ON DYNAMICS OF A DUMBBELL SATTELLITE WITH A SMALL LOAD ON THE LEIER

Alexander Rodnikov
Bauman Moscow State Technical University
Russia
avrodnikov@yandex.ru

Abstract

We study rotations about mass center of a dumbbell satellite that moves along the circular orbit in the Newtonian Central Force Field. Rotations are forced by a small load coasting along on the cable with ends fixed in the satellite endpoints. (We call this cable 'a leier'). We deduce criterion defining the direction of the dumbbell upturning in the orbit plane if the satellite initially is quasi-tangent to the orbit. Moreover, we study the satellite asymptotic motions consisting of one half-turn as the manifold dividing the system variables space into the areas of different motions

Key words

Space tether system, unilateral constraint, central newtonian field, Lagrangian, perturbed system

1 Introduction

Studying of the cable-connected systems remains one of the most interesting topics in the space dynamics during the last forty years. Hundreds papers concerning different aspects of these systems motion have been published after the pioneer works [Beletsky,Novikova, 1969; Beletsky, 1969].(See for example the bibliography in [Beletsky, levin, 1993; Cosmo, Lorenzini, 1997; Alpatov et all, 2006; Ivanov, Sitarsky, 1986]) In this paper we continue to analyse 'the leier system' that is some generalization of the classical couple of two particles jointed by a cable. Using assumptions and results from [Rodnikov, 2004; Rodnikov, 2006a; Rodnikov, 2006b; Rodnikov, 2006c; Rodnikov, 2008a; Rodnikov, 2008b] we study a dumbbell satellite rotations forced by a small load (or a cabin) moving along the cable with ends placed in the satellite endpoints.

It can easily be checked that the cabin sufficiently influence the dumbbell rotation only if the satellite motion is the sequense of half-turns (rotations on the angles close to flat) beginning in the vicinity of the 'horizontal' (tangent to the orbit) equilibrium. Reducing motion equations near such equilibrium we deduce criterion defining direction of any single half-turn from this sequence. Factually we construct the surface of the dumbbell asymptotic motions tending to librations about the horizontal equilibrium. This surface divide the problem phase space into areas of left-hand and right-hand half-turns.

Moreover, we study numerically the manifold of the satellite asymptotic motion consisting of only one half-turn in two-dimensional transections of the system phase space. This manifold allows to allocate areas of the dumbbell motions of the following four types: rotation on angle close to complete clockwise, similar rotation counter-clockwise, rotation consisting of left half-turn and right half-turn, rotation consisting of right half-turn and left half-turn.

2 Designations, variables, parameters and motion equations

We consider a space station consisting of three particles A_1, A_2, A_3 with masses m_1, m_2, m_3.(figure 1). A_1 and A_2 are jointed by a weightless rod of length $2c$, i.e. these particles compose a dumbbell. The third particle (or a cabin) A_3 can coast along on a cable of length $2a$. The ends of the cable are placed in A_1 and A_2. We call such cable 'a leier'. (This Dutch term means the rope with both fixed ends). We assume the dumbbell mass center C describes a circular orbit about attracting center O_1, $O_1C << a$ and $m_3 << m_1 + m_2$. Restricting to motions in the orbit plane we study the dumbbell rotations about C forced by the cabin. Let φ be an angle between O_1C and A_1A_2.

It is well known that if the cabin doesn't influence the satellite (or $m_3 = 0$) then there exist two types of the dumbbell steady motions in which $\varphi = \mathrm{const}$. There are stable 'vertical' relative equilibria $\varphi = 0, \pi$ and unstable 'horizontal' equilibria $\varphi = \pm\pi/2$ [Beletsky,1966; Beletsky, 1972]. Moreover motion of a dumbbell on the circular orbit about mass center is similar to the motion of the pendulum. So there are rotations about mass center, there are librations about vertical equilibrium and there are asymptotic (or separatrix) motions tending to the horizontal equilibrium [Belet-

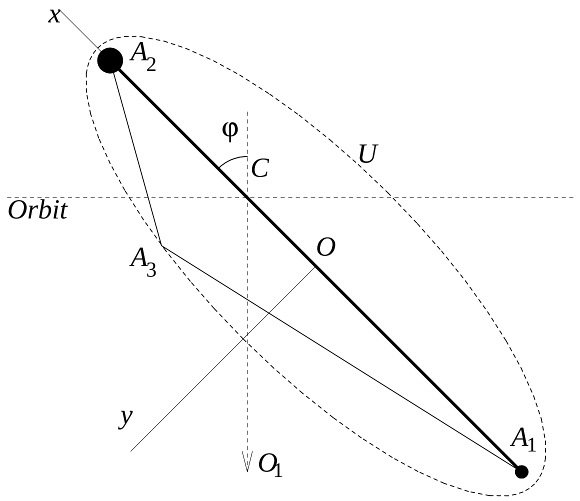


Figure 1.

