ADAPTIVE TRACKING OF MULTISINUSOIDAL SIGNAL FOR LINEAR SYSTEM WITH INPUT DELAY AND EXTERNAL DISTURBANCES

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Abstract

The problem of adaptive tracking of the output variable of a linear stationary plant behind a multiharmonic signal under conditions of control delay and external disturbances is considered. The state vector of the object is not available for direct measurements. The reference and disturbance signal parameters (amplitudes, phases, and harmonic frequencies) are a priori unknown, and a new algorithm is proposed to improve the performance of frequency estimation of a multisinusoidal signal. Examples are given that confirm the relevance of the proposed approach.

Key words

Adaptive tracking, identification algorithms, delay, frequency, multisinusoidal signal, regressor.

1 Introduction

In this paper, we propose an algorithm for tracking a multiharmonic signal for a linear stable system with an input delay and in the presence of external disturbances.

The problem of adaptive tracking of a reference signal, or the problem of a servomechanism, has been widely studied in recent decades. The first results date back to the 1970s. Some of these solutions were based on the principle of an internal model, which involves the use of an autonomous dynamic model excited by initial conditions, the output of which is used to simulate the driving force or disturbance signal. To achieve the required goal - to ensure zero steady-state tracking error or complete compensation of external disturbances - this model is built in a certain way into the control circuit.

To date, a considerable number of algorithms for tracking a multisinusoidal signal have been developed. The use of the identification approach for tracking a multisinusoidal signal was implemented for linear systems [Gromov *et al.*, 2016], [Borisov *et al.*, 2017].In [Gerasimov *et al.*, 2019a], an algorithm for tracking a multisinusoidal signal by a linear multichannel plant is presented. The method of direct adaptive control [Gerasimov *et al.*, 2019b] based on the principle of an internal model can be used to compensate for an external disturbing action with a delay.

In the automatic control theory, a system with delay is an important and urgent problem, the task of controlling which has always attracted the attention of many researchers [Olbrot, 1978], [Manitius and Olbrot , 197)], [Anderson and Spong, 1989], [Vlasov *et al.*, 2019], [Krstic, 2009], [Krstic and Smyshlyaev, 2008], [Bresch-Pietri and Krstic, 2009]. The selection of objects with a delay in a separate class is first of all the complexity of their study in comparison with objects that do not contain a time delay. A characteristic feature of control systems for objects with delay is the dependence of the state of the controlled process on the history and neglecting the influence of delay leads to a deterioration in the quality of the system.

Compensation an external unknown disturbance is one of the fundamental and actual problems in the theory of automatic control. The study of disturbed systems with delay in the control channel is very important for the wide-common practical application and implementation of such systems in various fields.

Compensation of disturbances can be carried out using methods based on the organization of sliding modes [Utkin, 1992], [Utkin, 1978], [Bandyopadhyay *et al.*, 2013]. In the work [Gerasimov *et al.*, 2016], a disturbance compensation method is proposed using the internal model method. In this case, the external disturbance is considered as the output of an autonomous dynamic model (disturbance generator). To compensate for such a disturbance, the structure of the disturbance generator transient processes of the object in the process of functioning, as should be appropriately reproduced in control algorithm, which explains the name of this method. The use of the identification approach to compensate for polyharmonic disturbances has been successfully implemented for linear [Pyrkin *et al.*, 2014], [Pyrkin and Bobtsov, 2015], [Vlasov *et al.*, 2018] and nonlinear systems [Bobtsov *et al.*, 2011]. In [Wang *et al.*, 2015], an algorithm for controlling the output of a linear multichannel plant is presented.

Developing the results of [Gromov *et al.*, 2016],[Borisov *et al.*, 2017], this article considers the problem of tracking a multisinusoidal signal under conditions of input delay and external disturbances, and a new algorithm is proposed to improve the performance of frequency estimation of a multisinusoidal signal.

The rest of this paper is organized as follows. The problem statement is described in Section 2. The frequency estimation algorithm of the disturbance signal is developed in Section 3. In Section 4, a predictive compensation algorithm of a multisine reference signal is given. In section 5, the computer simulation results of the proposed algorithms are included to confirm the efficiency of the approach. The conclusion is given in Section 6.

