CONTROLLED SYNCHRONIZATION-DESYNCHRONIZATION TRANSITIONS IN OSCILLATORY NETWORKS

A.S.Korneev and G.V.Osipov

***Abstract*— The automatic control method for synchronization - desynchronization is used in ensembles of interconnected oscillators. Our approach is based on the principles of automatic feedback control for locally diffusive coupled elements. This way automatic control is achieved for low coupling and this is demonstrated for coupled regular oscillators.**

I. INTRODUCTION

In the recent years, the synchronization - desynchronization in spatially extended regular and chaotic systems in nature and technology has attracted wide interest [1]. Construction of modern communication systems, radio-location complexes, networks of coupled power generators and lasers, etc., is impossible without making use of synchronization-desynchronization. In this connection the problem of design of optimal inter-element coupling schemes is very important. Despite on the great variety of possible applications three main cases of synchronization can be distinguished: (i) synchronization of oscillatory systems by an *external* signal; (ii) *mutual* synchronization of bidirectionally coupled oscillators; (iii) synchronization of coupled oscillators with the help of a *feedback loop* performing automatic phase and (or) frequency control. In the latter case the presence of a special control loop and bidirectional coupling makes the resulting automatic synchronization-desynchronization scheme rather versatile and reliable, making it widely applicable in technology.

In this paper we demonstrate an automatic control method of phase locking [1] in the network of regular non-identical oscillators, when the pairs of elements interact by a feedback [2] and by a local diffusive coupling. This method is basing on the well known principle of feedback control used in phase-locked loops (PLL) [3]. Our approach supposes the existence of a diffusive coupling and the special controllers, which allow to change the parameters of the controlled systems. First we present general principles of automatic phase synchronization-disynchronization (PS) for arbitrary coupled oscillators with diffusive coupling and controllers whose inputs are given by the quadratic forms of coordinates of the individual systems and its outputs are the results of the application of a linear differential operators. Next we give a simple example - coupled periodic oscillators - where we approve analytically that these principles work. Then we demonstrate numerically the effectiveness of our approach for locally coupled regular oscillators.

This work is done at financial support RFBR-NSC, project N 05-02-90567 and RFBR-MF project N 05-02-19815 and RFBR project N 06-02-16596.

A.S.Korneev and G.V.Osipov are with Radiophysical department, University of Nizhny Novgorod, Gagarin ave. 23, 603950 Nizhny Novgorod, Russia; askorneev@gmail.com, osipov@rf.unn.ru

II. GENERAL PRINCIPLES OF AUTOMATIC SYNCHRONIZATION-DESYNCHRONIZATION

First, we describe the automatic control method for the case of ensemble of arbitrary regular or chaotic oscillators given by the system:

$$\dot{x}_j = F_j(x_j, \omega_j), \quad j = 1, ..., N, \tag{1}$$

where x_j and F_j are n-vectors, ω_j are parameters defining the time dependence rate (in some cases, frequencies) of oscillations $x_j(t)$ and N is a number of oscillators. Our purpose is to control synchronization - desinchronization of the elements in a such ensemble using feedback control of the time scales of coupled oscillators and diffusive coupling in such a way that the new characteristic time scales Ω_j^{-1} become equal (synchronization) or different (desynchronization). Here Ω_j are the mean observed frequencies of the oscillators being controlled. In order to synchronize - desynchronize coupled subsystems we apply feedback control and diffusive coupling between nearest neighbors in the following form [2]:

$$\begin{array}{l} \dot{x}_j = F_j(x_j, \omega_j + \alpha_j u_j) + d(x_{j+1} - 2x_j + x_{j-1}), \\ Lu_j = Q_j(x_1, ..., x_N), \\ j = 1, ..., N, \end{array} \tag{2}$$

where L is a linear operator (e.g., $L = a_k \frac{d^k}{dt^k} + a_{k-1}\frac{d^{k-1}}{dt^{k-1}} + ... + a_1 \frac{d}{dt} + a_0$) acting as a low-pass filter; function $Q_j(x_1, ..., x_N)$ is the following:

$$Q_j(x_1, ..., x_N) = \sum_{k=1, k\neq j}^{N} Q_k(x_j, x_k), \tag{3}$$

where Q_k is a quadratic form $Q_k = x_j^T H x_k$ characterized the coupling between j-th and k-th oscillators. H is a $n \times n$ matrix; α_j are feedback controlling coefficients;d is a diffusive coupling coefficient; and $u_j(t)$ are the control variables added in (1) in such a way that it is able to change the characteristic time scales of the interacting subsystems. The spectrum of oscillations of $Q_k(x_j, x_k)$ consists of a "low" part defined by the difference $\Omega_k - \Omega_j$ and a "high" part defined by the sum $\Omega_k + \Omega_j$, which is damped by the low-pass filter due to a specially designed transfer function. Hence the control variable $u_j(t)$, being filtered, becomes a slow-varying time function, whose spectrum is located in the band $[0, 2|\Omega_k - \Omega_j|]$. We add $u_j(t)$ to the basic system (2) in such a way that it may provide a balance of the new time scales, i.e. $\Omega_k = \Omega_j$ or vice versa this may desynchronize our system if it was synchronized by local diffusive coupling, i.e. $\Omega_k \neq \Omega_j$.

