & -\mathsf{x}_3 & -\mathsf{x}_4 \end{bmatrix}.$$

The GDs' precession angles vary within the full range, but the domain $\boldsymbol{\mathcal{S}}$ of the GMC's AM vector $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})$ variations is limited. The "control" $u_p(t)$ of each GD is module-limited by *given* positive parameter $\mathrm{u}^{\rm m}$:

$$|u_p(t)| \le \mathrm{u}^{\rm m}, \ p = 1:4, \quad \forall t \in \mathrm{T}_{t_0}. \qquad (4)$$

These constrains are converted into β-*dependent* convex variation domain for a control torque $\mathbf{M} = \mathbf{M}^{\rm g}$.

4. THE PROBLEM STATEMENT

Considering the model (1), let denote an AM vector of a RB at initial time moment t_0 as $\mathbf{K}_0$. Let the vector of a GMC's total AM at the initial time be equal to zero, i.e. $\boldsymbol{\mathcal{H}}_0 \equiv \boldsymbol{\mathcal{H}}(t_0) = \mathbf{0}$. A norm of the vector $\mathbf{K}_0$ is assumed to be limited with the *given* constant, i.e. $\|\,\mathbf{K}_0\,\| \le k_{\rm o}^*,\ k_{\rm o}^* > 0$, but the direction of this vector within the BRF is *arbitrary*. Therefore, at the time $t = t_0$ the total AM vector related to the whole mechanical system $\mathbf{G}_0 = \mathbf{K}_0$ with $\|\,\mathbf{G}_0\,\| \equiv g_{\rm o} \le g_{\rm o}^* = k_{\rm o}^*$.

The inertial parameters of the RB are assumed to be known, the same for the possibility to measure the vector $\boldsymbol{\omega}(t)$ and the vector $\boldsymbol{\mathcal{H}}(t)$. Let establish of a fixed unit vector $\mathbf{f} = \mathbf{e}_y = \{0, 1, 0\}$ or $\mathbf{f} = -\mathbf{e}_y = \{0, -1, 0\}$ is given within the BRF — the unit of a RB having the *largest* moment of inertia or the one opposite.

The problem consists in designing required GMC control law which enable achieving such condition of a gyrostat (1) with the *specified accuracy* by any time moment $t = T_{\mathrm{f}}$:

$$\mathbf{K}_{\mathrm{f}} = \mathbf{J}\,\boldsymbol{\omega}_{\mathrm{f}};\ \boldsymbol{\omega}_{\mathrm{f}} = \omega_{\mathrm{f}}\mathbf{f};\ \boldsymbol{\mathcal{H}}_{\mathrm{f}} = \mathcal{H}_{\mathrm{f}}\mathbf{f}, \quad (5)$$

where $\mathbf{K}_{\mathrm{f}} \equiv \mathbf{K}(T_{\mathrm{f}})$; $\boldsymbol{\mathcal{H}}_{\mathrm{f}} \equiv \boldsymbol{\mathcal{H}}(T_{\mathrm{f}})$; $\boldsymbol{\omega}_{\mathrm{f}} \equiv \boldsymbol{\omega}(T_{\mathrm{f}})$ and module $\mathcal{H}_{\mathrm{f}}$ of the total GMC AM's is established, in particular, as $\mathcal{H}_{\mathrm{f}} = 0$. Taking into account the identity $J_y\,\omega_{\mathrm{f}} + \mathcal{H}_{\mathrm{f}} = g_{\mathrm{o}}$, where the value $\mathcal{H}_{\mathrm{f}}$ shall meet some constrains, one can find the obvious relation $\omega_{\mathrm{f}} = (g_{\mathrm{o}} - \mathcal{H}_{\mathrm{f}})/J_y$.

After solving this *vital* problem, it is necessary to ensure the *distribution* of the AM $\boldsymbol{\mathcal{H}}$ and control torque $\mathbf{M} = \mathbf{M}^{\mathrm{g}}$ vectors between four GDs. It is desirable to have the *explicit* distribution law (DL) allowing to obtain all movement characteristics for each electro-mechanical actuator based on the *analytical* relations. The GMC with *collinear* GD gimbal axes obtains a significant advantage: all its *singular* states are *passable* (Somov et al., 2003b). At 4 GDs the same approach is possible only for *2-SPE* scheme. The DL for such GMC was early presented in Somov (2002) and Somov et al. (2003b). It is also necessary to consider a respinuping the flexible spacecraft structure through using four GDs.

