The Exploitation Effect on Both Genetic Variety and Dynamic Behavior of Mendelian Limited Population: Non-Stationary and Stationary Strategy of Harvest ♣

© Oksana L. Zdanova
Institute for Automation and Control Processes
axanka@iacp.dvo.ru

© Efim Ya. Frisman
Complex Analysis of Regional Problems Institute
frisman@mail.ru

Abstract

The consequences of optimal harvest in the model of density-depended natural selection are considered. Also comparing of stationary and non-stationary strategy of harvest is conducted. It is shown with some examples, that non-stationary strategy of harvest sometimes may be more preferable then stationary one.

1 Introduction

The two types of issues appeared when attempts of ecological and population-genetics theory combination were made. The first type of issues is investigation of evolution factors effect (most of all the natural selection) on genetic structure changes and dynamic behavior of population under the limited ecological resources. The second one is analyses necessity of evolutionary-ecological consequences of harvest.

The conception of maximal equilibrated catch asserts that harvested populations are not in the same ecological conditions as non-harvested ones. So the conditions of selection and hence fitnesses of genotypic groups can changes in harvested populations.

The investigation of consequences of optimal stationary harvest with constant quota even in very simple model situation (equations set 1, 2) shows that such exploitation can change the stability of model equilibriums and consequently can result in not only dynamics regime character changes but even in essential changes of population's genetic structure. At that when reproductive potentials of genotypic groups are not large the optimal harvest only decreases the population number, and dynamic regime of population and its genetic structure remains the same. But if there are genotypes with large reproductive potentials in population, then optimal harvest can result in highly unexpected effects. It has been shown in [1], [2], that optimal stationary harvest can preserve genetic variety as well as eliminate existing one. Usually such exploitation of population results in stabilization of population dynamics. Nevertheless in some cases exploitation with constant quota doesn't able to stabilize the population dynamics. In that case one can see stable fluctuation of population size, catch volume and even genetic structure of population. So the population equilibrium is not attainable. Then the question appears: is exploitation with constant quota the optimal strategy of harvest? It seems reasonable to catch different portions from population depending on whether its size is in depression or on the rise.

In this research we compare stationary harvest strategy (exploitation with constant quota) with non-stationary one for those cases generally, when exploitation with constant quota doesn't result in stabilization of population dynamics.

2 The models

Let's consider the evolution model of Mendelian one-locus di-allelic limited population of diploid organisms:

$$\begin{cases} x_{n+1} = \bar{W}_n(x_n, q_n)x_n, \\ q_{n+1} = q_n\left(W_{AA}(n)q_n + W_{Aa}(n)(1-q_n)\right)/\bar{W}_n(x_n, q_n), \end{cases}$$
(1)

here n is a number of generation, $\bar{W}_n(x_n,q_n)=W_{AA}(n)q_n^{\,2}+2W_{Aa}(n)q_n(1-q_n)+W_{aa}(n)(1-q_n)^2$ - means average fitness of population in the n-th generation, x_n - population size in n-th generation, q_n - frequency of allele A in n-th generation, $(1-q_n)$ - frequency of allele a in n-th generation, $W_{AA}(n)$, $W_{Aa}(n)$, $W_{aa}(n)$ - fitness of genotype AA, Aa and aa accordingly in n-th generation. The fitness of each

♣ This work was supported by the Far Eastern Branch of Russian Academy of Sciences and the Russian Foundation for Basic Research (grant 06-04-96025) and by the Programs of Presidium of RAS "Origin and evolution of Biosphere" (grant FEB RAS 06-I-P18-082).

genotype is exponential function of population size: $W_{ij} = exp(R_{ij} (1 - \frac{x_n}{K_{ij}}))$. So, each genotype is specified by its resort (K_{ij}) and Malthusian (R_{ij}) parameters.

The Malthusian parameter (R_{ij}) means fertility rate in population without any resort limitation, in other words it describes rate of population increasing to emptiness. The resort parameter (K_{ij}) means that equilibrium value which would be reached by population size, if only ij-genotype individuals are presented in population.

The dynamic consequences of natural selection in Mendelian non-exploited population were investigated in detail [3]. The study of mechanisms and character of complicated dynamic behavior of population genetic structure and of population number was carried out. Conditions of genetic polymorphism existing also have been found.

2.1 Constant quota: stationary strategy of harvest

Then we added harvest with constant catch proportion from population size u into model (1):

$$\begin{cases} x_{n+1} = x_n \overline{W}_n (1-u), \\ q_{n+1} = q_n (W_{AA} q_n + W_{Aa}(1-q_n)) / \overline{W}_n, \end{cases}$$
$$\overline{W}_n = W_{AA} q_n^2 + 2W_{Aa} q_n (1-q_n) + W_{aa}(1-q_n)^2.$$

(2)

In general case $u \in [0,1]$; the arbitrary choice of catch proportion u can be followed by the underfishing or overfishing. The optimal harvest [4], [5] means exploitation with such catch volume, under which the income from harvested population is stable and maximal, provided that population doesn't endanger.