2 Problem statement

Consider the LTI system

$$\dot{x}(t) = Ax(t) + Bu\left(t-\tau\right) + D\delta\left(t\right), \qquad (1)$$

$$y(t) = C^T x(t), \qquad (2)$$

$$e\left(t\right) = g\left(t\right) - y\left(t\right). \qquad (3)$$

where $x \in R^n$ is an unmeasured state vector; $u \in R^q$ is the control signal; y is the controlled variable; $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{q\times n}$ are the matrices of corresponding dimensions; τ is a constant known delay, $\delta \in \mathbb{R}^q$ is an unmeasured bounded disturbance, e is the tracking error.

The disturbance $\delta(t)$ is represented as

$$\delta\left(t\right) = \sum_{j=1}^{l} A_j^\delta \sin\left(\omega_j^\delta t + \varphi_j^\delta\right), \qquad (4)$$

and is the sum of l sinusoids $\delta_j\left(t\right)$ with unknown amplitudes A_j^δ, frequencies ω_j^δ and phases φ_j^δ.

The reference signal $g(t)$ is represented as a biased multisinusoidal signal of the form

$$g\left(t\right) = \sum_{m=1}^{k} A_m^g \sin\left(\omega_m^g t + \varphi_m^g\right) \qquad (5)$$

and is the sum of k sinusoids $g_m\left(t\right)$ with unknown amplitudes A_m^g, frequencies ω_m^g and phases φ_m^g.

The control objective is to design $u(t)$ such that the tracking error $e(t)$ is asymptotically converged to zero

$$\lim_{t\to\infty} \left| e\left(t\right) \right| = 0. \qquad (6)$$

Take into account the following assumptions.

Assumption 1: Parameters A, B, C, D are known.

Assumption 2: The triple of matrices (A, B, C) is completely controllable and observable.

Assumption 3: The number of harmonics l, k is known.

Assumption 4: The known lower frequency limit ω_0 for the driving signal $g(t)$ and the disturbing action $\delta(t)$

$$\omega_j^\delta \geq \omega_0, j = \{1, 2, \ldots, l\},$$

$$\omega_m^g \geq \omega_0, m = \{1, 2, \ldots, k\}.$$

3 Frequency estimation in finite time

Consider a linear observer of an object of the following form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu\left(t-\tau\right), \qquad (7)$$

$$\hat{y}(t) = C^T \hat{x}(t), \qquad (8)$$

$$z\left(t\right) = \hat{y}\left(t\right) + e\left(t\right). \qquad (9)$$

where $\hat{x}$ is the state vector of the observer, $\hat{y}$ is the output signal of the observer, z is the estimate of the reference signal.

Consider the output residual

$$\widetilde{x}\left(t\right) = x\left(t\right) - \hat{x}\left(t\right), \qquad (10)$$

$$\dot{\widetilde{x}}\left(t\right) = A\widetilde{x}\left(t\right) + D\delta(t), \qquad (11)$$

Taking into account (11) and expressions (10), (3), rewrite $z\left(t\right)$ as

$$z\left(t\right) = \hat{y}\left(t\right) + g\left(t\right) - y\left(t\right) = g\left(t\right) - \widetilde{y}\left(t\right),$$

$$z\left(t\right) = -C\widetilde{x}\left(t\right) + g\left(t\right). \qquad (12)$$

From expression (12) we see that, in the signal $z\left(t\right)$, in addition to the reference signal $g\left(t\right)$, there is a perturbation component $y^\delta\left(t\right) = C\widetilde{x}\left(t\right)$. Then the signal $z\left(t\right)$ can be represented as

$$z\left(t\right) = g\left(t\right) - y^\delta\left(t\right) + \varepsilon\left(t\right), \qquad (13)$$

$$z\left(t\right)=\sum_{m=1}^{k}A_{m}^{g}\sin\left(\omega_{m}^{g}t+\varphi_{m}^{g}\right)- \\ -\sum_{i=1}^{n}A_{i}^{\delta}\sin\left(\omega_{i}^{\delta}t+\varphi_{i}^{\delta}\right)+\varepsilon\left(t\right) \quad (14)$$

Rewrite equation (14) as

$$z\left(t\right)=\sum_{i=1}^{n}A_{i}\sin\left(\omega_{i}t+\varphi_{i}\right) \quad (15)$$

where $\varepsilon\left(t\right)$ is an exponentially decaying component, the signal (15) is a multisinusoidal signal with harmonic frequencies ,$i=\{1,2,\ldots,n\}$, $n=l+k$.

