Analytical Synthesis of Precise Control Algorithms for a Space Purpose Object

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Abstract: This paper deals with the problem of decomposition and precise control of complex objects. Decomposition is based on concepts of an object technical controllability and model reference adaptive control. Precise accuracy is attained with the help of adaptive and optimal control. Computer simulation demonstrates good results.

Keywords: Complex Object, Lagrangian System, Technical Controllability, Decomposition, Adaptive Control, Lyapunov Function, Simulation.

1. INTRODUCTION

The theory for precise control of complex Lagrangian systems was presented in Zemlyakov and Glumov (2007); Zemlyakov and Krivoruchko (2007). This investigation demonstrated good results but these results were obtained under some restrictions. Restrictions have to be put on every concrete object under consideration. In this paper, we try to ensure some of these restrictions for an object of a space purpose.

As a complex, we assume an object with some interconnected subsystems (Šiljak, 1991; Zemlyakov and Krivoruchko, 2007). A mathematical model (MM) of such an object is usually a multi-connected nonlinear and non-stationary one. Synthesis of control algorithms for such an object is not a simple problem. The situation appears more difficult for the case of precise control.

Qualitatively, under precise control we mean the situation when the motion of any subsystem and the system in the whole coincide with the prescribed motions within the prescribed accuracies.

The usual method for such an object control is decomposition (Šiljak, 1991). In this paper, we assume that an object in the whole could be represented as a set of interconnected subsystems. For each subsystem a component of interconnections is selected and compensated on the base of adaptive or optimal control (Zemlyakov and Krivoruchko, 2007). For this goal we use two approaches to the decomposition. The first one is based on the decomposition of an object MM. In this case, the object has to satisfy the condition of technical controllability (Rutkovsky and Zemlyakov, 2003). The second approach is based on the decomposition of a system control MM. In this case, special adaptive control algorithms are derived (Zemlyakov and Krivoruchko, 2007). A space robotic module (SRM) (Glumov et al., 2006) is taken as an example for a space purpose object.

2. MODEL OF A SPACE ROBOTIC MODULE

SRM is intended for a large space stations service. It consists of a supporting body and one or some manipulators. Let our SRM have only one manipulator with *m* links. In an inertial space the SRM position in common case is determined by $N = 6(m+1)$ coordinates, but connections which are imposed on the relative positions of $(m+1)$ bodies reduce this number to a value $n \le N$.

For the SRM object we assume that the supporting body position is determined by six coordinates q_j and the position of each link is determined by the coordinates q_{6+i} that shows the position of a link with respect to the preceding link. Let us consider these coordinates as the generalized ones $q^T = (q_1, q_2, ..., q_n)$, where $n = 6 + m$ is a maximum number of generalized coordinates, *T* is the transposition sign. In turn, the number *n* can be decreased by superposing additional connections.

As the generalized coordinates of the supporting body, we consider the Euler angles $q_{00}^T = (q_1, q_2, q_3)$ and coordinates of the supporting body pole in an inertial coordinate system $q_{0t}^T = (q_4, q_5, q_6)$. The mechanical system MM will be derived on the base of Lagrange's equation

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = Q, \tag{1}$$

where $T = T(q, \dot{q})$ is the kinetic energy of the system in the whole and Q is the vector of generalized forces.

For purposes of simplification let us assume that the system potential energy is missing. It is correct for SRM which serves as an orbital station in weightlessness conditions.

Let us assume that the mechanical system position is controlled by a control vector $M^T = (M_1, M_2, ..., M_n)$.

The MM is necessary for the goal of its control so it is more reasonable to present (1) in the following form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = S(q)M\ , \qquad (2)$$

where the matrix $S(q)$ is determined by the equation $Q = S(q)M$.

