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HYBRID CONTROL OF UNDERACTUATED SYSTEMS WITH DISCONTINUOUS FRICTION

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Abstract
We propose a hybrid controller for a class of 2-DOF underactuated mechanical systems with discontinuous friction in the unactuated joint. The control objective is the regulation of both unactuated and actuated variables. The design of the control law is divided in two parts. First we design a discontinuous control for the unactuated joint, then we propose another discontinuous control for the actuated joint. The proposed controller guarantees the convergence of the position error to zero, and it is robust with respect to some uncertainty in the discontinuous friction coefficients. We illustrate the technique with its application to a physical system.

Key words
Discontinuous friction, mechanical systems, stability, uncertainty, underactuation.

1 Introduction

Control synthesis for underactuated systems is more complex than it is for systems with full control [Seto and Baillieul, 1994]. A few representative papers analyzing some problems about underactuated systems include the study of accessibility [Reyhanoglu, van der Shaft, McClamroch and Kolmanovsky, 1999], stabilization of equilibria through passivity techniques [Ortega, Spong, Gomez-Estern, 2002] and energy shaping [Bloch, Leonard and Marsden, 2000], stabilization and tracking via backstepping control [Seto and Baillieul, 1994], the use of virtual constraints to produce stable oscillations [Shiriaev, Perram and Canudas-de-Wit, 2005], path planning [Bullo and Lynch, 2001], and control of mechanical systems with an unactuated cyclic variable [Grizzle, Moog and Chevallereau, 2005], among others.

Most papers in the field of underactuated mechanical control have commonly neglected a fundamental issue in modeling and control, as it is the friction effect. Friction terms have been repeatedly left unmatched, and the classical approach reduces to solve the control problem for an undamped, open-loop model. Thus, in recent years several papers have addressed the problem of friction in the underactuated mechanical systems.

In [Woolsey, Bloch, Leonard and Marsden, 2001; Woolsey, Bloch, Leonard, Reddy, Chang and Marsden, 2004], the effect of linear damping (viscous friction) on the stability of equilibria which have been stabilized previously in the absence of damping, with the method of controlled Lagrangians, is described. The Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) technique for underactuated mechanical systems was extended to incorporate open-loop damping in [Gomez-Estern and van der Schaft, 2004]. Here, in addition, the conditions for recovering stability via damping injection in the presence of highly uncertain, smooth approximations of Coulomb friction effects, are presented.

For underactuated mechanical systems with discontinuous friction only in the actuated joints, the problem of compensation, in some cases, can be solved [Riachy, Floquet, Orlov and Richard, 2006]. However, the design of control algorithms for underactuated mechanical systems with discontinuous friction in the unactuated joints, without neglecting or approximating the discontinuous terms, to the best of our knowledge, is still open.

In this paper we propose a hybrid controller for a class of 2-DOF underactuated mechanical systems with discontinuous friction in the unactuated joint. The pendulum with inertial disk and the inverted pendulum are included in this class of systems. The control objective is the regulation of both unactuated and actuated joints. The control design is divided in two parts; first, the former joint is steered to the desired position in finite time, then the actuated joint is also driven to the desired position. We make use of a central result presented in [Orlov, 2005], about the conditions to have finite time stability of a class of uncertain switched systems. The proposed controller guarantees the convergence of the position error to zero, and it is robust with respect to some uncertainty in the discontinuous friction coefficients. We illustrate this procedure with the application to a physical system.

2 Problem Statement

Consider a 2-DOF underactuated mechanical system. Its state-space representation is given by

$$I_1\ddot{q}_1 + I_2\ddot{q}_2 + \frac{\partial V_1(q_1)}{\partial q_1} + C(\dot{q}_1)\,\alpha(\dot{q}_1) = 0$$
$$I_3\ddot{q}_1 + I_4\ddot{q}_2 = u. \quad (1)$$