As a result, we got the signal $z\left(t\right)$, which will be used further as a carrier of information about the reference signal that the object must follow. The second step is the development of an algorithm for estimating the signal frequency (15).

Consider the signal (15), for $n=1$

$$z\left(\mathrm{t}\right)=A\sin\left(\omega t+\varphi\right). \quad (16)$$

Along with the measured signal $z(t)$, consider delayed signals:

$$z\left(t\right)=\begin{cases}z\left(t-d\right),t\geq d,\\ 0,t<d\end{cases} \quad (17)$$

Proposition 1. The signal (16) is described by the relation [Khac *et al.*, 2022a],[Khac *et al.*, 2022b]

$$2\cos\left(\omega d\right)z\left(t-d\right)-z\left(t-2d\right)=z\left(t\right), \quad (18)$$

With n harmonics.

Consider the problem of constructing a regression model for the general case (15) with n signals.

The general case of a harmonic signal with constant parameters (15) is defined where is the number of signal.

$$z\left(t\right)=\sum_{i=1}^{n}A_{i}\sin\left(\omega_{i}t+\varphi_{i}\right). \quad (19)$$

Signals with multiple delays can be represented by using this delay operator, as

$$\begin{cases}z\left(t-d\right)=\Omega z\left(t\right),\\ z\left(t-2d\right)=\Omega^{2}z\left(t\right),\\ \vdots\\ z\left(t-2nd\right)=\Omega^{2n}z\left(t\right),\end{cases}$$

Rewrite equation (18) as

$$\left(\Omega^{2}-2\Omega+1\right)z\left(t\right)=0. \quad (20)$$

where $c=\cos\left(\omega d\right)$, Ω is the delay operators.

Proposition 2. The following relation holds for any signal $z(t)$ with the sinusoids number n [Khac *et al.*, 2022a]

$$\left[\Omega^{2}-2\Omega c_{1}+1\right]\cdot\ldots\cdot\left[\Omega^{2}-2\Omega c_{n}+1\right]z(t)=0. \quad (21)$$

where $c_{i}=\cos\left(\omega_{i}d\right), i=\overline{1,n}$.

Now we are constructing from (21) the regression model for the general case as

$$\Xi\left(t\right)=\phi^{T}\left(t\right)\Theta_{i}, \quad (22)$$

where $\Xi\in\mathbb{R}^{1}$ - is a dependent function, $\phi=\left[\phi_{1}\ \phi_{2}\ ...\ \phi_{n}\right]\in R^{n}$ - is regressor, $\Theta=\left[\Theta_{1}\ \Theta_{2}\ ...\ \Theta_{n}\right]\in\mathbb{R}^{n}$ - is vector of unknown parameters, or more specifically

$$\left[\Omega^{2}+1\right]^{n}z\left(t\right)=\Theta_{1}\phi_{1}\left(t\right)+\Theta_{2}\phi_{2}\left(t\right)+...+\Theta_{n}\phi_{n}\left(t\right). \quad (23)$$

The $\Xi\left(t\right)$ component is obtained using the Newton binomial

$$\Xi\left(t\right)=\left[\Omega^{2}+1\right]^{n}z(t), \quad (24)$$

The components of the vector of unknown parameters Θ are related to $c_{i}, i=\overline{1,n}$ by Vieta's formulas

$$\begin{cases}\Theta_{1}=c_{1}+c_{2}+\ldots+c_{n},\\ \Theta_{2}=-c_{1}c_{2}-c_{1}c_{3}-\ldots-c_{n-1}c_{n},\\ \vdots\\ \Theta_{n}=(-1)^{n+1}c_{1}c_{2}\cdot\ldots\cdot c_{n}.\end{cases}$$