5. SYNTHESIS OF MAIN CONTROL LAW

An AM vector $\mathbf{G}(t) = \mathbf{J}\ \boldsymbol{\omega}(t) + \boldsymbol{\mathcal{H}}(t)$ of the *whole* mechanical system with no external torques has its value unchanged within any *inertial* reference frame (IRF), in accordance with the theoretical mechanics principles. The unit vector $\mathbf{g}(t) \equiv \{\mathrm{g}_i(t)\} = \mathbf{G}(t)/g_{\mathrm{o}}$ is also a *fixed* one within the IRF, but within the BRF this unit is *moving* in accordance with equation

$$\dot{\mathbf{g}}(t) = -\,\boldsymbol{\omega}(t) \times \mathbf{g}(t). \quad (6)$$

Let the following be calculated within the BRF when the system moves as per the *measured values* of the $\boldsymbol{\omega}(t)$ and $\boldsymbol{\mathcal{H}}(t)$ vectors:

- position of an AM unit vector $\mathbf{g}(t)$;
- position of a vector $\boldsymbol{\xi}(t) = \mathbf{g}(t) \times \mathbf{f}$;
- for $\parallel \boldsymbol{\xi}(t) \parallel = S_{\varphi}(t) \equiv \sin\varphi(t) \geq \varepsilon_1 = \mathrm{const}$ the *unit* vector value $\mathbf{e}_{\xi}(t) = \boldsymbol{\xi}(t)/\parallel \boldsymbol{\xi}(t) \parallel$;
- a cosine of angle between the units $\mathbf{g}$ and $\mathbf{f}$, namely
$$C_{\varphi}(t) \equiv \cos\varphi(t) = \langle \mathbf{f}, \mathbf{g}(t)\rangle.$$

A mismatch between the actual and required values of the SC rate vector is presented as

$$\boldsymbol{\eta}(t) = \delta\boldsymbol{\omega}(t) \equiv \boldsymbol{\omega}(t) - \omega_{\mathrm{f}}\,\mathbf{f}. \quad (7)$$

Let assume that at time t_0 there is also calculated an indicator $a_{\mathrm{f}} = \mathrm{Sgn}\ C_{\varphi}(t_0)$ of the unit vector direction $\mathbf{f}$ by the *definition*

$$\mathrm{Sgn}\ x = 1 \text{ for } x \geq 0 \quad \text{and} \quad \mathrm{Sgn}\ x = -1 \text{ for } x < 0,$$

and then we determine the unit vector $\mathbf{f} = a_f\mathbf{e}_y$. At the denotation $\boldsymbol{\zeta}(t) = \mathbf{g}(t) - \mathbf{f}$ as a nearby measure for the unit vectors $\mathbf{g}$ and $\mathbf{f}$, it is suitable to use a scalar function

$$v_p(t) \equiv v_p(\boldsymbol{\zeta}(t)) = |\boldsymbol{\zeta}(t)|^2/2 = 1 - \langle \mathbf{f}, \mathbf{g}(t)\rangle >> 0. \quad (8)$$

This function have positive values under $\mathbf{g}(t) \neq \mathbf{f}$ and obtains zero value at the above vectors coincided only. With the above selection of the unit vector $\mathbf{f} = a_f\{0, 1, 0\}$, we *always* have $v_p(t_0) \leq 1$. Taking into account standard vector *identities* $\langle \mathbf{a}, (\mathbf{b} \times \mathbf{c})\rangle \equiv \langle \mathbf{b}, (\mathbf{c} \times \mathbf{a})\rangle \equiv \langle \mathbf{c}, (\mathbf{a} \times \mathbf{b})\rangle$ and $\dot{\boldsymbol{\zeta}}(t) \equiv -\,\boldsymbol{\omega}(t) \times \mathbf{g}(t)$ by (6), we have derivative of the function v_p (8) as follows

$$\dot{v}_p = \langle \boldsymbol{\zeta}(t), \dot{\boldsymbol{\zeta}}(t)\rangle = \langle \boldsymbol{\xi}(t), \boldsymbol{\eta}(t)\rangle. \quad (9)$$