Hereinafter, $q_1 \in \mathbb{R}$, $q_2 \in \mathbb{R}$, I_1, I_2, I_3 and I_4 are positive constants and $I_1 > \frac{I_2 I_3}{I_4}$; $\frac{\partial V_1(q_1)}{\partial q_1}$ is a gravitational torque and it is a smooth function satisfying $\left.\frac{\partial V_1(q_1)}{\partial q_1}\right|_{q_1=0} = 0$ and $V_1(0) = 0$,

$$\alpha(z) = \begin{cases} 1 & z > 0 \\ [-1,1] & z = 0 \\ -1 & z < 0 \end{cases} \quad (2)$$

and u is a control input. Throughout, we assume a relatively strong friction level in the unactuated joint link whereas friction forces, enforced the actuated link, are assumed to be negligible. Following frequently used static friction models (see, e.g., [Bartolini and Punta, 2000; Olsson, Astrom, Canudas de Wit, Gafvert, and Lischinsky, 1998]), the modulus of the friction term $C(\dot{q}_1)$ is governed by

$$C(\dot{q}_1) = C_c + (C_s - C_c)\exp\left(-\frac{\dot{q}_1^2}{v_s^2}\right), \quad (3)$$

where C_c and C_s are the Coulomb friction level and the level of stiction, with $0 < C_c \le C_s$, and v_s is the Stribeck velocity. Note that $C_c \le C(\dot{q}_1) \le C_s$.

The control objective is to steer to zero q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$. Since the system in question is underactuated the hybrid control synthesis is invoked. Due to the presence of the friction, our hybrid controller is composed of discontinuous control laws which are capable of rejecting friction influence on the system.

If we define $x_1 = q_1$, $x_2 = \dot{x}_1$, $x_3 = q_1 + \frac{I_4}{I_3}q_2$, $x_4 = \dot{x}_3$ and

$$\psi = \left[(C_s - C_c)\exp\left(-\frac{\dot{q}_1^2}{v_s^2}\right) - (C_s - C_c)\right]\alpha(\dot{q}_1), \quad (4)$$

then the system (1) is described by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{-1}{I_1 - I_2 I_3/I_4}\left(\frac{\partial V_1(x_1)}{\partial x_1} + C_s\alpha(x_2)\right. \\ &\quad \left. + \psi + \frac{I_2}{I_4}u\right), \quad (5) \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{1}{I_3}u. \quad (6) \end{aligned}$$

Note that $|\psi| \le C_s - C_c < C_s$. The control aim is that of steering to zero the state $x = (x_1, x_2, x_3, x_4)^T$ with a hybrid control $u = u(x_1, x_2, x_3, x_4)$.

We proceed as follows. Let us consider a hybrid control law u of the form

$$u = \begin{cases} u_1(x_1, x_2) \text{ if } x_1^2 + x_2^2 \ne 0 \\ u_2(x_3, x_4) \text{ if } x_1 = x_2 = 0. \end{cases} \quad (7)$$

First we will design a discontinuous controller u_1 for subsystem (5) ensuring the convergence of x_1 and x_2 to zero in finite time. Then we propose a discontinuous controller u_2 for subsystem (6) steering x_3 and x_4 to zero.

To this end we note that the meaning of the resulting differential equation (4)–(7) with the discontinuous right-hand side is viewed in the Filippov sense [Filippov, 1988].

3 Main Result

We recall first a result presented in [Orlov, 2005]. Consider the uncertain second order switched system

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= -a\,\mathrm{sign}(y_1) - b\,\mathrm{sign}(y_2) - hy_1 - py_2 \\ &\quad + \omega(y_1, y_2, t), \quad (8) \end{aligned}$$

where a and b are constants, h and p are parameters of the linear gain, $\omega(y_1, y_2, t)$ is a piecewise continuous nonlinear perturbation, uniformly bounded

$$|\omega(y_1, y_2, t)| \le M \quad (9)$$

for all continuity points (y_1, y_2, t) and some $M > 0$, and

$$\mathrm{sign}(z) = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \end{cases}. \quad (10)$$

Here, solutions of this system are defined in the Filippov sense [Filippov, 1988].

Theorem 1. [Orlov, 2005] Let conditions

$$\begin{aligned} &0 < M < b < a - M, \\ &h \geq 0, \qquad p \geq 0 \end{aligned} \quad (11)$$

be satisfied. Then the uncertain switched system (8) with (9) is globally equiuniformly finite time stable around the origin.