The components of the $\phi\left(t\right)$ regressor are as follows

$$\begin{cases}\phi_{1}=2\Omega\left[\Omega^{2}+1\right]^{n-1}z\left(t\right),\\ \phi_{2}=2^{2}\Omega^{2}\left[\Omega^{2}+1\right]^{n-2}z\left(t\right),\\ \vdots\\ \phi_{n}=2^{n}\Omega^{n}z\left(t\right).\end{cases}$$

Estimation algorithm.

Parameters estimations of the first order regression model (22) can be obtained using method DREM ([Aranovskiy *et al.*, 2017]).

Applying the delay block $\upsilon_{i}, i=\overline{1,n-1}$ for the known elements of the regression model (22), then for (22) we get

$$\Xi\left(t-\upsilon_{i}\right)=\phi^{T}\left(t-\upsilon_{i}\right)\Theta_{i}. \quad (25)$$

Denote

$$\chi_{e}=\varpi_{e}\Theta_{i}, \quad (26)$$

where $\chi_{e}=\left[\Xi\left(t\right)\ \Xi\left(t-\upsilon_{1}\right)\ ...\ \Xi\left(t-\upsilon_{i}\right)\right]^{T}, \varpi_{e}=\left[\phi^{T}\left(t\right)\ \phi^{T}\left(t-\upsilon_{1}\right)\ ...\ \phi^{T}\left(t-\upsilon_{i}\right)\right]$.

Multiplying (26) by adj$\varpi_e(t)$, gives

$$\chi_i(t) = \Delta(t)\Theta_i, \tag{27}$$

where $\Delta(t) = \det\varpi_e(t) \in \mathbb{R}^1, \chi_i(t) = \mathrm{adj}\varpi_e\chi_e(t) \in \mathbb{R}^n$.

Algorithm for estimating parameters Θ_i can be presented as

$$\dot{\hat{\Theta}}_i(t) = -\kappa_i\Delta(t)\left(\chi_i(t) - \Delta(t)\hat{\Theta}_i\right), \tag{28}$$

where κ_i is any positive number.

To obtain an estimate in finite time, we replace the estimation error $\tilde{\Theta}_i(t)$ by definition with $\Theta_i - \hat{\Theta}_i(t)$

$$\Theta_i - \hat{\Theta}_i(t) = \Theta_i W(t) - \hat{\Theta}_i(0)W(t), \tag{29}$$

where $\dot{W}(t) = -\kappa\Delta^2(t)W(t), W(0) = 1$ or $W(t) = e^{-\kappa\int_0^t \Delta^2(s)ds}$.

Express the value of the parameter $\Theta_i = \hat{\Theta}_i^{ft}(t)$ explicitly from the relation (29)

$$\hat{\Theta}_i^{ft}(t) = \frac{\hat{\Theta}_i(t) - W(t)\hat{\Theta}_i(0)}{1 - W(t)}. \tag{30}$$

Frequency Estimation

To estimate the frequency, use the function $\arccos(.)$ based on the parameter $\hat{\Theta}_i^{ft}(t)$ from (30)

$$\hat{\omega}_i(t) = \frac{1}{d}\arccos\left(\hat{c}_i^{ft}(t)\right). \tag{31}$$

4 Adaptive tracking of multisinusoidal signal

Consider a linear filter

$$\zeta(t) = \frac{\lambda^{2n}}{(p+\lambda)^{2n}}z(t), \tag{32}$$

where $(p+\lambda)^{2n}$ is a Hurwitz polynomial.

From equation (32), $\zeta(t)$ is represented as

$$\zeta(t) = \sum_{i=1}^{l}\zeta_i(t) + \varepsilon_1(t), \tag{33}$$

$\zeta_a(t)$ is a harmonic function of time dependent on frequency ω_i, $\varepsilon_1(t)$ is a function of time decreasing exponentially.