In (Glumov et al., 2006) it was shown that the MM could be presented as

$$A(q)\ddot{q} + \sum_{s=1}^{n}\left[\dot{q}^T D_s(q)\dot{q}\right]e_s = S(q)M \qquad (3)$$

where matrices

$$A(q) = \left(a_{ij}(q)\right)\ \ (i,j=\overline{1,n}),$$
$$D_s(q) = \left(d_{it}^s(q)\right)\ \ (i,t,s=\overline{1,n}),\ S(q)$$

are $A(q) = C^T BC$,

$$d_{it}^s(q) = \frac{1}{2}\left[\frac{\partial a_{si}(q)}{\partial q_t} + \frac{\partial a_{st}(q)}{\partial q_i} - \frac{\partial a_{it}(q)}{\partial q_s}\right], \qquad (4)$$

$$S(q) = C^T ZL\ .$$

In (4) the matrices B, C, Z, L are determined by the SRM concrete constructive parameters.

As an example, we will take an SRM kinematic scheme of a simple structure: a carrying body and a manipulator with one link. An object moves in a space with the inertial coordinate system. The carrying body has the main central coordinate system $O_0x_0^1x_0^2x_0^3$. A manipulator is hinged to the carrying body at the point $(x_m^1,\ x_m^2,\ x_{in}^3)$. The manipulator has only one link. The link has the frame system $O_1x_1^1x_1^2x_1^3$ and the rotational degree of freedom relative to the $O_1x_1^1$ axis. The moments of inertia for the carrying body are relatively 150, 120, 100 kgm^2; the mass is 300 *kg*. The length and the mass of the link are relatively 1 *m* and 1 *kg*. The coordinates for the link center of mass are $(0,\ 0.8,\ 0)$. As generalized coordinates, we take the Euler angles: q_1 – the nutation, q_2 – the precession, q_3 – the roll in the whole angles, q_4, q_5, q_6 are the coordinates of the point O_0 in the inertial system relative to the axis $O_{in}x_{in}^1,\ O_{in}x_{in}^2,\ O_{in}x_{in}^3$. The angle position of the link relative to the carrying body is determined by the coordinate q_7.

Using formulas (4) and the their realization in the Maple system (Glumov et al., 2006), we derive the following matrices

$$A(q) = \left(a_{ij}(q)\right)\ \ (i,j=\overline{1,n}),$$
$$D_s(q) = \left(d_{it}^s(q)\right)\ \ (i,t,s=\overline{1,n}),\ S(q)$$

in symbolic–numerical forms. For each numerical vector q these matrices take also numerical forms. So we consider that object MM (3) is in our disposal with the known symbolic – numerical matrices.

Let the control vector $M = (M_i)\ (i=\overline{1,\ 7})$ consist of seven components: the first three M_1, M_2, M_3 are the moments that act on the carrying body relative to the axes $O_0x_0^1,\ O_0x_0^2,\ O_0x_0^3$; M_4, M_5, M_6 are the forces that act relative to the same axis; M_7 is the moment that acts to change the angle position of the manipulator link.

3. THE PROBLEM STATEMENT

Let for every generalized coordinate $q_i\ (i=\overline{1,\ 7})$ there exist a desired function $q_i^{ref}(t)$ and the differential equation

$$\ddot{q}_i + d_i\dot{q}_i + k_iq_i = k_iq_i^{ref}(t)\ , \qquad (5)$$

where the functions $q_i^{ref}(t)$ and numbers $d_i,\ k_i$ are prescribed in advance; $\left|q_i^{ref}(t)\right| \le w_i,\ \ w_i = \text{const} > 0$.

It is necessary to derive a control law $M = M(t)$ that guarantees the motion for every generalized coordinate $q_i\ (i=\overline{1,\ 7})$ with respect to Eq. (5).

To solve analytical problems we will simplify some conditions for a solution. We propose that:

- non-inertial measuring devices make possible to get vectors $q = q(t),\ \dot{q} = \dot{q}(t)$;
- non-inertial acting devices make possible to get

$$M(t) = U(t)\ , \qquad (6)$$

where $U(t)$ is a vector that is realized by a controller;

- efficient computer is used in a controller to solve relatively complex algebraic terms.

