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NONLINEAR OBSERVER-ESTIMATOR APPLICATION: QUARTER-CAR SUSPENSION SYSTEM UNDER ROAD DISTURBANCES

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Abstract
The aim of this article is to present the application of a useful technique that allows to design a nonlinear observer capable to estimate an unknown dynamics in the system under analysis. In order to have a practical example, the controller design for a quarter-car suspension system under road perturbations is presented. The lack of exactitude for system parameters values and the necessity to identify the variables measurement makes this system ideal to apply the observer-identifier.

Key words
Identification, modeling, linear system, control design.

1 Introduction

Nowadays exists a great number of mechanical systems that allow to make different kinds of tasks in order to improve the human comfort. However, the mathematical models developed for this systems are ideal and some dynamics and parameter values are not contemplated when the control signal is designed. Even the systems are usually perturbed for unknown external signals, causing an undesired behavior. On the other hand, the automotive industry has as main goal to improve the driving experience and the passengers safety due to the damage on roads that causes many changes in the ideal conditions for which the cars were designed. The new designs for suspension systems are mainly based on maintaining the car body in horizontal position without allowing rotations about the mass center even under the effects of the road.

The full suspension system is composed by the front mechanism and rear mechanism, each one with a left subsystem and a right subsystem, usually called quarter-car suspension system. A suspension subsystem should be able to compensate the perturbations induced by road conditions and due to the forces produced by the interaction with the other suspension subsystems, in other words, compensate the dynamics induced by the full-car suspension system.

The proposed linear mathematical model in the literature for the quarter-car suspension system represents passive, semi-active or active behavior for typical car suspensions. This model represents the dynamics of tire and car body using springs, dampers, masses and an actuator located between the car body and the wheel. This actuator, allows to modify the damping rates (semi-active) or the force applied to car body (active) according to the control objective and the kind of controller used.

Some of the control design approach includes adaptive control [Nugroho *et al.*, 2012], fuzzy control [Ranjbar-Sahraie, Soltani, and Roopaie, 2011], optimal control [Paschedag, Giua, and Seatzu, 2010], sliding mode control [Alvarez-Sanchez, 2013; Ahmed and Taparia, 2013] and skyhook control [Chen, 2009]. However, the requirement of knowledge the parameters values make almost impossible the implementation of the controller designed without a parameter identification [Zarringhalam *et al.*, 2012], even a robust one.

The control scheme presented here is a traditional space state control design using a novel technique proposed by [Rosas, Alvarez, and Fridman, 2007] that avoid the necessity of knowing all the parameters system values and the full state vector, resulting in a robust and a feasible control law. The numerical values used for simulations, correspond to a real car system parameter values.

2 Nonlinear Observer-Identifier

Consider the next second order system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax_1 - bx_2 + \xi(\cdot) - csign(x_1)\end{aligned} \quad (1)$$

where a, b and c are positive constants and $\xi(\cdot)$ is a upper bounded perturbation, i.e. $|\xi(\cdot)| \leq \rho$, with ρ being a constant. According to [Rosas, Alvarez, and Fridman, 2007] the system (1) has an origin that is an exponentially stable equilibrium in the sense of Lyapunov.

Now, consider a system in a state variable form as follows

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= f(z) + g\tau + \gamma(t)\end{aligned} \quad (2)$$

where g is a constant, $f(z)$ is a Lipschitz function, $\gamma(t)$ is an external perturbation and τ is the control signal, the main assumption is that $\gamma(t)$ and τ are both bounded such that (2) behavior is also bounded. A nonlinear observer-estimator for (2) is given by

$$\begin{aligned}\dot{\hat{z}}_1 &= c_1(z_1 - \hat{z}_1) + w_1 \\ \dot{w}_1 &= c_2(z_1 - \hat{z}_1) + c_3 sign(z_1 - \hat{z}_1) \\ \dot{\hat{z}}_2 &= c_4(w_1 - \hat{z}_2) + w_2 + f(z) + g\tau \\ \dot{w}_2 &= c_5(w_1 - \hat{z}_2) + c_6 sign(w_1 - \hat{z}_2)\end{aligned} \quad (3)$$

where c_1, c_2, c_3, c_4, c_5 and c_6 are positive constants such that they assure an exponentially stable origin.