Neglecting the exponentially decaying component $\varepsilon_1(t)$, differentiating (33) $2n$ times, we obtain

$$\begin{cases} \zeta^{(1)}(t) = \dot{\zeta}_1(t) + \dot{\zeta}_2(t) + \cdots + \dot{\zeta}_n(t), \\ \zeta^{(3)}(t) = \theta_1\dot{\zeta}_1(t) + \theta_2\dot{\zeta}_2(t) + \cdots + \theta_n\dot{\zeta}_n(t), \\ \vdots \\ \zeta^{(2n-1)}(t) = \theta_1^{n-1}\dot{\zeta}_1(t) + \theta_2^{n-1}\dot{\zeta}_2(t) + \cdots + \theta_n^{n-1}\dot{\zeta}_n(t), \end{cases} \tag{34}$$

$$\begin{cases} \zeta^{(2)}(t) = \theta_1\zeta_1(t) + \theta_1\zeta_2(t) + \cdots + \theta_l\zeta_n(t), \\ \zeta^{(4)}(t) = \theta_1^2\zeta_1(t) + \theta_2^2\zeta_2(t) + \cdots + \theta_n^2\zeta_n(t), \\ \vdots \\ \zeta^{(2n)}(t) = \theta_1^n\zeta_1(t) + \theta_2^n\zeta_2(t) + \cdots + \theta_n^n\zeta_n(t), \end{cases} \tag{35}$$

Rewrite (34) and (35) in matrix form

$$\begin{bmatrix} \zeta^{(1)}(t) \\ \zeta^{(2)}(t) \\ \vdots \\ \zeta^{(2n-1)}(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \dots & \theta_n \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^{n-1} & \theta_2^{n-1} & \dots & \theta_n^{n-1} \end{bmatrix} \begin{bmatrix} \dot{\zeta}_1(t) \\ \dot{\zeta}_2(t) \\ \vdots \\ \dot{\zeta}_n(t) \end{bmatrix} \tag{36}$$

$$\begin{bmatrix} \zeta^{(2)}(t) \\ \zeta^{(4)}(t) \\ \vdots \\ \zeta^{(2n)}(t) \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_l \\ \theta_1^2 & \theta_2^2 & \dots & \theta_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^n & \theta_2^n & \dots & \theta_n^n \end{bmatrix} \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \vdots \\ \zeta_n(t) \end{bmatrix} \tag{37}$$

From expressions (36) and (37) we have

$$\begin{bmatrix} \dot{\zeta}_1(t) \\ \dot{\zeta}_2(t) \\ \vdots \\ \dot{\zeta}_n(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \dots & \theta_l \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^{n-1} & \theta_2^{n-1} & \dots & \theta_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} \zeta^{(1)}(t) \\ \zeta^{(2)}(t) \\ \vdots \\ \zeta^{(2n-1)}(t) \end{bmatrix} \tag{38}$$

$$\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \vdots \\ \zeta_n(t) \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_l \\ \theta_1^2 & \theta_2^2 & \dots & \theta_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^n & \theta_2^n & \dots & \theta_n^n \end{bmatrix}^{-1} \begin{bmatrix} \zeta^{(2)}(t) \\ \zeta^{(4)}(t) \\ \vdots \\ \zeta^{(2n)}(t) \end{bmatrix} \tag{39}$$

The implemented algorithm for estimating the variables $\zeta_i(t)$ and $\dot{\zeta}_i(t)$ takes

$$\begin{bmatrix} \dot{\hat{\zeta}}_1(t) \\ \dot{\hat{\zeta}}_2(t) \\ \vdots \\ \dot{\hat{\zeta}}_n(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \hat{\theta}_1 & \hat{\theta}_2 & \dots & \hat{\theta}_n \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\theta}_1^{n-1} & \hat{\theta}_2^{n-1} & \dots & \hat{\theta}_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} \zeta^{(1)}(t) \\ \zeta^{(2)}(t) \\ \vdots \\ \zeta^{(2n-1)}(t) \end{bmatrix} \tag{40}$$

$$\begin{bmatrix} \hat{\zeta}_1(t) \\ \hat{\zeta}_2(t) \\ \vdots \\ \hat{\zeta}_n(t) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \dots & \hat{\theta}_n \\ \hat{\theta}_1^2 & \hat{\theta}_2^2 & \dots & \hat{\theta}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\theta}_1^n & \hat{\theta}_2^n & \dots & \hat{\theta}_n^n \end{bmatrix}^{-1} \begin{bmatrix} \zeta^{(2)}(t) \\ \zeta^{(4)}(t) \\ \vdots \\ \zeta^{(2n)}(t) \end{bmatrix} \tag{41}$$

