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DYNAMIC-EXPERIMENTAL EVALUATION OF THE BUCKLING OF CANTILEVERED BAR UNDER GEOMETRIC NONLINEARITY

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Abstract: The matrix stiffness of structures under axial compression loads is different from the conventional matrix stiffness. In slender structures, the stiffness changing due to geometric nonlinearity is one very important point in bulking analysis. For these structures, the stability analysis by dynamic method is usually required. The Rayleigh's method is a simple and easy way to consider the geometric nonlinearity effect on slender cantilevered structure stability analysis.

1 Introduction

Structures as chimneys, high reservoirs and telecommunication poles are examples of structures that have fundamental frequency influenced by the axial load. The axial load modifies the structure stiffness and, consequently, the structure dynamic behavior.

It is important to point out that the stiffness matrix of the structures with elements under the axial loads is different from the conventional stiffness matrix. Moreover, there is an important aspect that appears when considering the geometric nonlinearity. That is, once changed the structure stiffness by the axial load effect, and depending on the loading, the structure can reach the equilibrium trajectory bifurcation point and will buckle. The buckling failure is potentially dangerous and can initiate the collapse of many types of engineering structures. The stability analysis consists of determining the structure stability loss and the corresponding critical load. The structure remains in rest before and after buckling, except in cases when the stability loss is due to transitions from state of rest to motion called kinetic or dynamic instability. In dynamic instability analysis, the critical load is obtained from the equation of motion by means of a non-trivial eigenvalue solution. The stability loss occurs when the structure fundamental frequency of vibration goes to zero.

The dynamic instability investigation is broader than other analyses that reject the inertial forces, such as static analysis. Hence, since the dynamic method takes into account inertial forces in the structure stability analysis formulation, the way the elastic systems mass and stiffness are distributed becomes important, Gambir (2004).

Although in a subtle way, the consideration of the geometric stiffness is highly nonlinear, since the geometric stiffness depends on the internal stresses state in the structure, which is only obtained from the deformations caused by displacements suffered by the structure. Therefore, it is a linearization of a nonlinear problem of a certain configuration that is not the initial unloaded one.

This paper reports an dynamic-experimental research about geometric nonlinear cantilever bars buckling. A mathematical formulation based on the Rayleigh's Methods and a computational processing using the Finite Element Method follow the experimental procedure, allowing numerical checking the compression axial load influence on the structure stiffness changing, which leads to a fundamental frequency of vibration reduction until the limit in stability loss. In addition, the Rayleigh's Method allows the provision of a simple and easy way that can to be applied by professional engineers to consider the geometric nonlinearity effect on the determination of the slender cantilevered structures fundamental frequency of vibration. The Rayleigh's Method allows the provision of a simple and easy way that can to be applied by professional engineers to consider the geometric stiffness effect on the determination of slender cantilevered structures fundamental frequency of vibration.

To this purpose, a set of dynamic laboratory tests with reduced models was carried out. These tests were developed in the Laboratory of Nonlinear Dynamics of Structures of the Polytechnic School of the University of São Paulo.

2 Reyleigh's method

For a first approach of the nonlinearity effect, the structures can perfectly be modeled as a system composed of a cantilever bar supporting a representative body mass representative fixed at its top. Therefore, it is possible to understand these structures behavior by following basic concepts. A bar with length L is considered supporting a mass at its free end and fixed at its base. Once excited with a horizontal force p(t), this system will be subjected to both conservatives and non-conservative forces. When p(t) ceases acting, the system experiences free vibrations and the structure seeks its natural modes of vibration.

The analytical formulation adopted in this work is based on Rayleigh's Method, created by Lord Rayleigh. The basic concept behind the Rayleigh's Method in dynamic analysis is the principle of the energy conservation (Clough, 1993). References to the application of this method by several researchers to mechanical system stability and dynamic analyses can be found in D. Addessi (2005), P.A.A. Laura (2006), Selvakumar (2006), M.E. Biancolini (2005), X. X. Hu (2004), M. Chiba (2003), Y. K. Cheunga (2003).