We use this result to solve the problem described in the previous section. First we propose the discontinuous controller

$$u_1(x_1, x_2) = \frac{I_4}{I_2} k_1 \mathrm{sign}(x_1), \quad (12)$$

for subsystem (5)where k_1 is a constant. A lower bound on values k_1 forcing x_1 and x_2 converge to zero is established in the next Theorem.

Theorem 2. The subsystem (5) with $u = u_1$, where u_1 is given by (12), is globally equiuniformly finite time stable around $x_1 = x_2 = 0$ if

$$k_1 > 2C_s - C_c + \left| \frac{\partial V_1(x_1)}{\partial x_1} \right| \quad \text{for all} \quad x_1. \quad (13)$$

Proof. To begin with, note that $x_1 = x_2 = 0$ is the only equilibrium point of the subsystem (5) subject to (12),(13). Indeed, if $x_2 = 0$ on a trajectory of this subsystem, then due to (13), the second equation of (5) with (12) fails to hold for any $x_1 \neq 0$. This particularly means that the subsystem trajectories cross the axes $x_1 = 0$ and $x_2 = 0$ everywhere but the origin.

Thus, it is sufficient to analyze the behavior of the trajectories beyond the discontinuity surfaces $\mathcal{S}_1 = \left\{ (x_1, x_2)^T \in \mathbb{R}^2 : x_1 = 0 \right\}$ and $\mathcal{S}_2 = \left\{ (x_1, x_2)^T \in \mathbb{R}^2 : x_2 = 0 \right\}$. With this in mind, let us consider the function

$$V_2\left(x_1, x_2\right) = V_1(x_1) + k_1 \left|x_1\right| + \frac{1}{2}\left(I_1 - \frac{I_2 I_3}{I_4}\right) x_2^2, \quad (14)$$

where k_1 is given by (13), which is Lipschitz continuous, radially unbounded, and positive definite. The time-derivative of V_2 along the trajectories of (5) with (12) and (13) outside the discontinuity surfaces $\mathcal{S}_1$ and $\mathcal{S}_2$ satisfies

$$\dot{V}_2(x_1, x_2) \leq -C_c \left|x_2\right| < 0. \quad (15)$$

By applying the extended version of the invariance principle, made in [Alvarez, Orlov and Acho, 2000], to the discontinuous right-hand side subsystem (5) subject to (12),(13), we conclude that all the subsystem trajectories converge to $(x_1, x_2) = (0, 0)$.

Thus, initialized in an arbitrarily small vicinity

$$D_\epsilon = \left\{ (x_1, x_2)^T \in \mathbb{R}^2 : V_2(x_1, x_2) \leq \epsilon \right\}, \quad (16)$$

where $\epsilon > 0$, of $x_1 = x_2 = 0$, the subsystem (5) with (12) and (13) cannot leave this vicinity. Moreover, due to (13), $k_1 > \left|\frac{\partial V_1(x_1)}{\partial x_1}\right| + C_s + |\psi|$ and in finite time t_1, $C_s \geq C_c > \left|\frac{\partial V_1(x_1)}{\partial x_1}\right| + |\psi|$. Therefore, if $y_1 = x_1$, $y_2 = x_2$, $a = \frac{k_1}{I_1 - I_2 I_3/I_4}$, $b = \frac{C_s}{I_1 - I_2 I_3/I_4}$, $h = p = 0$, and $\omega = \frac{-1}{I_1 - I_2 I_3/I_4}\left(\frac{\partial V_1(x_1)}{\partial x_1} + \psi\right)$ for $t \geq t_1$, according to Theorem 1, the subsystem (5) under conditions (12) and (13) is globally equiuniformly finite time stable and $x_1(t) = x_2(t) = 0$ at the settling time $t = T(x_1^0, x_2^0)$, dependent on the initial conditions $x_1(0) = x_1^0, \ x_2(0) = x_2^0$.

Now we propose the discontinuous control law

$$u_2(x_3, x_4) = -k_2 \mathrm{sign}(x_3) - k_3 \mathrm{sign}(x_4), \quad (17)$$

for subsystem (6) where $k_2 > 0$ and $k_3 > 0$ are constants.