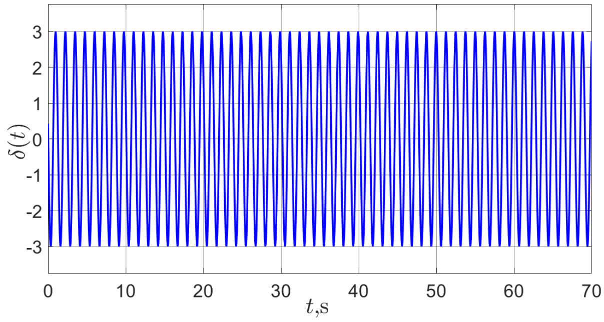

Figure 1. Time diagram of disturbance signal $\delta(t)$

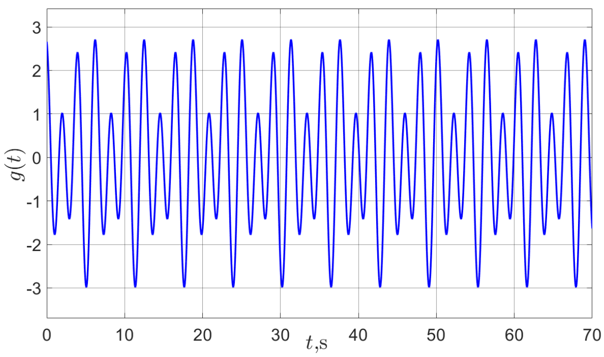

Figure 2. Time diagram of reference signal $\hat{\omega}(t)$

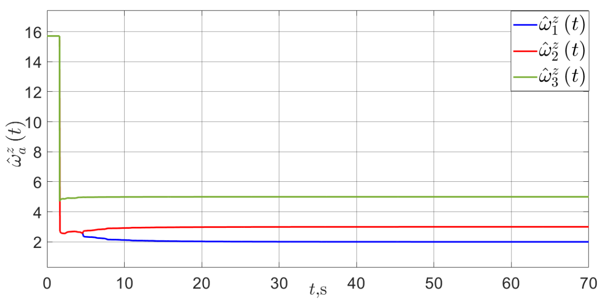

Figure 3. Time diagram of frequency estimate $\hat{\omega}_i(t)$

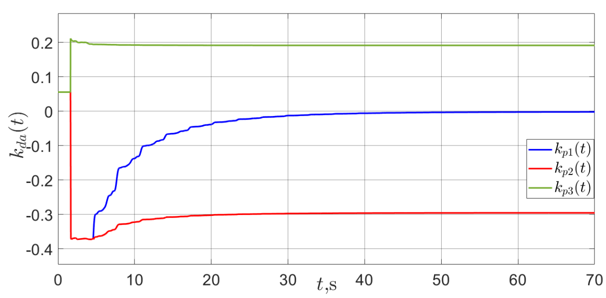

Figure 4. Time diagram of estimate k_{di}

Finally, write the control law as

$$u(t) = -\sum_{i=1}^{n} \frac{1}{\hat{L}_i(t)} \left(k_{pi}(t)\hat{\zeta}_i(t) + k_{di}(t)\widehat{\dot{\zeta}}_i(t) \right) \tag{42}$$

where the proportional and differential gains are given by

$$k_{pi}(t) = \cos(\tau\hat{\omega}_i(t) - \hat{\mu}_i(t)), \tag{43}$$

$$k_{di}(t) = \frac{\sin(\tau\hat{\omega}_i(t) - \hat{\mu}_i(t))}{\eta_i(t)}. \tag{44}$$