4. THE OBJECT DECOMPOSITION ON THE BASE OF "PHYSICAL" PRINCIPLE

The MM (3) can be presented as a system of two subsystems

$$A_{11}(q)\ddot{q}_0 = S_{11}(q)U_0 + [S_{12}(q)U_7 - A_{12}(q)\ddot{q}_7 - N_0(q,\dot{q})], \qquad (7)$$

$$A_{22}(q)\ddot{q}_7 = U_7 + [-A_{21}(q)\ddot{q}_0 - N_7(q,\dot{q})]\ . \qquad (8)$$

5. CONTROL OF THE CARRYING BODY MOTION

Here we will take into consideration MM (7) which we rewrite in the following form

$$\ddot{q}_0 = R_0(q)U_0 + f_0(*)\ , \qquad (9)$$

where $R_0(q) = (r_{0ij}(q))\ \ (i,j=\overline{1,\ 6})$ and

$$R_0(q) = A_{11}^{-1}(q)S_{11}(q);$$
$$f_0(*) = A_{11}^{-1}(q)[S_{12}(q)U_7 - A_{12}(q)\ddot{q}_7 - N_0(q,\dot{q})] \qquad (10)$$

For this subsystem we use an adaptive suboptimal relay control (Rutkovsky and Zemlyakov, 2003).

5.1. Conditions of the technical controllability

Definition 1 (Rutkovsky and Zemlyakov, 2003). The system that consists of the object with MM (7) or (9) and a controller with a control algorithm $U_0 = U_0(t,q,\dot{q})$ will be named as a technically controllable one with respect to requirement (5) if this requirement is fulfilled during the system operation.

Statement 1 (Rutkovsky and Zemlyakov, 2003). Necessary conditions for the technical controllability of the object with MM (9) are as follows:

1) $$r_{0ii}(q) > 0 \quad (i = \overline{1,6}), \tag{11}$$

2) the matrix $R_0(q)$ has to be belong to the class of matrices with diagonal domination, that is,

$$r_{0ii}(q) > \sum_{j=1(j\neq i)}^{6} |r_{0ij}(q)|. \tag{12}$$

It is easy to check that for the SRM with constructive parameters under consideration the necessary conditions do not take place.

5.2. The technical controllability of the SRM

Let us take the control algorithm $U_0 = U_0(t,q,\dot{q})$ in the form

$$U_0(t,q,\dot{q}) = S_{11}^{-1}(q)U_0^*(t,q,\dot{q}). \tag{13}$$

It is possible if the matrix $S_{11}(q)$ is not singular. Then Eq. (9) is rewritten in the following form

$$\ddot{q}_0 = R_0^*(q)U_0^* + f_0(*) \tag{14}$$

where $R_0^*(q) = A_{11}^{-1}(q)$.

It is easy to check that for the SRM with constructive parameters under consideration and MM in form (14) the necessary conditions take place for

$$20^0 < q_1 < 160^0;\ -1^0 < q_2 < 1^0;\ -1^0 < q_3 < 1^0. \tag{15}$$

Let us take the control algorithm $U_0^*(t,q,\dot{q}) = (u_{0i}^*(t,q,\dot{q}))$ $(i = \overline{1,6})$ in the relay form

$$u_{0i}^*(t,q,\dot{q}) = U_i^* sign(u_{0i}(t,q,\dot{q})) \tag{16}$$

where $U_i^* = \text{const} > 0$. In (Rutkovsky and Zemlyakov, 2003) it is shown that for such a case the MM (14) can be decomposed to a system of nonstationary equations

$$\ddot{q}_i = \rho_i(t) sign(u_{0i}(t,q,\dot{q})) \tag{17}$$

where

$$\rho_i^{\max} \geq \rho_i(t) \geq \rho_i^{min},\ \rho_i^{\max},\ \rho_i^{min} = \text{const} > 0. \tag{18}$$

In (Rutkovsky and Zemlyakov, 2003) it is shown that for every equation in (17) there exists an adaptive suboptimal control in the form

$$u_{0i}(t,q_i,\dot{q}_i) = \varepsilon_i + \frac{(\dot{\varepsilon}_i)^2}{2\rho_i^{\min}} sign(\dot{\varepsilon}_i) \quad (i = \overline{1,6}), \tag{19}$$

where $\varepsilon_i = q_i^{ref}(t) - q_i(t)$.