Vectors $\boldsymbol{\xi}(t)$ and $\boldsymbol{\zeta}(t)$ are connected by identities

$$\boldsymbol{\xi}^2 \equiv \boldsymbol{\zeta}^2(1 - \boldsymbol{\zeta}^2/4); \boldsymbol{\zeta}^2 \equiv 2\boldsymbol{\xi}^2/(1 + (1 - \boldsymbol{\xi}^2)^{1/2}), \quad (10)$$

moreover the vector $\boldsymbol{\xi}(t)$ is moving by equation

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\eta} - \boldsymbol{\phi};\ \boldsymbol{\phi} \equiv \omega_{\mathrm{f}}\boldsymbol{\zeta} + \mathbf{g}\langle \mathbf{f}, \boldsymbol{\eta}\rangle + (\boldsymbol{\eta} + \omega_{\mathrm{f}}\mathbf{f})\zeta^2/2. \quad (11)$$

Taking into account that due to (7) $\dot{\boldsymbol{\omega}}(t) = \dot{\boldsymbol{\eta}}(t)$ and the relations

$$\mathbf{G}(t) = g_{\mathrm{o}}\mathbf{g} = g_{\mathrm{o}}\mathbf{f} + g_{\mathrm{o}}(\mathbf{g}(t) - \mathbf{f}) = \mathbf{K}_{\mathrm{f}} + \boldsymbol{\mathcal{H}}_{\mathrm{f}} + g_{\mathrm{o}}\boldsymbol{\zeta}(t);$$

$$\boldsymbol{\nu} \equiv \mathbf{J}\boldsymbol{\eta} - g_{\mathrm{o}}\boldsymbol{\zeta} = -(\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_{\mathrm{f}}); \quad \dot{\boldsymbol{\nu}} = \mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{G},$$

the equation (1) is presented in simplest form

$$\dot{\boldsymbol{\nu}} \equiv \mathbf{J}\dot{\boldsymbol{\eta}} - g_{\mathrm{o}}\dot{\boldsymbol{\zeta}} = \mathbf{M} = -\dot{\boldsymbol{\mathcal{H}}}. \quad (12)$$

The function $v_e(\boldsymbol{\nu}) \equiv \boldsymbol{\nu}^2/(2j_h) = (\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_{\mathrm{f}})^2/(2j_h)$ defines a GMC kinetic energy at its motion with respect to required equilibrium in the BRF, where any constant $j_h > 0$ presents its inertia properties.

The RB movement required $\mathbf{O}_{\eta} \equiv \{\boldsymbol{\xi} = \mathbf{0}; \boldsymbol{\eta} = \mathbf{0}\}$ is the same $\mathbf{O}_{\nu} \equiv \{\boldsymbol{\xi} = \mathbf{0}; \boldsymbol{\nu} = \mathbf{0}\}$ due to the identities (10). For denotation $\rho^2(t) \equiv \parallel \boldsymbol{\xi}(t) \parallel^2 + \parallel \boldsymbol{\eta}(t) \parallel^2$ in the first let consider any small domain

$$\boldsymbol{\mathcal{O}} \equiv \{\parallel \boldsymbol{\xi} \parallel < \varepsilon_1\} \cap \{\parallel \rho \parallel < \varepsilon_{\rho} = \mathrm{const}\},$$

within which *no constrains* for the control torque $\mathbf{M}$ vector have occurred. To justify the structure of the control torque $\mathbf{M}$ law into the equation (12), we introduce the *Lyapunov* function

$$\mathrm{V}(\boldsymbol{\zeta}, \boldsymbol{\xi}, \boldsymbol{\eta}) = a\,b\,v_p(\boldsymbol{\zeta}) + (a/j_h)\langle \boldsymbol{\nu}, \mathbf{P}\boldsymbol{\xi}\rangle + v_e(\boldsymbol{\nu}), \quad (13)$$

where scalar parameters $a > 0$, $b > 0$ and $\mathbf{P}$ is a constant definitely-positive matrix. For *large* value of parameter b the function V (13) is definitely positive with respect to the vector variables $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$. The derivative of this function with (9) and (12) taken into account have the form

$$\dot{\mathrm{V}} = ab\langle \boldsymbol{\xi}, \boldsymbol{\eta}\rangle + [\langle \mathbf{M}, \boldsymbol{\mu}\rangle + \langle \boldsymbol{\nu}, \mathbf{P}\dot{\boldsymbol{\xi}}\rangle]/j_h, \quad (14)$$

where vector $\boldsymbol{\mu} \equiv \boldsymbol{\nu} + a\mathbf{P}\boldsymbol{\xi}$. For domain $\boldsymbol{\mathcal{O}}$ the GMC control law is selected in the form

$$\mathbf{M} = \mathbf{M}_{\xi} \equiv -qj_h\mathbf{D}\boldsymbol{\mu} = -m\,[\,\boldsymbol{\xi} + k\mathbf{D}\boldsymbol{\nu}] \quad (15)$$

with parameters $q > 0$, $m = qj_ha > 0$, $k = 1/a > 0$ and definitely-positive matrix $\mathbf{D} = \mathbf{P}^{-1}$.