For the application of that method, the calculation of the cantilever bars natural frequencies will be carried out considering a one-degree-of-freedom mass-spring system with uniform cross-section, for which a trigonometric function is adopted as shape (1), function similar to the first buckling mode. This approximation suggests that the analysis will only be accurate at a neighborhood of the buckling load.

$$\phi(x) = 1 - \cos\left(\frac{\pi x}{2L}\right) \tag{1}$$

The generalized mass M related to top of the bar will be given by

$$M = \int_0^L m(x)\phi(x)^2 + \sum m_i \phi(i) \tag{2}$$

where m(x) is the mass for unit of length and m_i is the lumped mass at one determined position i along the bar. The system generalized elastic stiffness, considering material elastic linearity will be given by

$$K_e = \int_0^L EI(x)\left(\frac{d^2\phi(x)}{dx^2}\right)^2 dx \tag{3}$$

and the generalized geometric stiffness will be given by

$$K_g = \int_0^L N(x)\left(\frac{d\phi(x)}{dx}\right)^2 dx \tag{4}$$

Making $N(x) = F_0 + F(x)$, F_0 e F(x) will be, respectively, an external force applied at the top and the distributed internal axial force. Thus,

$$N(x) = \left[m_0 + m_1\left(L - x\right)\right]g \tag{5}$$

with m_0 representing the lumped mass and m_1 the uniformly distributed mass. The generalized total mass, in this context, is then given by

$$M = m_0 + \frac{1}{2}Lm_1\frac{3\pi - 8}{\pi} \tag{6}$$

The system total generalized stiffness, for configurations at the reference configuration neighborhood, and considering the compression axial force as positive, is given by

$$K = K_e - K_g \tag{7}$$

Computing the previous expressions and simplifying the solution to consider only the degree of freedom relative to the horizontal motion, one can find the stiffness matrix of the model by Rayleigh's Method as

$$K = \frac{\pi^4}{32}\frac{EI}{L^3} - \left[\frac{\pi^2}{16}\left(\frac{2m_0 + m_1 L}{L}\right) + \frac{1}{4}m_1\right]g \tag{8}$$

where *E* is the material modulus of elasticity, *L* is the bar length, *I* is the section moment of inertia around the perpendicular axes to the motion. The first term of the previous equation refers to the elastic stiffness matrix and second term of the same equation refers to the geometric stiffness matrix.

A simple expression for the calculation of the fundamental frequency of vibration under geometric non-linearity, in Hertz, can finally be obtained by square root the stiffness mass division, which can be written as

$$f = \frac{1}{2\pi}\left(\frac{\frac{\pi^4}{32}\frac{EI}{L^3} - \left[\frac{\pi^2}{16}\left(\frac{2m_0 + m_1 L}{L}\right) + \frac{1}{4}m_1\right]g}{m_0 + \frac{1}{2}Lm_1\frac{3\pi - 8}{\pi}}\right)^{\frac{1}{2}} \tag{9}$$

3 Laboratory tests

Two sets of tests in reduced models have been carried out. The first group (*Set 1*) was subjected to a compression force F_0 beyond the self-weight, while the second group (*Set 2*) was submitted exclusively to its self-weight. For the *Set 1*, electric strain gages manufactured by Excel Sensory (120 Ohm resistance and factor 2.1) were used. The adopted set up for the strain gages linking to the acquisition system has been composed of a 1/4 bridge and three wires. The employed accelerometers have been manufactured by Bruel & Kjaer, models 4393 and 4371, with characteristics of: sensitivity - 3.1 pC/g and 10 pC/g, interval of frequency - 0.1 the 16,500 Hz to Hz and 0.1 the 12,600 Hz to Hz. For the Set 2, only the 4371 accelerometer type has been used.