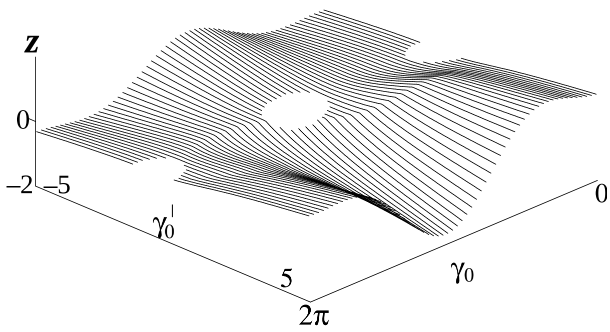


Figure 2.

sky,1966; Beletsky, 1972]. Evidently, the cabin doesn't leave the ellipse U of eccentricity $e = c/a$ with foci in A_1 and A_2. So we have some generalization of the classical space tether system suggested for the first time in [Beletsky,Novikova, 1969; Beletsky, 1969]. (In the classic case a sounder doesn't leave a circle with center in a spacecraft). Let Oxy be a coordinate system with origin in the dumbbell midpoint O such that A_1 and A_2 belong to Ox and $Oy \perp Ox$. Note that the cabin influences the dumbbell rotation only if the cable is tensed and the cabin moves along the ellipse U boundary. In this case the cabin coordinates can be represented by formulae $x_{A_3} = a\cos\gamma$, $y_{A_3} = b\sin\gamma$, $b = \sqrt{a^2 - c^2}$. Here γ is eccentric anomaly of the cabin on the ellipse U.

One of the considered system motion equations has a form

$$\varphi'' + 3/2\sin 2\varphi + \kappa f(\varphi', \varphi, \gamma', \gamma) = 0,$$

where $\kappa = m_3(m_1 + m_2)a^2/(4c^2 m_1 m_2)$. In our case $\kappa << 1$. By $()'$ the derivative w.r.t. dimensionless time $\tau = \omega t$ is designated, here ω is the orbital angular velocity.

Hence the cabin sufficiently influence the dumbbell rotation only for the motions in the separatrix $\varphi' = \pm\sqrt{3}\cos\varphi$ vicinity with radius of order $\sqrt{\kappa}$. In this case we have a sequence of left-hand and right-hand half-turns of the satellite.

3 Determination of a single half-turn direction

Direction of each half-turn can be determined by the following criterion. Let $\varphi_0', \varphi_0, \gamma_0', \gamma_0$ be values of $\varphi', \varphi, \gamma', \gamma$ in the beginning of the considered half-turn. Evidently, φ_0' is close to zero and φ_0 is close to $\pm\pi/2$. (Without loss of generality we can assume $\varphi_0 = -\pi/2 + \sqrt{\kappa}\psi_0$). Note that the dumbbell remains in the vicinity of the horizontal equilibrium for some values of $\varphi_0', \varphi_0, \gamma_0', \gamma_0$. (Factually, in this case an asymptotic motion tending to librations about horizontal steady motion takes place.) These values compose a surface that can be represented by formula

$$z = A(\gamma_0', \gamma_0) + O(\gamma), \qquad (1)$$

where $z = \kappa^{-1/2}(\sqrt{3}\psi_0 + \psi_0') = \kappa^{-1}(\sqrt{3}\varphi_0 + \sqrt{3}\pi/2 + \varphi_0')$ and $O(\kappa) \sim \kappa$ as $\kappa \to 0$ (The surface (1) example for $e = 1/3$ and $m_2 = 2m_1$ is depicted in figure 2, where the holes correspond to motions with un-tense cable). This surface divides the space of $\varphi_0', \varphi_0, \gamma_0', \gamma_0$ into areas of up-turnings counter-clockwise ($z > A$) and of up-turnings clockwise ($z < A$).