Next we demonstrate the automatic control method for phase synchronization - desynchronization for ensemble of locally coupled regular oscillators.

III. Synchronization-desinchronization of locally coupled regular oscillators

As the simplest case we consider feedback control of phase synchronization in ensemble of *locally* mutually coupled Poincaré systems:

$$\begin{array}{l}\dot{x}_j = -(\alpha_j u_j + \omega_j) y_j - \lambda(x_j^2 + y_j^2 - 1)x_j + \\ +d(x_{j+1} - 2x_j + x_{j-1}), \\ \dot{y}_j = (\alpha_j u_j + \omega_j)x_j - \lambda(x_j^2 + y_j^2 - 1)y_j, \\ \dot{u}_j = -u_j + \beta_{j+1}x_j y_{j+1} + \beta_{j-1}x_j y_{j-1}, \\ j = 1, ..., N,\end{array} \quad (4)$$

where ω_j are the frequencies and $\lambda > 0$ is a damping parameter of oscillators, u_j is the control variable, α_j and β_j are the parameters of the j-th controller, d are the parameters of diffusive coupling. We assume free-end boundary conditions: $\beta_0 = \beta_{N+1} = 0$. For the quadratic form Q_j we take the simplest form of coupling with nearest neighbors

$$Q_j = \beta_{j+1}x_j y_{j+1} + \beta_{j-1}x_j y_{j-1} \quad (5)$$

In this example we take the linear operator L in the form $L = \frac{d}{dt} + 1$. Using polar coordinates $x_j = \rho_j cos\phi_j, y_j = \rho_j sin\phi_j$, we rewrite (4) in the form:

$$\begin{array}{l}\dot{\rho}_j = \lambda\rho_j(1-\rho_j^2) + d(\rho_{j+1}cos\phi_{j+1}cos\phi_j - \\ -2\rho_j cos^2\phi_j + \rho_{j-1}cos\phi_{j-1}cos\phi_j), \\ \dot{\phi}_j = \alpha_j u_j + \omega_j - \frac{d}{\rho_j}(\rho_{j+1}cos\phi_{j+1}sin\phi_j - \\ -2\rho_j cos\phi_j sin\phi_j + \rho_{j-1}cos\phi_{j-1}sin\phi_j), \\ \dot{u}_j = -u_j + \beta_{j+1}\rho_j\rho_{j+1}cos\phi_j sin\phi_{j+1} + \\ \beta_{j-1}\rho_j\rho_{j-1}cos\phi_j sin\phi_{j-1}, \\ j = 1, ..., N\end{array} \quad (6)$$

Let us take the gradient distribution of individual frequencies $\omega_j = \omega_1 + \Delta(j-1)$, and $\alpha_j = \alpha$, $\beta_j = \beta$, $\rho_j = \rho$. Then introducing the phase difference variable $\theta_j = \phi_j - \phi_{j+1}$, $\frac{\beta}{2}\rho^2 = \hat{\beta}, \frac{d}{2} = \hat{d}, \frac{\alpha\beta}{2}\rho^2 = \hat{\alpha}$ and averaging the system (6) we obtain:

$$\begin{array}{l}\dot{\phi}_1 = \alpha u_1 + \omega_1 - \hat{d}sin\theta_1, \\ \dot{u}_1 = -u_1 - \hat{\beta}sin\theta_1, \\ \dot{\theta}_j = \alpha(u_j - u_{j+1}) + \Delta + \hat{d}(sin\theta_{j-1} - 2sin\theta_j + sin\theta_{j-1}), \\ \dot{u}_j = -u_j - \hat{\beta}(sin\theta_j - sin\theta_{j-1}), \\ j = 1, ..., N\end{array} \quad (7)$$

with the boundary conditions: $\theta_0 = \theta_N = 0$. Stable steady state $(\bar{\theta}_1, ..., \bar{\theta}_j, ...\bar{\theta}_{N-1})$ in system (7) corresponds to a regime of the global synchronization in the chain.The system of equations for the stationary phase differences $\bar{\theta}_j$

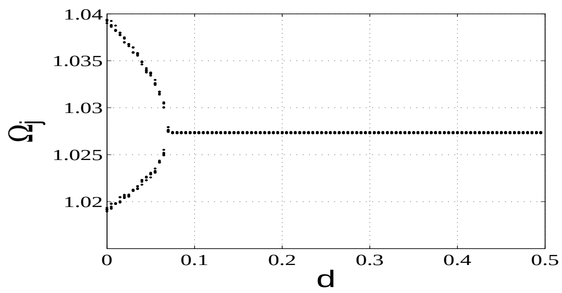

Fig. 1. Mean frequencies Ω_j in a chain of Poincaré systems with linear distribution of individual frequencies versus d. The parameter of a feedback control $\alpha = 0.2$. The number of elements $N = 10$, $\omega_1 = 0.98$, $\Delta = 0.01, \beta = 1$.