The first test samples have been made of a flat metallic bar with 1/2"(12.7 mm) x 1/8" (3.175 mm) nominal section, to which two masses have been fixed by lateral pressure at the free end. Those masses have been added to the the accelerometers masses and their magnetic bases gives a total value of 1.595 g at the rod top. For the second test, a pipe, with 1/2"(12.7 mm) x 3/8" (3.175 mm) nominal section and 1.2 mm thickness, has been used. The material longitudinal modulus of elasticity "E" has been assumed to be 205 GPa for both tests. The connecting rod material density has been determined by experiments. The obtained density is 8190 kg/m^3, while the known density value of 7850 kg/m^3 has been assumed for the other components.

The metallic masses added to the rod simulate the compression axial load action at the system stiffness. Thus, the rod-mass system can be compressed by the bar self-weight and by the vertical load produced by the mass at the top. The apparatus has been fixed to the support in order to keep the same standard fixing procedure for all models. After being fixed, the support devices as well as the models have been horizontally leveled by a bubble level. The experimental reference length has been visually controlled and its determination has been done by a metallic metric ribbon. The length has varied by 5 cm steps. A random force, with magnitude necessary and enough to put the system in motion, has excited the models with several lengths. After the excitation, the systems oscillated about the deformed initial equilibrium position. It is interesting to mention that in the tests the models with longer length of the set 1 have presented a static equilibrium configuration rather than the expected deformed configuration with respect to the bar axis original straight position. This phenomenon has indicated that the beam has buckled, and the oscillations have occurred about the buckled position.

The time histories have been recorded for further analyses. The analysis of the vibration system has been carried out in the frequency domain using the *Fast Fourier Transform*, provided by the AqDAnalysis 7 software. A *Hanning* type compensation window has been used in the auto-spectra computation, with the maximum allowed resolution for the amount of acquired samples.

4 Modeling by finite elements

It is important to remember that, in the finite element procedure discretization technique, the domain is divided in small, but finite, regions with simple format, joined by nodal points, which has the generalized displacements as problem unknown. And, this is the main difference between the Finite Element Method (FEM) and the Rayleigh's Method previously described in Section 3. In the first method, the interpolation functions are only valid for small regions, while in the second method, the shape function is valid for the entire element.

Two analyses utilizing the FEM have been carried out with the SAP 2000 software. On one hand, the first analysis has been carried out under linear conditions (LFEM). On the other hand, the second analysis has been carried out under nonlinear conditions (NLFEM), considering the so called P-Delta effect on the structure static geometric nonlinearity. The analysis under P-Delta effect has been carried out for comparative purposes because SAP 2000 utilizes a simplified calculation process that exactly takes into account the normal force effect on the system stiffness. This viable technique can capture second order effects because the P-Delta effect can be linearized and the problem solution can be directly and accurately obtained without iterations. This technique is only valid for structures that suffer small lateral displacements as compared to their dimensions. Also, the technique requires that the applied vertical force to the structure due to the structure self-weight remain constant during the structure motion. Moreover, this technique only allows the inclusion of the structure weight in the negative part of the geometric stiffness, discarding the lateral force effect, which is worthless, Cook (2002), Wilson (1987) and Rutemberg (1982). Finally, this technique can easily be programmed in the FEM environment demanding reduced computational efforts.

5 Obtained results

The obtained test results are shown below, as well as the results from the Rayleigh and FEM.

Table 1. Experimental and Rayleigh – Set 1

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	Solution of Rayleigh	Hz	%
0.20	6.3477	6.3276	0.020	0.32
0.25	4.4556	4.4729	-0.017	0.39
0.30	3.2959	3.3520	-0.056	1.67
0.35	2.5024	2.6122	-0.110	4.20
0.40	1.9836	2.0925	-0.109	5.20
0.45	1.6479	1.7096	-0.062	3.61
0.50	1.3428	1.4167	-0.074	5.21
0.55	1.1292	1.1855	-0.056	4.75
0.60	0.9155	0.9983	-0.083	8.29
0.65	0.7935	0.8429	-0.049	5.86
0.70	0.6104	0.7110	-0.101	14.15
0.75	0.4883	0.5965	-0.108	18.14
0.80	0.3662	0.4946	-0.128	25.95
0.85	0.3052	0.4011	-0.096	23.90