$A(\gamma_0', \gamma_0)$ can be expressed through definite integrals. For instance, if $h = (1 - e^2\cos^2\gamma_0)\gamma_0'^2 - 3(1 - e^2)\sin^2\gamma_0 < 0$, then

$$A = \frac{\exp\left(-\sqrt{3}(T/2 \pm t_1)\right)}{1 - \exp(-\sqrt{3}T)}\cdot$$

$$\cdot \int_{\gamma_2}^{\gamma_1} \left[f_-(y)\exp\left(\sqrt{3}W(\gamma_2, y)\right) \pm\right.$$

$$\left.\pm f_+(y)\exp\left(-\sqrt{3}W(\gamma_2, y)\right)\right] dy \pm$$

$$\pm \int_{\gamma_0}^{\gamma_1} f_\pm(y)\exp\left(\pm\sqrt{3}W(y, \gamma_0)\right) dy,$$

where $W(x, y) = \int_x^y V(\xi)d\xi$, $T = 2W(\gamma_2, \gamma_1)$, $t_1 = W(\gamma_0, \gamma_1)$, $\gamma_2 = \arccos H + \pi k$, $\gamma_1 = \pi(k+1) - \arccos H$,

$$H = \sqrt{1 + \frac{h}{3(1 - e^2)}},$$

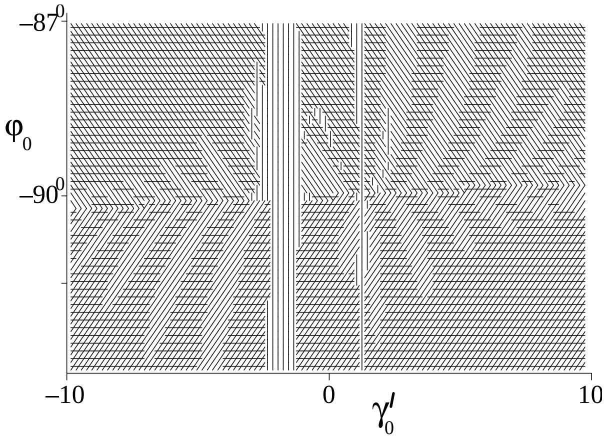


Figure 3.

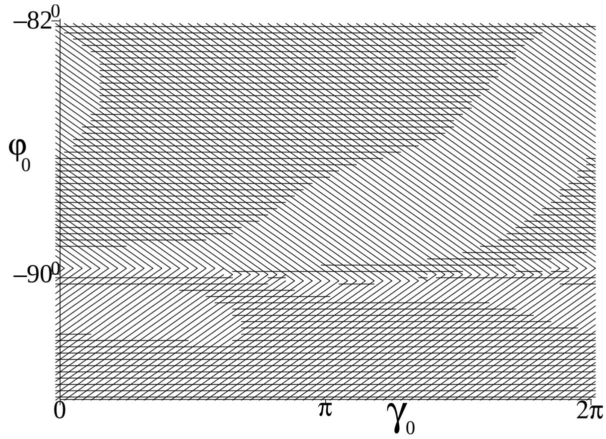


Figure 4.

$$V(\gamma) = \sqrt{\frac{1 - e^2\cos^2\gamma}{h + 3(1-e^2)\sin^2\gamma}},$$

$$f_{\pm}(y) = e\sin y(\mu - e\cos y)\cdot$$

$$\cdot\left(\frac{\sqrt{1-e^2}}{1-e^2\cos^2\gamma}\left(\frac{1}{V(y)} + 3V(y)\sin^2\gamma\right) \pm 2\right).$$

Here '$+$' must be chosen if $\gamma_0' > 0$ and '$-$' must be chosen if $\gamma_0' < 0$. $k = 0$ if $0 < \gamma_0 < \pi$ and $k = 1$ if $\pi < \gamma_0 < 2\pi$.

4 Examples of numerical studying for the two first half-turns

Considered model of the dumbbell motion isn't intended for a long time intervals. So we can study only a few first half-turns. Consider only two first half-turns. There are the following four types of such motions:

a) one turn clockwise on an angle close to complete (or two right-hand half-turns),

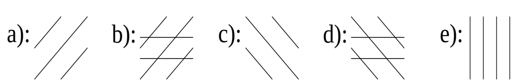
b) the first half-turn clockwise and the second half-turn counterclockwise,

c) one turn counterclockwise on an angle close to complete(or two left-hand half-turns),

d) the first half-turn counterclockwise and the second half-turn clockwise.