Theorem *For the RB movement required* $\mathbf{O}_{\eta}$ *of the system's model (11), (12) with the control law (15) the property of exponential stability*

$$\rho(t) \leq \beta\,\rho(t_0)\exp(-\alpha(t - t_0)),\ \ \alpha, \beta = \mathrm{const} > 0 \quad (16)$$

is guaranteed for arbitrary vector of initial conditions $\{\boldsymbol{\xi}(t_0), \boldsymbol{\eta}(t_0)\} \in \boldsymbol{\mathcal{O}}_0 \subseteq \boldsymbol{\mathcal{O}}$ *at chosen large value* $q(g_{\mathrm{o}})$.

Proof The derivative (14) of function (13) by the relation (11) taken into account is presented as

$$\begin{aligned}\dot{\mathrm{V}} = &-qa^2\langle \boldsymbol{\xi}, \mathbf{P}\boldsymbol{\xi}\rangle + a(b\,\langle \boldsymbol{\xi}, \boldsymbol{\eta}\rangle - 2q\langle \boldsymbol{\xi}, \mathbf{J}\boldsymbol{\eta}\rangle)\\ &-q\langle \boldsymbol{\nu}, \mathbf{D}\boldsymbol{\nu}\rangle + (a/j_h)\langle \boldsymbol{\nu}, \mathbf{P}(\boldsymbol{\eta} - \boldsymbol{\phi}(\boldsymbol{\eta}, \boldsymbol{\zeta}))\rangle,\end{aligned} \quad (17)$$