Defining two error variables as $e_1 = z_1 - \hat{z}_1$ and $e_2 = \dot{z}_1 - \dot{\hat{z}}_1$, the first error system dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= \xi_1(\cdot) - c_1 e_2 - c_2 e_1 - c_3 sign(e_1)\end{aligned} \quad (4)$$

where

$$\xi_1(\cdot) = f(z) + g\tau + \gamma(t).$$

The system (4) has the form of (1), so the origin of (4) is an exponential stable equilibrium point. Furthermore, $e_1 \to 0$ implies that $\hat{z}_1 \to z_1$ and $w_1 \to z_2$.

The error variables $e_3 = z_2 - \hat{z}_2$ and $e_4 = \dot{z}_2 - \dot{\hat{z}}_2$, produces a second error dynamics system given by

$$\begin{aligned}\dot{e}_3 &= e_4 \\ \dot{e}_4 &= \xi_2(\cdot) - c_4 e_4 - c_5 e_3 - c_6 sign(e_3)\end{aligned} \quad (5)$$

where

$$\xi_2(\cdot) = \dot{\gamma}(t).$$

The system (5) also has the form of (1) so that $\hat{z}_2 \to z_2$ and $w_2 \to \gamma(t)$. The convergence of term w_2 implies that the external perturbation is identified, which is an advantage of this methodology at the moment of control design. A more detailed explanation for the analysis of convergences is found in [Rosas, Alvarez, and Fridman, 2007] and [Rosas and Alvarez, 2011].

In order to show the feasibility of the methodology proposed by [Rosas, Alvarez, and Fridman, 2007], a quarter-car suspension system model is ideal to design a controller that uses the nonlinear observer-identifier.

3 Quarter-Car Suspension Dynamics

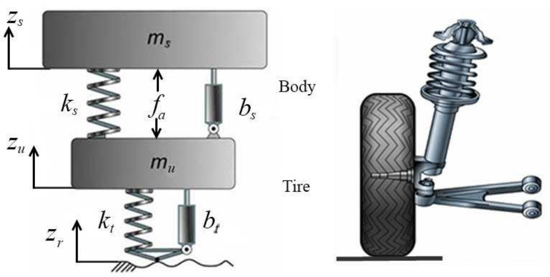

Figure 1. Quarter-Car Suspension System

The figure 1 shows the representation for a quarter-car suspension system that is used to obtain a mathematical model for the system dynamics. This model allows to design a controller capable to fulfill the control objective: passengers comfort. The subscript *s* is for the sprung elements, the subscript *u* represents the unsprung subsystem while the subscript *t* refers to tire.

3.1 Mathematical Model

According to Newton methodology, the equations that describe the system dynamics are

$$m_s \ddot{z}_s = -b_s(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) + f_a \quad (6)$$

$$\begin{aligned}m_u \ddot{z}_u = {} & b_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) - f_a \\ & + b_t(\dot{z}_r - \dot{z}_u) + k_t(z_r - z_u)\end{aligned} \quad (7)$$

where m_s and m_u denotes the mass of the sprung and unsprung elements, respectively. While b_s, b_t, k_s and

k_t are the damping rate and stiffness of the car body (sprung mass) and tire, respectively. The linear actuator is represented by means of f_a and the term z_r denotes the road perturbations.

3.2 State Space System

Using $z_1 = z_s$, $z_2 = \dot{z}_s$, $z_3 = z_u$, $z_4 = \dot{z}_u$, (6) and (7) can be rewritten in the next state space form

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{1}{m_s}\left[-b_s(z_2 - z_4) - k_s(z_1 - z_3) + f_a\right] \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= \frac{1}{m_u}\left[b_s(z_2 - z_4) + k_s(z_1 - z_3) - f_a \right. \\ &\quad \left. + b_t(\dot{z}_r - z_4) + k_t(z_r - z_3)\right] \end{aligned} \tag{8}$$

The first two expressions of (8) describe the car body dynamics, which is the variable of interest to control. Using the second equation of (8), a new equation in the form of a perturbed one can be expressed as

$$\dot{z}_2 + \frac{b_s}{m_s}z_2 + \frac{k_s}{m_s}z_1 = \frac{1}{m_s}\left(b_s z_4 + k_s z_3 + f_a\right) \tag{9}$$

This last equation can be used to design the controller that fulfills the comfort objective.