Table 2. Experimental and NLFEM – Set 1

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	EFM Nonlinear	Hz	%
0.20	6.3477	6.2810	0.067	1.06
0.25	4.4556	4.4405	0.015	0.34
0.30	3.2959	3.3281	-0.032	0.97
0.35	2.5024	2.5940	-0.092	3.53
0.40	1.9836	2.0783	-0.095	4.56
0.45	1.6479	1.6983	-0.050	2.97
0.50	1.3428	1.4077	-0.065	4.61
0.55	1.1292	1.1783	-0.049	4.17
0.60	0.9155	0.9925	-0.077	7.76
0.65	0.7935	0.8383	-0.045	5.34

0.70	0.6104	0.7073	-0.097	13.70
0.75	0.4883	0.5936	-0.105	17.74
0.80	0.3662	0.4924	-0.126	25.62
0.85	0.3052	0.4011	-0.096	23.90

Table 3. Experimental and LFEM – Set 1

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	EFM Linear	Hz	%
0.20	6.3477	6.3989	-0.051	0.80
0.25	4.4556	4.5733	-0.118	2.57
0.30	3.2959	3.4749	-0.179	5.15
0.35	2.5024	2.7543	-0.252	9.15
0.40	1.9836	2.2517	-0.268	11.90
0.45	1.6479	1.8848	-0.237	12.57
0.50	1.3428	1.6073	-0.265	16.46
0.55	1.1292	1.3915	-0.262	18.85
0.60	0.9155	1.2198	-0.304	24.95
0.65	0.7935	1.0805	-0.287	26.56
0.70	0.6104	0.9657	-0.355	36.79
0.75	0.4883	0.8698	-0.381	43.86
0.80	0.3662	0.7886	-0.422	53.56
0.85	0.3052	0.7192	-0.414	57.56

Table 4. Experimental and Rayleigh – Set 2

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	Solution of Rayleigh	Hz	%
1.00	8.1790	8.1717	0.007	0.09
1.05	7.4770	7.4809	-0.004	0.05
1.10	6.6220	6.8693	-0.247	3.60
1.15	6.3170	6.3259	-0.009	0.14
1.20	5.7680	5.8414	-0.073	1.26
1.25	5.1880	5.4080	-0.220	4.07
1.30	4.8830	5.0191	-0.136	2.71
1.35	4.5170	4.6690	-0.152	3.26
1.40	4.3030	4.3529	-0.050	1.15
1.45	3.9060	4.0666	-0.161	3.95
1.50	3.6930	3.8066	-0.114	2.98
1.55	3.5100	3.5699	-0.060	1.68
1.60	3.2960	3.3537	-0.058	1.72
1.65	3.0820	3.1559	-0.074	2.34
1.70	2.8990	2.9745	-0.075	2.54
1.75	2.7470	2.8076	-0.061	2.16
1.80	2.5940	2.6539	-0.060	2.26
1.85	2.4720	2.5120	-0.040	1.59
1.90	2.2990	2.3806	-0.082	3.43
1.95	2.1970	2.2589	-0.062	2.74
2.00	2.1060	2.1458	-0.040	1.86
2.05	1.9840	2.0406	-0.057	2.77
2.10	1.8920	1.9426	-0.051	2.60
2.15	1.8010	1.8511	-0.050	2.71
2.20	1.7240	1.7655	-0.042	2.35
2.25	1.6480	1.6854	-0.037	2.22
2.30	1.5560	1.6103	-0.054	3.37
2.35	1.4950	1.5398	-0.045	2.91
2.40	1.4340	1.4735	-0.039	2.68
2.45	1.3730	1.4111	-0.038	2.70
2.50	1.3120	1.3522	-0.040	2.97
2.55	1.2510	1.2967	-0.046	3.52
2.60	1.2210	1.2442	-0.023	1.86
2.65	1.1600	1.1945	-0.034	2.89
2.70	1.1290	1.1475	-0.018	1.61
2.75	1.0680	1.1029	-0.035	3.16
2.80	1.0380	1.0605	-0.023	2.13
2.85	0.9920	1.0203	-0.028	2.78
2.90	0.9770	0.9821	-0.005	0.52
2.95	0.9160	0.9457	-0.030	3.14
3.00	0.8850	0.9110	-0.026	2.85
3.05	0.8540	0.8779	-0.024	2.72
3.10	0.8240	0.8463	-0.022	2.64
3.15	0.7930	0.8162	-0.023	2.84
3.20	0.7630	0.7873	-0.024	3.09
3.25	0.7320	0.7597	-0.028	3.64
3.30	0.7170	0.7332	-0.016	2.21
3.35	0.6920	0.7079	-0.016	2.24
3.40	0.6590	0.6835	-0.025	3.59
3.45	0.6410	0.6602	-0.019	2.91
3.50	0.6100	0.6377	-0.028	4.35
3.55	0.5900	0.6162	-0.026	4.25
3.60	0.5700	0.5954	-0.025	4.26
3.65	0.5650	0.5754	-0.010	1.80
3.70	0.5340	0.5561	-0.022	3.97
3.75	0.5190	0.5375	-0.018	3.44
3.80	0.5040	0.5195	-0.016	2.99
3.85	0.4880	0.5022	-0.014	2.83
3.90	0.4730	0.4854	-0.012	2.56