In general case the model of harvested population (2) has three stationary points; there are two monomorphic and one polymorphic point. So, there are three catch proportions: u_{AA}, u_{aa} and u_{Aa} – that are optimizing yield in each equilibrium accordingly.

2.2 Non-stationary strategy of harvest

Then we suppose that there is stable 2-cycle in population, i.e. population size shows stable two-years fluctuations; it reaches value x_1 in uneven years and x_2 in even ones. So, there are two catch portions for even and uneven years accordingly: u_1 and u_2. In order to determine values of u_1 and u_2 we resolve the sat of equations optimizing two-year catch volume.

3 Results and Discussion

Let's consider some interesting outcomes of our study.

Population which loses its genetic variety as a result of natural selection one can see on figure 1. This population hasn't polymorphic equilibrium, so there are two catch proportions: u_{AA} and u_{aa} – that are optimizing yield in each monomorphic equilibrium accordingly. Exploitation with constant quota couldn't preserve genetic variety in this population. It is much unexpected result, that genetic variety will be preserved under exploitation with non-stationary strategy.

On figure 2 one can see population, which size is catastrophically fluctuating. Exploitation of this population doesn't result in fast stabilization of its dynamics. Choice of constant catch proportion u_{aa}=0.950 results in average catch volume R=78.8; another constant catch proportion u_{AA}=0.999 results in $R=3.14*10^{-8}$, and non-stationary exploitation with u_1=0.974 and u_2=0.0 gives R = 174.9 (all catch volumes are averaged for 40 generations of exploitation beginning). So non-stationary strategy of harvest in this case allows the fastest grows of population size from depression and as direct effect of this growth – the fastest attainment of considerable catch volume.

References

[1] E.Ya. Frisman, O.L. Zhdanova. The Trade Effects on Both Genetic Variety and Dynamic Behavior of Mendelian Limited Population // The International Conference "PhysCon'2005" Proceedings. St.-Petersburg, 2005, pp. 832-836.

[2] O. L. Zhdanova, E. A. Kolbina, and E. Ya. Frisman. Effect of Harvesting on the Genetic Diversity and Dynamics of a Mendelian Limited Population // Doklady Biological Sciences, 2007, Vol. 412, pp. 49–52. © Pleiades Publishing, Ltd., 2007. (Original Russian Text © O.L. Zhdanova, E.A. Kolbina, E.Ya. Frisman, 2007, published in Doklady Akademii Nauk, 2007, Vol. 412, No. 4, pp. 564–567.)

[3] O. L. Zhdanova, E. A. Kolbina., E. Ya. Frisman. Problems of regular behavior and determined chaos in the mathematical model of Mendelian limited population evolution. In *Dal'nevost. Matematich. Zhurnal*, volume 4(2), pages 289-303, 2003. (In Russian.)

[4] E.I. Skaletskaya, E.Ya. Frisman, A.P. Shapiro. Discrete models of populations' number dynamics and harvest optimization. M: Nauka, 1979. (In Russian.)

[5] P.D.N. Srinivasu, S. Ismail. Global dynamics and controllability of a harvested prey-predator system. In *Journal of Biological Systems*, volume 9(1), pages 67-79, 2001.