4 Controller-Estimator Design

In order to fulfill the control objective, $z_1 \to z_d$. The desired value for the sprung mass, z_d, is given by

$$z_d = Asin(\omega t)$$

where ω is the frequency required for comfort purposes. This frequency needs to be selected according to [Guglielmino *et al.*, 2008] in the range of 0.75 Hz and 4 Hz in order to avoid nausea, vertigo, fatigue and even column damage. Using (9) is easy to design the next controller that cancels the undesired dynamics and impose a new one

$$\begin{aligned} f_a = &-(b_s z_4 + k_s z_3) + b_s z_2 + k_s z_1 \\ &+ m_s\left[-k_d(z_2 - z_2 d) - k_p(z_1 - z_1 d) - \dot{z}_{2d}\right] \end{aligned} \tag{10}$$

where k_d and k_p are positive constants. Substituting (10) in (9), the closed loop system is obtained, in function of z_1, as

$$(\ddot{z}_1 - \ddot{z}_{1d}) + k_d(\dot{z}_1 - \dot{z}_{1d}) + k_p(z_1 - z_1 d) = 0. \tag{11}$$

However, the controller (10) needs to know the dynamics of the unsprung mass and all the parameters of the entire system, which is the disadvantage of this kind of control design.

4.1 Estimator Design

The subsystem composed by the first two equations of (8) can be rewritten as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \Phi(\cdot) + \frac{1}{m_s}f_a \end{aligned} \tag{12}$$

where

$$\Phi(\cdot) = \frac{1}{m_s}\left[-b_s(z_2 - z_4) - k_s(z_1 - z_3)\right]$$

represents the dynamics to be estimated. Using (3), an observer-estimator for (12) is given by

$$\begin{aligned} \dot{\hat{z}}_1 &= c_1(z_1 - \hat{z}_1) + w_1 \\ \dot{w}_1 &= c_2(z_1 - \hat{z}_1) + c_3 sign(z_1 - \hat{z}_1) \\ \dot{\hat{z}}_2 &= c_4(w_1 - \hat{z}_2) + w_2 + \frac{1}{m_s}f_a \\ \dot{w}_2 &= c_5(w_1 - \hat{z}_2) + c_6 sign(w_1 - \hat{z}_2). \end{aligned} \tag{13}$$

The convergence for the variables is obtained by means of (13) as

$$\begin{aligned} \hat{z}_1 &\to z_1 \\ w_1 &\to z_2 \\ \hat{z}_2 &\to z_2 \\ w_2 &\to \Phi(\cdot). \end{aligned} \tag{14}$$

Using (14) the controller (10) can be rewritten as

$$f_a = m_s(-w_2 - k_d e_2 - k_p e_1 - \ddot{z}_1) \tag{15}$$

where

$$\begin{aligned} e_1 &= z_1 - z_{1d} \\ e_2 &= z_2 - \dot{z}_{1d}. \end{aligned}$$

Substituting the controller (15) in (12) the closed loop system obtained is

$$\ddot{e}_1 + k_d\dot{e}_1 + k_p e_1 = 0 \tag{16}$$

which is a stable equation that converges to zero according the values of k_d and k_p.

Table 1. Quarter-car Honda Civic 2005 parameters

Parameter	Value	Units
Sprung mass (m_s)	315	kg
Unsprung mass (m_u)	51	Kg
Spring stiffness (k_s)	43.3	KN/m
Damping constant (b_s)	3.9	KN·s/m
Tire stiffness (k_t)	210	KN/m
Tire damping (b_t)	1.1	KN·s/m
Distance floor to tire center	0.311	m
Distance floor to car body center of mass	0.518	m

5 Simulation Results

The system parameters used for simulations correspond approximately to the real values for a quarter-car Honda Civic 2005. These values are listened in table 1.