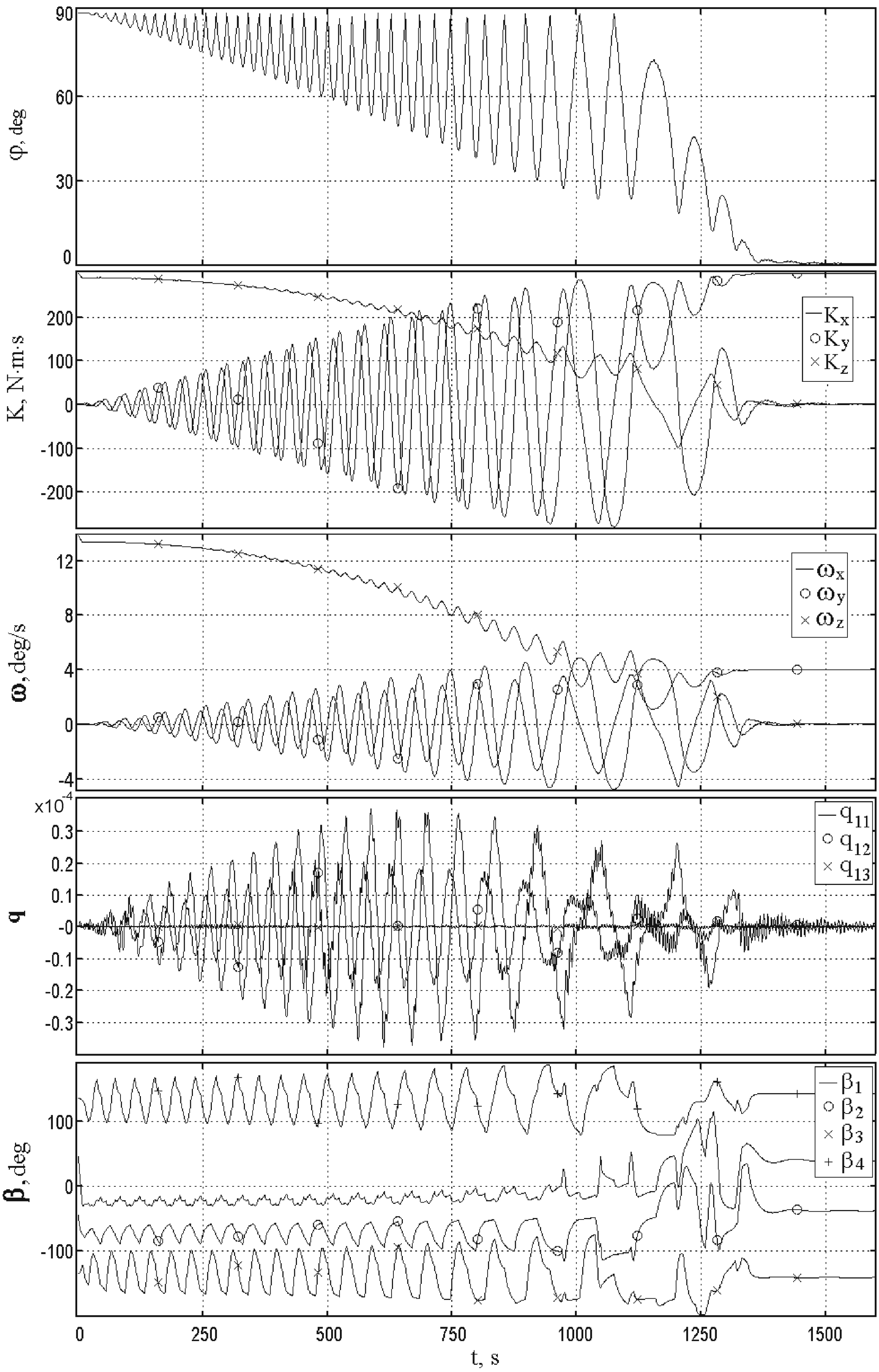

Fig. 9. Dynamics of the flexible SC respinup by 4 GDs with $h_g = 7.5$ Nms and constrains $u^m = 10$ deg/s.

where vector $\boldsymbol{\nu} = \mathbf{J}\boldsymbol{\eta} - g_o\boldsymbol{\zeta}$ and the function $\boldsymbol{\phi}(\cdot)$ was defined in (11). Taking into account

$$\langle \boldsymbol{\nu}, \mathbf{D}\boldsymbol{\nu}\rangle = \langle \mathbf{D}\mathbf{J}\boldsymbol{\eta}, \mathbf{J}\boldsymbol{\eta}\rangle - 2g_o\langle \mathbf{D}\mathbf{J}\boldsymbol{\eta}, \boldsymbol{\zeta}\rangle + g_o^2\langle \mathbf{D}\boldsymbol{\zeta}, \boldsymbol{\zeta}\rangle$$

and analogous representations of the terms $\langle \boldsymbol{\nu}, \mathbf{P}\boldsymbol{\eta}\rangle$, $\langle \boldsymbol{\nu}, \mathbf{P}\boldsymbol{\zeta}\rangle$, $\langle \boldsymbol{\nu}, \mathbf{P}\boldsymbol{\phi}\rangle$ in (17), and also identities (10), one makes sure of the majoring $\dot{\mathrm{V}} \le -\mathrm{W}(\boldsymbol{\xi}, \boldsymbol{\eta})$, where scalar function $\mathrm{W}(\cdot)$ is definitely positive with respect to variables $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ for *large* values of parameters b and q, *depending* on total AM value g_o.

By analogy with Smirnov (1981) there is proved that $\mathrm{W}(t) \to 0$ at $t \to \infty$ and function $\mathrm{V}(t)$ is decreased *monotonically.* Standard estimates (Smirnov and Yurkov, 1989; Yurkov, 1999) are derived from majoring functions V and W by quadratic forms

$$a_1\rho^2 \le \mathrm{V} \le a_2\rho^2, a_1 > 0; \quad b_1\rho^2 \le \mathrm{W} \le b_2\rho^2, b_1 > 0,$$

from where the condition (16) is appeared with the parameters $\alpha = b_1/(2a_2)$ and $\beta = (a_2/a_1)^{1/2}$. □