6. THE MODEL REFERENCE ADAPTIVE CONTROL

From the MM (8) it is evident that a programmed computer aided adaptive control can be presented in the following form

$$u_7(t,q_i,\dot{q}_i) = A_{22}(q)[k_7(q_7^{ref} - q_7) - d_7\dot{q}_7 + L]. \tag{20}$$

Equation (8) together with (20) can be represented by the equation

$$\ddot{q}_7 + d_7\dot{q}_7 + k_7 q_7 = k_7 q_7^{ref}(t) + [f_7(*) + L]. \tag{21}$$

From Eq. (21) it is evident that if the equality

$$[f_7(*) + L] \equiv 0 \tag{22}$$

is valid then the problem for the generalized coordinate q_7 is solved. To solve equality (22) we use the principle of model reference adaptive control (Petrov et al., 1980). Let us take a reference model in the form

$$\ddot{q}_{7m} + d_i\dot{q}_{7m} + k_i q_{7m} = k_i q_7^{ref}(t). \tag{23}$$

From (21) and (23) we receive an equation with respect to the error $\varepsilon_7 = q_7 - q_{7m}$ in the form

$$\ddot{\varepsilon}_7 + d_7\dot{\varepsilon}_7 + k_7\varepsilon_7 = [f_7(*) + L]. \tag{24}$$

Equation (24) can be rewritten in a matrix form

$$\dot{x} = A_7 x + \rho(y),\quad \dot{y} = \psi + \mu(t) \tag{25}$$

where $\varepsilon_7 = x_1$, $\dot{\varepsilon}_7 = x_2$, $f_7(*) + L = y$, $\dot{f}_7(*) = \mu(t)$, $\dot{L} = \psi$, $x^T = (x_1\ x_2)$, $\rho^T(y) = (0\ y)$ and matrix

$$A_7 = \begin{pmatrix} 0 & 1 \\ -k_7 & -d_7 \end{pmatrix}.$$

Now we can choose an algorithm for L purposeful variation from the condition of an asymptotical convergence of system (25) with respect to the movement

$$x \equiv 0,\quad y \equiv 0. \tag{26}$$

For this purpose, we take Lyapunov's function in the form

$$V(x,y) = \kappa x^T P x + y^2$$

where P is a positive definite matrix, $\kappa = \text{const} > 0$. The derivative of $V(x,y)$ with respect to time taking into account system (25) is determined by the equality

$$\dot{V}(x,y) = \kappa x^T Q x + 2y[\sigma + \mu(t) + \psi] \tag{28}$$

where Q is the prescribed negative definite matrix, $\sigma = (p_{21}x_1 + p_{22}x_2)$, p_{jk} are elements of the matrix $P = (p_{jk})$ $(j,k = 1,2)$.