The Observer-Estimator and the control parameters are shown in table 2.

Table 2. Observer-Estimator and Control Parameters

Parameter	Value
c_1, c_2, c_3	5.5, 1.2, 0.1
c_4, c_5, c_6	150, 150, 1055
k_d, k_p	110, 22

The simulations results were obtained by means of SIMNON® with a fixed integration step of 1 ms. The road perturbation profile is shown in figure 2. One can notice three different amplitudes and frequencies acting directly over the tire. The first two signals represents a bumpy road, with 5 cm and 10 cm of deep, and the last signal is a speed reducer of 10 cm of high.

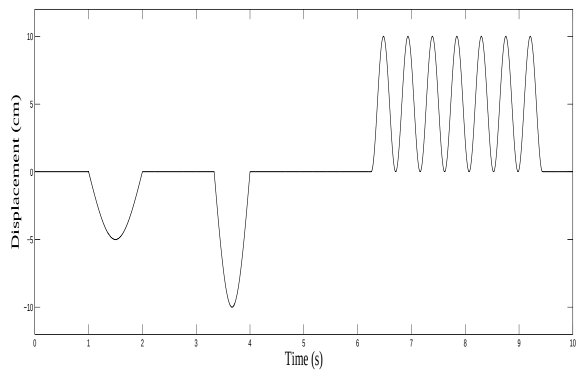

Figure 2. Road perturbations

From real comfort purposes, the desired behavior for body car (sprung mass) is

$$z_d = 0.1sin(0.5t)$$

The $\omega = 0.5$ represents a frequency of π Hz, which is between the recommended values. The free displacement (segmented line) versus the controlled displacement (continuous line) is shown in figure 3, where the values are displaced the distance from the floor to the car body center of mass.

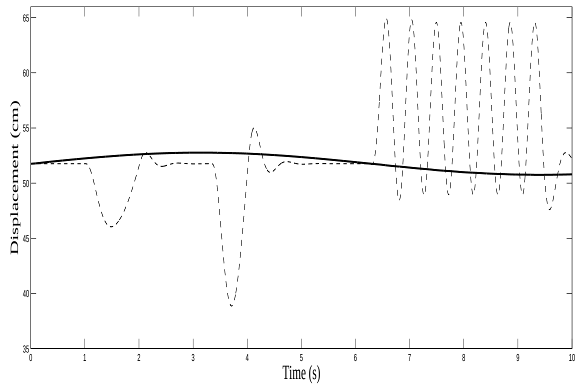

Figure 3. Sprung mass: controlled vs free

The force required to control the sprung mass and avoid the road perturbations is shown in figure 4. Where the effect of the observer-estimator is clear due the high frequency oscillations in the control signal.

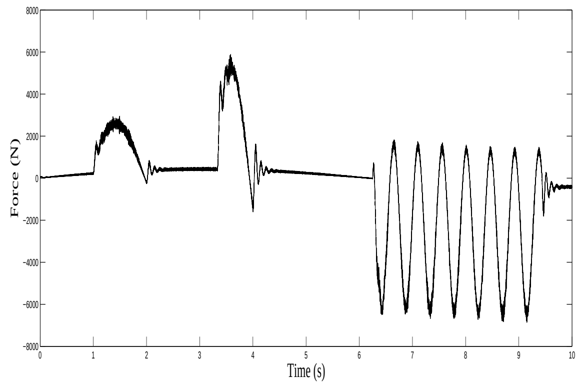

Figure 4. Force control

Figure 5 shows the comparison between the dynamics $\Phi(\cdot)$ and its estimation ω_2. The continuous line represents the real dynamics meanwhile the segmented line the estimated one. Due that the estimated signal is around the real signal, the zoom shown in figure 6 is necessary.