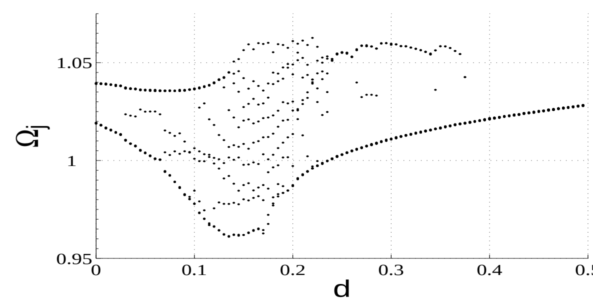

Fig. 2. Mean frequencies Ω_j in a chain of Poincaré systems with linear distribution of individual frequencies versus d. The parameter of a feedback control $\alpha = -0.2$. The number of elements $N = 10$, $\omega_1 = 0.98$, $\Delta = 0.01, \beta = 1$.

can be written as:

$$\begin{array}{c}\Delta + (\hat{\alpha} + \hat{d})(\sin\ \bar{\theta}_2 - 2\sin\ \bar{\theta}_1) = 0, \\ \Delta + (\hat{\alpha} + \hat{d})(\sin\ \bar{\theta}_{j+1} - 2\sin\ \bar{\theta}_j + \sin\ \bar{\theta}_{j-1}) = 0, \\ j = 2, ..., N-2, \\ \Delta + (\hat{\alpha} + \hat{d})(\sin\ \bar{\theta}_N - 2\sin\ \bar{\theta}_{N-1}) = 0\end{array} \quad (8)$$

The distribution of $\bar{\theta}_j$ is [4]:

$$\sin\ \bar{\theta}_j = \frac{\Delta}{2(\hat{\alpha} + \hat{d})}(Nj - j^2). \quad (9)$$

From (9) follows that the system (7) can have 2^{N-1} steady states. Only one of then ($\bar{\theta}_j \in [-\pi/2; \pi/2]$ for all $j = 1, ..., N-1$) is stable. As the frequency mismatch Δ is increased, the condition of the existence of steady states:

$$|\frac{\Delta}{2(\hat{\alpha} + \hat{d})}(N\ j - j^2)| < 1 \quad (10)$$

is violated first for $j = N/2$ at even N, i.e. for the middle element in the chain. Thus, the condition for the existence of a stable steady state (8) in the N-element chain is given by the inequality

$$|\frac{\Delta N^2}{8(\hat{\alpha} + \hat{d})}| < 1. \quad (11)$$

Using conditions (10) we can control synchronization - desynchronization transitions by the variation of the single control parameter α or d.

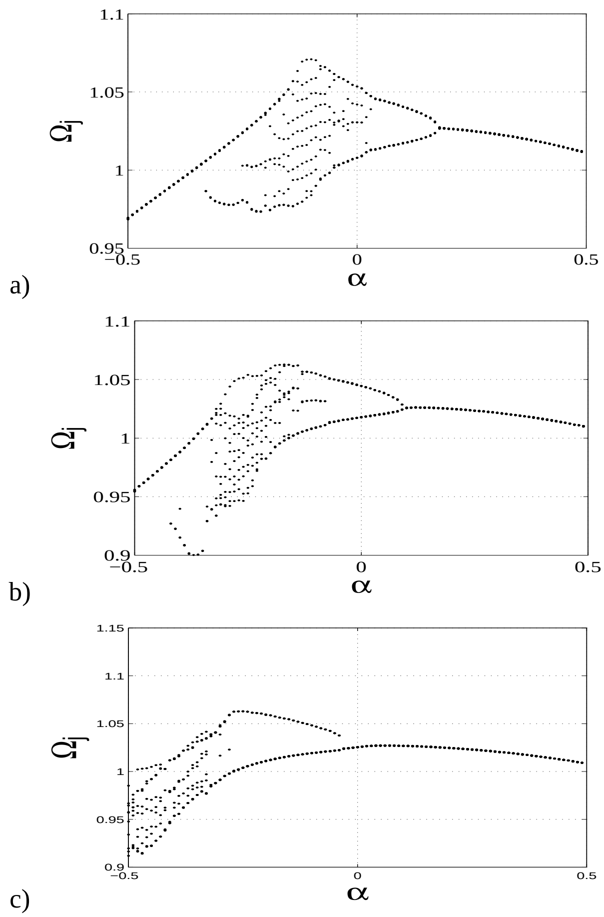

Fig. 3. Mean frequencies Ω_j in a chain of Poincaré systems with linear distribution of individual frequencies versus α. The parameter of a diffusive coupling (a)$d = 0.1$, (b) $d = 0.2$, (c) $d = 0.3$ The number of elements $N = 10$, $\omega_1 = 0.98$, $\Delta = 0.01, \beta = 1$.