Estimates of the transmission coefficients $\hat{L}_i(t)$ and phase shifts $\hat{\mu}_i(t)$ are given by the formulas

$$\hat{L}_i(t) = \left| \frac{b(j\hat{\omega}_i(t))\lambda^{2n}}{a(j\hat{\omega}_i(t))(j\hat{\omega}_i(t) + \lambda)^{2n}} \right|, \tag{45}$$

$$\hat{\mu}_i(t) = arg\frac{b(j\hat{\omega}_i(t))\lambda^{2n}}{a(j\hat{\omega}_i(t))(j\hat{\omega}_n(t) + \lambda)^{2n}}. \tag{46}$$

where $j = \sqrt{-1}$.

5 Simulation

Let us consider the results of numerical simulation illustrating the efficiency of the proposed algorithm for estimating the frequency of an unbiased harmonic signal with constant parameters. The simulation was performed using the MATLAB Simulink software environment.

The control object will be described by differential equations (1), (2), (3) in which A, B, C, D are $A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tau = 1s.$

The perturbing effect has the form

$$\delta(t) = 3\sin(5t + 3).$$

The reference signal $g(t)$ has the form

$$g(t) = sin(2t + 1) + 2sin(3t + 2).$$

Delay parameter for disturbance signal parameterization $d = 0.1s$.

DREM algorithm parameters: $\nu_1 = 0.1, \nu_2 = 0.2, \kappa_i = 10000$.

The time diagram of the perturbation function $\delta(t)$ is shown in Fig.1.

The time diagram of the perturbation function $g(t)$ is shown in Fig.2.

On Fig. 3 shows transients for signal frequency estimation

On Fig. 4 shows transients for signals k_{di}.

On Fig. 5 shows transients for signals k_{pi}.

Figure 6 demonstrates that the tracking error tends to zero asymptotically.

The results of modeling adaptive stabilization processes are shown in Fig. 1 - 6. Analysis of transient modeling results reveals the ability of the composite system to provide an exponential convergence to zero of the tracking error.

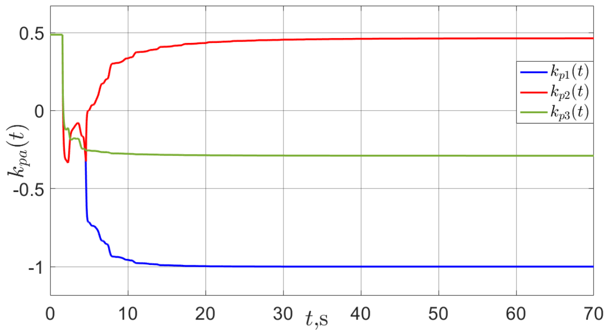

Figure 5. Time diagram of estimate k_{pi}

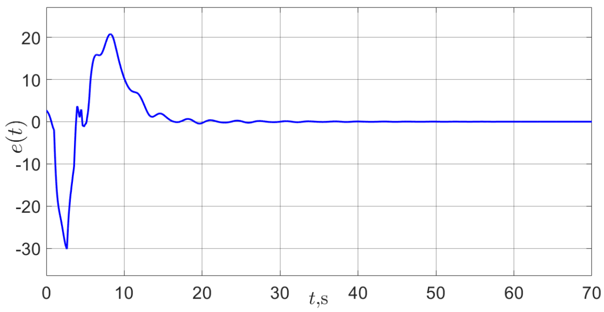

Figure 6. Time diagram of tracking error $e(t)$

6 Conclusion

A solution to the problem of adaptive tracking of the output of a linear plant with a delay in control and in the presence of a perturbation behind a multisinusoidal signal with unknown amplitudes, frequencies and phases of harmonics is presented. The solution of the problem is based on the use of the indirect control method. A new approach is proposed to determine the frequencies of a multisinusoidal signal in finite time, provides rapid convergence to zero of the estimation errors. The proposed idea can be extended to the case of an object with unknown parameters and an unknown delay, which is proposed as a direction for further research by the authors.

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