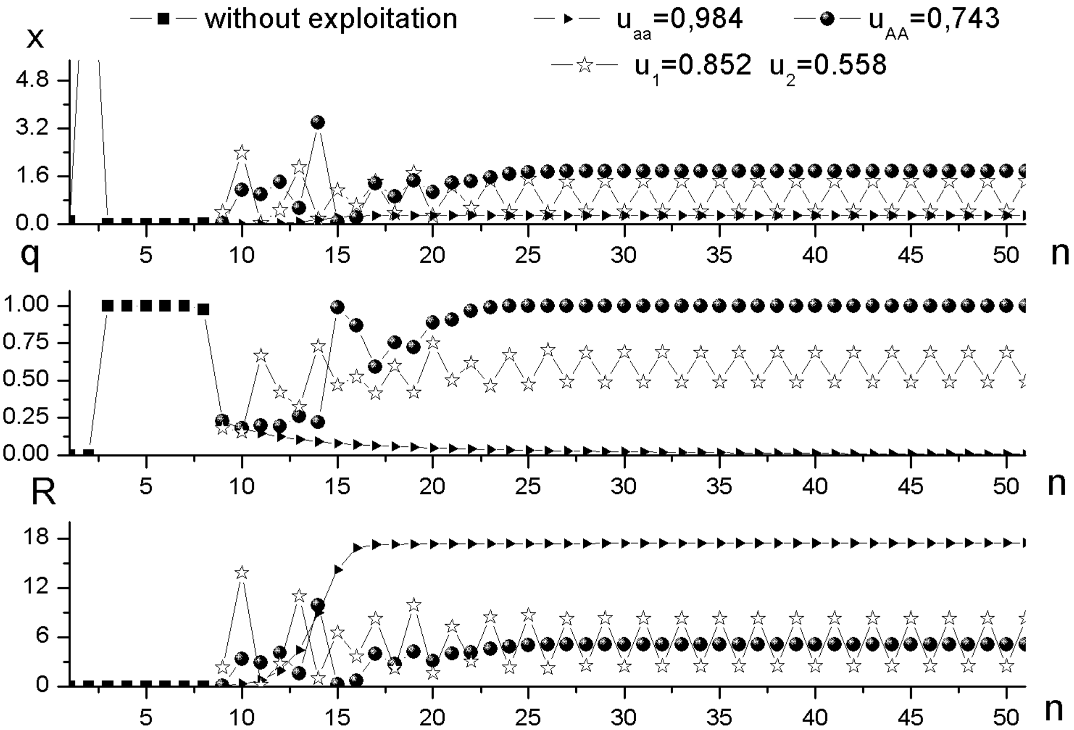

Fig. 1 Dynamics of population's number (x), allele A frequency (q) and catch volume (R) under the constant catch proportions (according two monomorphic (u_{AA}, u_{aa}) equilibriums) and under the non-stationary strategy of harvest (u_1 and u_2). The harvest begins after 8[th] generation; there would be a stable two-years fluctuations of population size and fixation of allele A in non-exploited population. Population parameters are: K_{AA}=5, K_{Aa}=1.5, K_{aa}=1.49, R_{AA}=2.1, R_{Aa}=5.04, R_{aa}=5.11, x_0=0.1, q_0=0.00001.

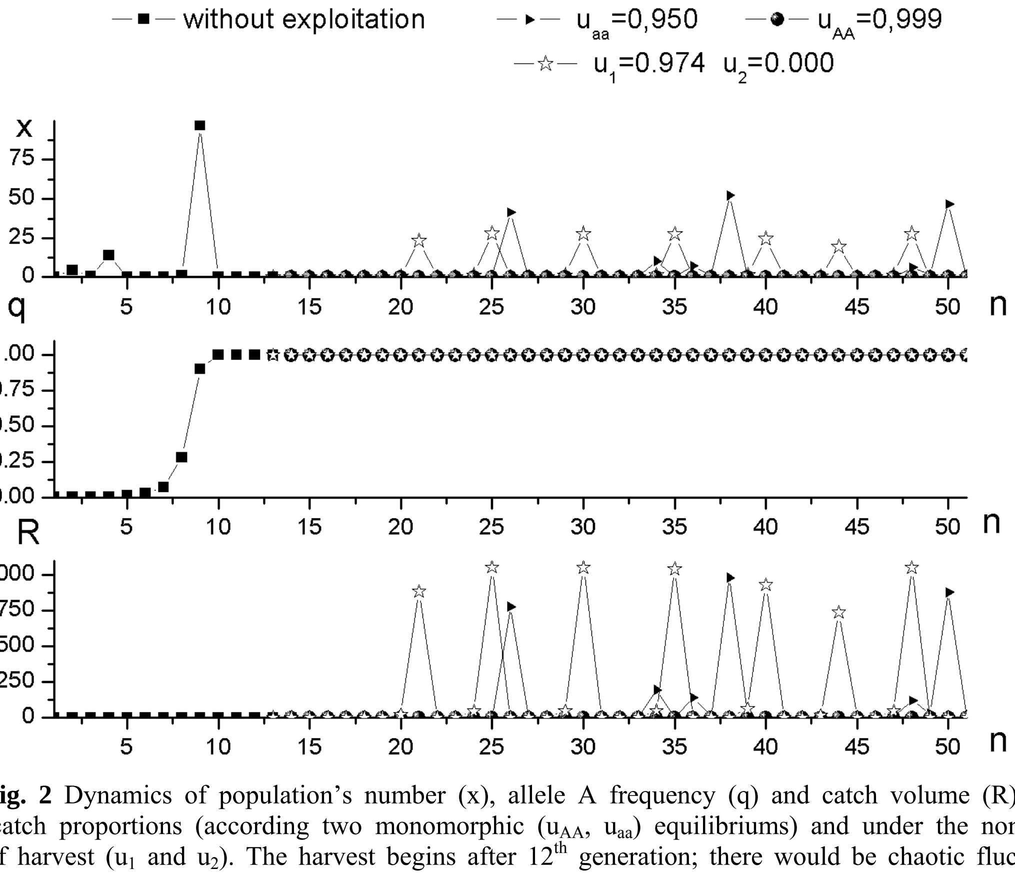

Fig. 2 Dynamics of population's number (x), allele A frequency (q) and catch volume (R) under the constant catch proportions (according two monomorphic (u_{AA}, u_{aa}) equilibriums) and under the non-stationary strategy of harvest (u_1 and u_2). The harvest begins after 12[th] generation; there would be chaotic fluctuations of population size and fixation of allele A in non-exploited population. Population parameters are: K_{AA}=8, K_{Aa}=3.1, K_{aa}=3, R_{AA}=8, R_{Aa}=3.88, R_{aa}=3.94, x_0=0.1, q_0=0.001.