In order to avoid escaping subsystem (5) from the origin after the settling time instant $T(x_1^0, x_2^0)$, the action of controller (7) on the subsystem should not exceed the Coulomb friction level, i.e., the values k_2 and k_3 are to be such that

$$k_2 + k_3 \leq C_c \frac{I_4}{I_2}. \quad (18)$$

To reproduce this conclusion it suffices to compute the time derivative of the function $V_2(x_1, x_2)$, governed by (14), and to make sure that

$$\dot{V}_2(x_1, x_2) \leq -[C_c - \frac{I_2}{I_4}(k_2 + k_3)] \left|x_2\right| \leq 0 \quad (19)$$

on the solutions of subsystem (5), driven by controller (17).

Moreover, by applying Theorem 1 to the closed-loop subsystem (6), (17), the latter proves to be finite time stable provided that

$$0 < k_3 < k_2. \qquad (20)$$

Summarizing, we arrive at the following.

Theorem 3. Let system (5), (6) be driven by the hybrid controller (7) composed of (12), (17) and let the controller parameters meet conditions (13), (18) and (20). Then the closed-loop system is finite time stable.

It is worth noticing that due to construction (cf. Theorem 1) the proposed controller is robust with respect to friction discrepancies. In the remainder, our development is supported by an application to a physical system.

4 Application

Let us consider the system shown in figure 1, whose motion is constrained to be in a horizontal plane. The first joint pivots around a point O, and a driver deplaces the second joint. This system is modelled by the equations

$$\begin{aligned} J_o\ddot{q}_1 + m_2 l_o \ddot{q}_2 + 2m_2 q_2 \dot{q}_2 \dot{q}_1 + C(\dot{q}_1)\,\alpha(\dot{q}_1) &= 0, \\ m_2 l_o \ddot{q}_1 + m_2 \ddot{q}_2 - m_2 q_2 \dot{q}_1^2 &= u, \end{aligned} \qquad (21)$$

where q_1 is the pendulum angle, q_2 is the position of mass 2, m_1 and m_2 are the masses, $J_o = J_1 + J_2 + m_2\left(l_o^2 + q_2^2\right) + m_1 l_c^2$ is the system moment of inertia about O, J_1 and J_2 are moments of inertia, $\alpha(\cdot)$ and $C(\cdot)$ are given by (2) and (3), l_c and l_o are positives constants, and u is the control input.

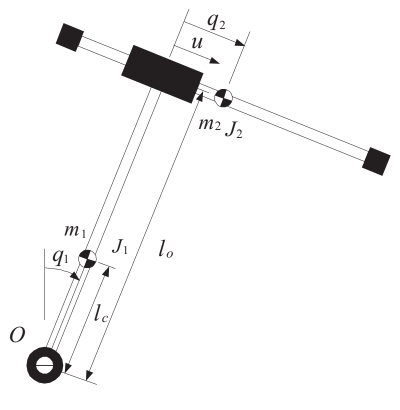

Figure 1. Horizontal Pendulum

The objective is to design a hybrid control law u which steers to zero q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$.

In order to apply the previous result we must simplify this model, assuming q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$ are small. Under this condition, system (21) can be simplified to

$$\begin{aligned} J_{oe}\ddot{q}_1 + m_2 l_o \ddot{q}_2 + C(\dot{q}_1)\,\alpha(\dot{q}_1) &= 0, \\ m_2 l_o \ddot{q}_1 + m_2 \ddot{q}_2 &= u, \end{aligned} \qquad (22)$$

with $J_{oe} = J_1 + J_2 + m_2 l_o^2 + m_1 l_c^2$.

For this system we have $I_1 = J_{oe}$, $I_2 = I_3 = m_2 l_o$, $I_4 = m_2$, $I_1 > \frac{I_2 I_3}{I_4}$, $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_1 + \frac{1}{l_o} q_2$, $x_4 = \dot{q}_1 + \frac{1}{l_o}\dot{q}_2$, and, according to (7) with (12), (17), we propose

$$\begin{aligned} u_1 &= \frac{1}{l_o} k_1 \mathrm{sign}(q_1), \\ u_2 &= -k_2 \mathrm{sign}\left(q_1 + \frac{1}{l_o} q_2\right) - k_3 \mathrm{sign}\left(\dot{q}_1 + \frac{1}{l_o}\dot{q}_2\right), \end{aligned} \qquad (23)$$

where, according to (13), (18) and (20), $k_1 > 2C_s - C_c$, $C_c > l_o(k_2 + k_3)$, and $k_2 > k_3 > 0$.