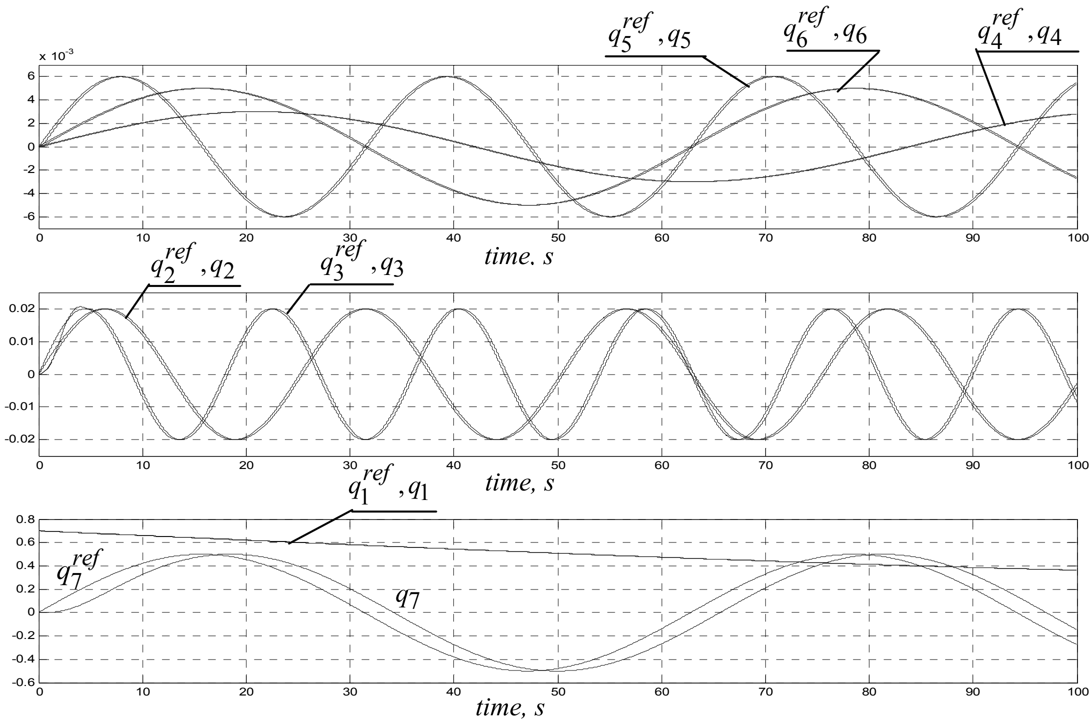

Fig. 1.

In this paper, we suppose that the sign of the coordinate y_i is known. Then we choose the desired algorithm in the form (27)

$$\psi = -\sigma - \bar{k}\, sign(y) \tag{29}$$

where $\bar{k} > 0$ and

$$\bar{k} > |\mu(t)|. \tag{30}$$

Then we have inequalities

$$V(x, y) > 0, \quad \dot{V}(x, y) < 0 \tag{31}$$

which ensure the solution of the problem.

7. THE SIMULATION RESULTS

Let for the SRM under consideration the prescribed functions be the following: $q_1^{ref}(t) = 0.7 - A_1 \sin(\omega_1 t)$; $q_i^{ref}(t) = A_i \sin(\omega_i t)\ (i = \overline{2,7})$, $A_1 = 0,4$; $A_2 = A_3 = 0,02$; $A_4 = 0,006$; $A_5 = 0,005$; $A_6 = 0,003$; $A_7 = 0,5$; $\omega_2 = 0,25 s^{-1}$; $\omega_3 = 0,35 s^{-1}$; $\omega_1 = 0,01 s^{-1}$; $\omega_4 = 0,2 s^{-1}$; $\omega_5 = \omega_7 = 0,1 s^{-1}$; $\omega_6 = 0,075 s^{-1}$. It is necessary to reproduce these motions by the generalized coordinates with the precise accuracy that is determined by (5) with the coefficients $k_i = 25$; $d_i = 7\ (i = \overline{1,6})$; $k_7 = 0,25$; $d_7 = 0,7$.

The matrix Q in (28) we choose as $Q = \begin{pmatrix} -0,05 & 0 \\ 0 & -0,05 \end{pmatrix}$.

Due to (28) the matrix P in (27) is $P = \begin{pmatrix} 0,12 & 0.1 \\ 0.1 & 0,18 \end{pmatrix}$.

For simulation in algorithm (29) was taken $\bar{k} = 1$. In equalities (16) we have $U_1^* = 1,5$; $U_2^* = 1,81$; $U_3^* = 1,71$; $U_4^* = U_5^* = U_6^* = 82$. In algorithms (19) we have

$$\rho_1^{\min} = \rho_2^{\min} = \rho_3^{\min} = 0,005;$$
$$\rho_4^{\min} = \rho_5^{\min} = \rho_6^{\min} = 0,1 .$$

Fig. 1 presents the functions $q_i^{ref}(t)\ (i = \overline{1,7})$ and relative coordinates $q_i(t)\ (i = \overline{1,7})$ of the SRM motion. From Fig. 1 it is clear that the problem that was formulated in this paper is solved.

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