The tracking error for the sprung mass is shown in figure 7, where one can notice that is in order of millimeters.

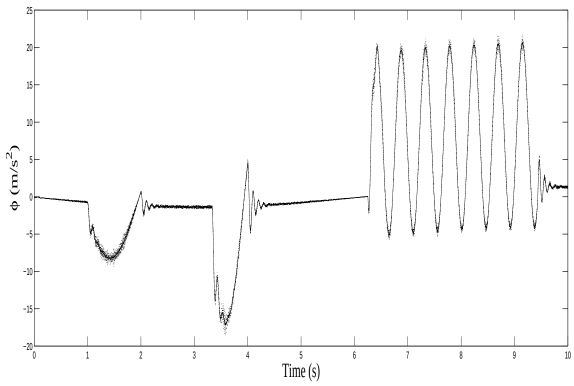

Figure 5. Dynamics $\Phi(\cdot)$ real vs estimated ω_2

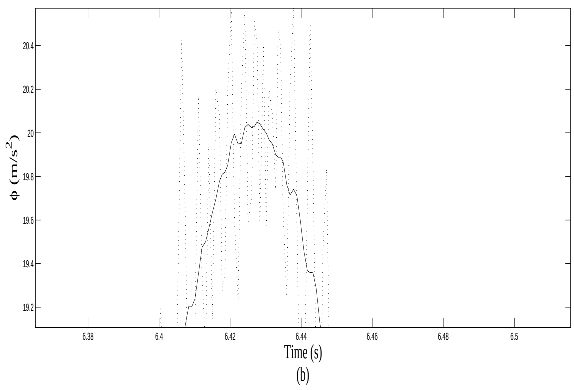

Figure 6. Zomm of signals $\Phi(\cdot)$ and ω_2

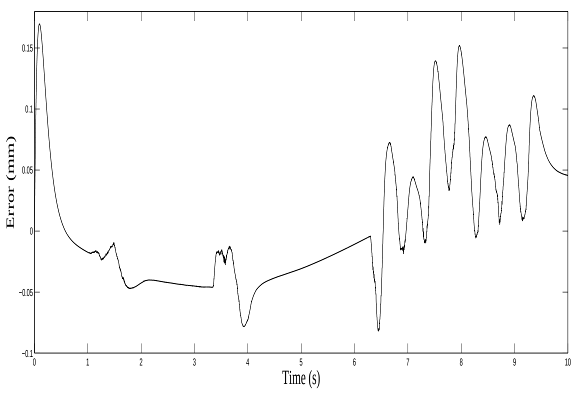

Figure 7. Sprung mass: tracking error

The behavior of the sprung mass (car body) and unsprung mass (tire) induced by the road profile are shown in figure 8, where the distance among the signals represents a real separation for the tire center to the road and the body car mass center to the road.

6 Conclusion

The methodology presented in this paper allows to have a useful estimator that could be implemented in a testing car in order to demonstrate the effectiveness of the designed controller, this because it is not necessary to know the parameters of the quarter-car, the

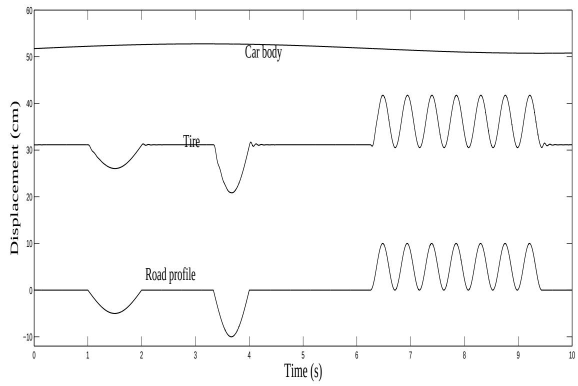

Figure 8. Displacement comparison: Road profile, tire and car body

road profile neither the tire dynamics. Even when the observer-estimator is nonlinear due the use of the sign function, the high frequencies are not induced to the car body and the control aim, passengers comfort, is fulfilled. The results obtained motivate to continue to the next step, the control of a half-car.

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