Due to the identity $\boldsymbol{\nu} \equiv \mathbf{J}\boldsymbol{\eta} - g_o\boldsymbol{\zeta} = -(\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_f)$ the control law (15) is appeared in very *simple* form

$$\mathbf{M}_\xi = -m[\,\boldsymbol{\xi}(t) - k\mathbf{D}(\boldsymbol{\mathcal{H}}(t) - \boldsymbol{\mathcal{H}}_f)\,]$$

interior to nearest neighborhood of required gyrostat state $\mathbf{O}_\eta$. Outside this neighborhood the control law is not effective because of various *equilibria manifolds* (Hall, 1995a) which exist at conditions $\mathbf{M}_\xi = \mathbf{J}\dot{\boldsymbol{\eta}} - g_o\dot{\boldsymbol{\zeta}} \equiv \mathbf{0}$ but $\mathbf{J}\boldsymbol{\eta} - g_o\boldsymbol{\zeta} = \mathbf{c}$ and $a\mathbf{P}\boldsymbol{\xi} = -\mathbf{c}$ with a *constant* vector $\mathbf{c} \neq \mathbf{0}$. Therefore other *simple* control laws are needed for fastest the SC respinuping without *sticking* its motion on any equilibria manifold differing from the state $\mathbf{O}_\eta$. For denotations

$$\mathbf{M}_\xi^{\mathrm{r}}(t) \equiv -m\,[\,\mathbf{e}_\xi(t)\mathrm{Sgn}C_\varphi(t) - k\mathbf{D}(\boldsymbol{\mathcal{H}}(t) - \boldsymbol{\mathcal{H}}_f)\,],$$

$$\mathbf{M}^{\mathrm{r}}(t) \equiv -\mathrm{M}^*\ \{\mathrm{Sgn}\,\mathrm{g}_i(t), \quad i = x, y, z\},$$

where M^* is a *large* constant parameter, developed control law has the form

$$\mathbf{M} = \begin{cases} \mathbf{M}_\xi(t) & \|\,\boldsymbol{\xi}(t)\,\| < \varepsilon_1; \\ \mathbf{M}_\xi^{\mathrm{r}}(t) & \varepsilon_1 \le \|\,\boldsymbol{\xi}(t)\,\| \le \varepsilon_2; \\ \mathbf{M}^{\mathrm{r}}(t) & \|\,\boldsymbol{\xi}(t)\,\| > \varepsilon_2, \end{cases} \qquad (18)$$

where for example, the parameters $\varepsilon_1 = 0.1$ (angle $\varphi = 6°$) and $\varepsilon_2 = 0.5$ (angle $\varphi = 30°$).

6. COMPUTER SIMULATION

Based on the above control laws, the SC motion have been simulated with the following parameter values: $J_x = 2900$, $J_y = 3600$ and $J_z = 870$ kgm^2 (Somov et al., 2003a).

Fig. 3 in Somov et al. (2004) summarizes the simulation results for initial position of the SC AM vector $\mathbf{G}(t_0)$ with module $g_o = 300$ Nms along the unit $\mathbf{g}(t_0) = \{0, 0, 1\}$ within the BRF and its final position coincided with the unit $\mathbf{f} = \{0, 1, 0\}$. For clearness in this paper the simplest canonical GMC schemes were applied:

- canonical scheme on 3 RWs with constrains $\mathrm{M}^{\mathrm{m}} = 0.15$ Nm and $\mathrm{H}^{\mathrm{m}} = 5$ Nms;
- the *2-SPE* scheme on 4 GDs, see Fig. 8, with own AM $\mathrm{h}_g = 7.5$ Nms, angle $\gamma^{\mathrm{g}} = \pi/4$ and constrain $\mathrm{u}^{\mathrm{m}} = 10$ deg/s.

Some results on the flexible spacecraft dynamics during its respinuping by four GDs with the same parameters, are presented in Fig. 9.

Optimization (Somov, 2000) and robust gyromoment control problems (Somov et al., 1999b; Somov, 2001; Matrosov and Somov, 2003) were also considered for respinuping the flexible spacecraft.

In addition to Somov et al. (2005a) and Somov et al. (2005b) problems of the SAPs guidance on the Sun were studied for mode of the SC body settled rotation. Moreover the SC inertia tensor is changed into the BRF and the GMC's control torque vector $\mathbf{M} = \mathbf{M}^{\mathrm{g}}$ is recalculated for the principle central axes for this inertia tensor.

7. CONCLUSIONS

Principle aspects of nonlinear dynamics related to the controlled coincidence of any SC body axis with the SC AM vector by the GDs were presented.

Methods for synthesis of nonlinear control and analytical proof for the required SC rotation stability were developed. Optimization and robust gyromoment control problems were considered for respinuping a flexible spacecraft.

Obtained results were verified by the careful computer simulation of strongly nonlinear processes.

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