In order to show our approach we present the results of our numerical experiments.

In all experiments we take a chain of 10 Poincaré oscillators with gradient distribution of individual frequencies ($\omega_j = \omega_1 + \Delta(j-1), \omega_1 = 0.98\Delta = 0.01$).

Experiment 1. Feedback coupling α is fixed ($\alpha = 0.2$) and diffusive coupling coefficient d is varied (Fig.1). For $d = 0$ whole chain is divided into two groups (clusters) of synchronized elements with mean frequencies $\bar{\Omega}_1 \approx 1.02$ and $\bar{\Omega}_2 \approx 1.04$. With increase of diffusive coupling d the frequencies become closer and for $d > 0.075$ the global synchronization regime takes place.

Experiment 2. Feedback coupling α is fixed ($\alpha = -0.2$) and diffusive coupling coefficient d is varied (Fig.2). As in the previous case frequencies tends to be different. For $d = 0$ we observe two clusters in this system with mean frequencies $\bar{\Omega}_1 \approx 1.02$ and $\bar{\Omega}_2 \approx 1.04$. Next we switch on diffusive coupling d. First we can see appearance of three clusters ($d = 0.04$), then four clusters ($d = 0.08$) etc. For $d = 0.15$ every oscillator has its own frequency and the global non-synchronous regime sets in. With further increase of diffusive coupling the collective behavior of units become more regular and synchronous clusters appear. For $d > 0.4$ regime of global synchronization is stable. So, by changing the coupling it is possible to desynchronize the synchronous behavior of coupled systems.

Experiment 3. Diffusive coupling coefficient d is fixed ($d = 0.1$) and feedback coupling parameter is varied (Fig.3 (a)). For $\alpha = -0.5$ system is synchronized and all oscillators have the same frequency $\bar{\Omega} \approx 0.97$. After increasing of feedback coupling the mean frequency increases. Only in the interval $-0.4 < \alpha < -0.2$ there is no global and cluster synchronization. For $-0.2 < \alpha < -0.1$ the global desynchronization regime takes place. With increasing α synchronous clusters appears again and for $\alpha > 0.2$ the global synchronization is observed.

Experiment 4. Diffusive coupling coefficient d is fixed $d = 0.2$ and feedback coupling parameter is varied (Fig.3 (b)). In this case the system has the same qualitative behavior as in the previous case. Only initially ($\alpha = 0$) there is two synchronous clusters.

Experiment 5. Diffusive coupling coefficient d is fixed $d = 0.3$ and feedback coupling parameter is varied (Fig.3 (c)). For $\alpha = 0$ the system is fully synchronized. As in the two previous cases increase of α leads to more coherent behavior but its decrease leads to break up of synchronous states first and then to transition to cluster and global synchronization at larger coupling strengths.

In such a way we can control the state of our system. We can only change the strength of diffusive coupling or (and) feedback control.

We tested our control scheme in the cases of locally forced ensembles. With appropriate chosen parameters of feedback control proposed method gives positive results.

IV. Conclusions

In conclusion, we have demonstrated for an ensembles of coupled regular oscillators an automatic control method for transitions (i) synchronization - desynchronization and (ii) desynchronization - synchronization of regular oscillators. This method can be used for control synchronization-desynchronization of oscillators of different nature (regular or chaotic), and different topology. The control of synchronization sets in at very small values of control parameters, which is very important from energetic point of view. We suppose that our approach can be helpful for the design of different schemes of automatic control method for synchronization-desynchronization and could be applied to communication, engineering and medicine.

References

[1] A. S. Pikovsky, M. G. Rosenblum, and J. Kurths, “ *Synchronization - A universal concept in Nonlinear Sciences* “, Cambridge University Press, Cambridge, 2001;

[2] V. N. Belykh, G. V. Osipov, N. Kuckländer, and J. Kurths Physica D, 200, pp.81-104,(2005)

[3] W. C. Lindsey, *Synchronization Systems in Communication and Control* (Prentice-Hall, Englewood Cliffs, NJ, 1972).

[4] V. S. Afraimovich, V. I. Nekorkin, G. V. Osipov, and V. D. Shalfeev, *“Stability, Structures and Chaos in Nonlinear Synchronization Networks”*, World Scientific, Singapore, 1994.