Initial points for motions of each type form the area in the space of $\varphi_0', \varphi_0, \gamma_0', \gamma_0$. Examples for $\kappa = .01$, $e = 1/3$ and $m_2 = 2m_1$ of this space transections by two-dimensional planes are depicted in figures 3-8 (planes $\gamma_0 = 12\pi/7, \varphi_0' = 0$ for figure 3, $\gamma_0' = 3, \varphi_0' = 0$ for figure 4, $\varphi_0 = -\pi/2, \varphi_0' = 0$ for figure 5, $\gamma_0 = -\pi/2, \gamma_0' = 0$ for figure 6, $\gamma_0 = \pi/2, \varphi_0 = -\pi/2$ for figure 7, $\gamma_0' = 0, \varphi_0 = -\pi/2$ for figure 8).

In these figures we use designations



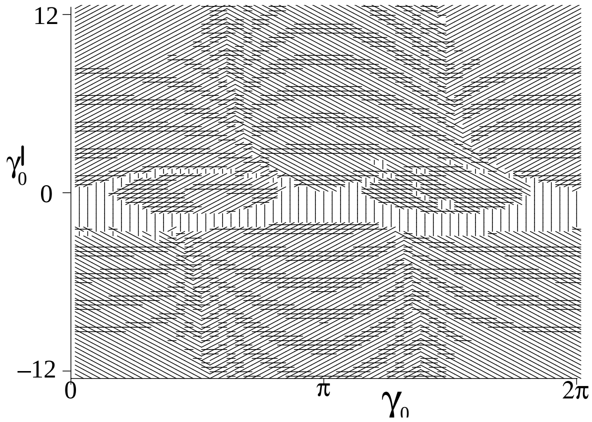


Figure 5.

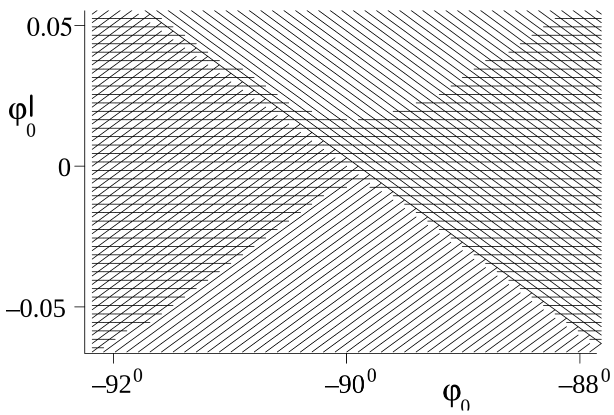


Figure 6.

(Areas e) correspond to the cabin motions inside ellipse U). Note that bounds between a)-b) and c)-d) areas are a trace of the surface (1). (In this case the dumbbell 'does not know the direction of rotation' and remains 'quasi-horizontal') The bound between a) and b) areas corresponds to the dumbbell motions consisting of only one right-hand half-turn (Factually, in this

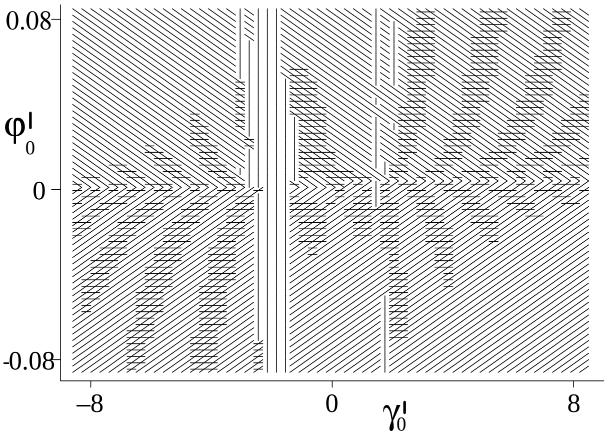


Figure 7.

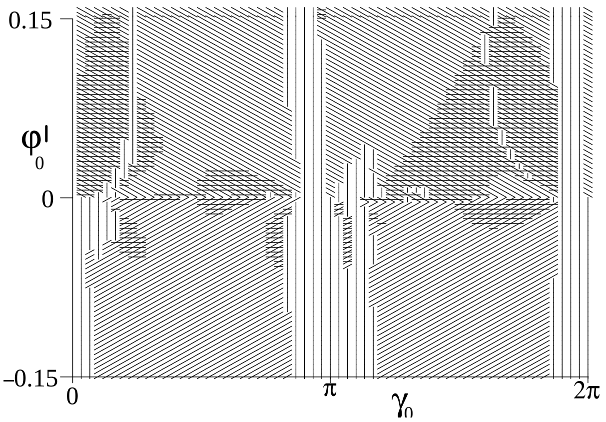


Figure 8.

case the dumbbell upturn clockwise and then tends to librations about the 'horizontal' equilibrium). Similarly, the bound between c) and d) areas corresponds to the dumbbell motion consisting of only one left-hand half-turn.