Figures 2 show an experimental result conducted on the system manufactured by ECP, model 505. We have set $l_o = 0.330\ m$, $m_1 = 0.785\ kg$, $m_2 = 0.213\ kg$, $J_{oe} = 0.0477957\ kg \cdot m^2$, $C_s \approx C_c \approx 0.0004\ N \cdot m$, $k_1 = 0.0008$, $k_2 = 0.0005$, and $k_3 = 0.0004$, with $q_1(t_0) = -0.045\ rad$, $\dot{q}_1(t_0) = 0\ rad/s$, $q_2(t_0) = 0\ m$, and $\dot{q}_2(t_0) = 0\ m/s$. The control law is applied in $t = 2.5$ sec.

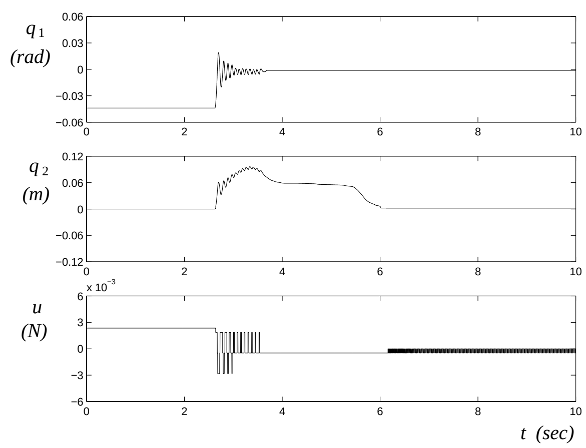

Figure 2. Positions and control of a Horizontal Pendulum.

The proposed control was designed assuming q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$ are small. However, we can take advantage of the finite time convergence if we wish steer to an arbitrary position the unactuated joint. Experimental result is shown in Figures 3, where now we have set $q_1(t_0) = 0\ rad$, $\dot{q}_1(t_0) = 0\ rad/s$, $q_2(t_0) = 0\ m$, $\dot{q}_2(t_0) = 0\ m/s$, and the output of a 2nd order filter to a step with a magnitude of $0.31416\ rad$ as reference of q_1. The step is applied in $t = 1.5$ sec.

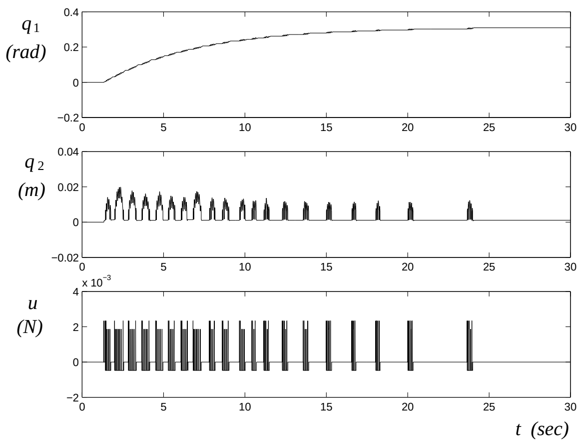

Figure 3. Positions and control of a Horizontal Pendulum.

5 Conclusions

In this paper we have proposed a hybrid controller for a class of 2-DOF underactuated mechanical systems with discontinuous friction in the unactuated joint. The control objective is the regulation of both the unactuated and actuated joints. The proposed controller guarantees the convergence of the position error to zero in finite time, and it is robust with respect to some uncertainty in the discontinuous friction coefficients. We illustrated the technique with its application to a physical underactuated system: a horizontal pendulum. Here, in order to apply the main result we must simplify this model, assuming q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$ are small; however, since the closed loop system converges to the desired position in a finite time, it is possible to drive it to an arbitrary position in a long enough time.

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