Analyzing transections we can say that if $|\varphi_0'|$ or $|\varphi_0|$ rise then 'the slice structure' (fig.3) decreases, that 'the petals' (figures 4,5) is bounded, that the dumbbell rotates for sufficiently big $|\varphi_0'|$ or $|\gamma_0'|$ and the dumbbell librations take place for sufficiently big $|\phi_0|$, 'petals' in figures 3,7 decrease for $e \to 0$.

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5 Conclusion

In this paper the dumbbell satellite rotations forced by a small cabin on the leier are considered. The criterion for the direction of the dumbbell rotation from the vicinity of the 'horizontal' equilibrium is deduced. Asymptotic dumbbell rotations consisting of one half-turn are studied numerically as boundaries between the areas of the satellite motion of four different types.

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References

Beletsky, V. V. (1966). *Motion of an Artificial Satellite-about its Center of Mass.* Israel Program for Scientific Translation. Jerusalem.

Beletsky, V. V. and Novikova, E.T. (1969). On the Relative Motion of Two Tethered Bodies on an Orbit. *CosmicReseach*, v. 7,No 3, pp. 377-384.

Beletsky, V. V. (1969). On the Relative Motion of Two Tethered Bodies on an Orbit II. *Cosmic Reseach*, v. 7, No 6, pp.

Beletsky, V. V. (1972). *Ocherki o dvizhenii kosmicheskih tel (Space Bodies Motion).* Nauka. Moscow (in Russian).

Ivanov V.A., Sitarsky Yu.S. (1986) *Dinamika polyota sistemy gibko svyazannykh kosmicheskih ob'ektov (Flight Dynamics of flexible-connected space objects).* Mashinostroenie. Moscow.(in russian)

Beletsky V.V., Levin E.M.(1993)*Dynamics of Space tether systems.* AAS Publ., Univelt Inc., San Diego.

Edited by M.L. Cosmo and E.C. Lorenzini.(1997) *Tethers In Space Handbook.* Smithsonian Astrophysical Observatory for NASA Marshall Space Flight Center Grant NAG8-1160 monitored by C.C. Rupp M.L. Cosmo and E.C. Lorenzini, Principal Investigators. 3rd Edition.

Rodnikov, A. V. (2004). Motion of a Load along a Cable Fastened to a Two-Body-Tethered Spacecraft. *Cosmic Reseach*,v. 42,No 4, pp. 427-431.

Alpatov A.P.,Beletsky V.V., Dranovsky V.I., Zakrzhevsky A.E., Pirozhenko A.V., Troger H., Khoroshilov V.S. (2006) *Dinamika kosmicheskih sistem c trosovymi i sharnirnymi soedineniyami (Dynamics of space systems with the cable and the hinge jointings).* RCD. Moscow-Izhevsk (in russian)

Rodnikov, A. V. (2006a). Equilibrium Positions of a Weight on a Cable Fixed to a Dumbbell-Shaped Space Station moving along a circular geocentric orbit. *Cosmic Reseach*,v. 44,No 1, pp. 58-68.

Rodnikov, A. V. (2006b). Existence of Nonimpact Motions along a Rail Tie Fixed to an Extended Spacecraft. *Cosmic Reseach*,v. 44,No 6, pp. 532-539.

Rodnikov, A. V. (2006c). The Algoritms for Capture of the Space Garbage Using 'Leier Constraint'. *Regular and Chaotic Dynamics*,v. 11,No 4, pp. 483-489.

Rodnikov A.V.(2008) On Systems with 'Leier Constraint' in the Central Newtonian Force Field. *Proceedings of ENOC-2008. June 30-July 4, 2008*, Saint Petersburg, Russia. (http://lib.physcon.ru).

Rodnikov A.V.(2008) Rotations of a Dumbbell Equipped with 'the Leier Constraint'.*Journal of Vibroengineering (JVE).* v.10, is. 4, pp. 557-561