Table 5. Experimental and NLFEM – Set 2

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	EFM Nonlinear	Hz	%
1.00	8.1790	8.2290	-0.050	0.61
1.05	7.4770	7.5029	-0.026	0.34
1.10	6.6220	6.8633	-0.241	3.52
1.15	6.3170	6.2981	0.019	0.30
1.20	5.7680	5.7969	-0.029	0.50
1.25	5.1880	5.3510	-0.163	3.05
1.30	4.8830	4.9528	-0.070	1.41
1.35	4.5170	4.5960	-0.079	1.72
1.40	4.3030	4.2752	0.028	0.65
1.45	3.9060	3.9859	-0.080	2.00
1.50	3.6930	3.7242	-0.031	0.84
1.55	3.5100	3.4867	0.023	0.67
1.60	3.2960	3.2705	0.025	0.78
1.65	3.0820	3.0733	0.009	0.28

1.70	2.8990	2.8928	0.006	0.21
1.75	2.7470	2.7273	0.020	0.72
1.80	2.5940	2.5751	0.019	0.74
1.85	2.4720	2.4348	0.037	1.53
1.90	2.2990	2.3053	-0.006	0.27
1.95	2.1970	2.1854	0.012	0.53
2.00	2.1060	2.0743	0.032	1.53
2.05	1.9840	1.9710	0.013	0.66
2.10	1.8920	1.8756	0.016	0.88
2.15	1.8010	1.7860	0.015	0.84
2.20	1.7240	1.7023	0.022	1.27
2.25	1.6480	1.6240	0.024	1.48
2.30	1.5560	1.5507	0.005	0.34
2.35	1.4950	1.4819	0.013	0.88
2.40	1.4340	1.4173	0.017	1.18
2.45	1.3730	1.3565	0.017	1.22
2.50	1.3120	1.2992	0.013	0.99
2.55	1.2510	1.2451	0.006	0.47
2.60	1.2210	1.1941	0.027	2.25
2.65	1.1600	1.1458	0.014	1.24
2.70	1.1290	1.1001	0.029	2.63
2.75	1.0680	1.0568	0.011	1.06
2.80	1.0380	1.0157	0.022	2.20
2.85	0.9920	0.9766	0.015	1.57
2.90	0.9770	0.9395	0.037	3.99
2.95	0.9160	0.9042	0.012	1.31
3.00	0.8850	0.8718	0.013	1.51
3.05	0.8540	0.8397	0.014	1.70
3.10	0.8240	0.8091	0.015	1.84
3.15	0.7930	0.7799	0.013	1.68
3.20	0.7630	0.7519	0.011	1.48
3.25	0.7320	0.7251	0.007	0.95
3.30	0.7170	0.6995	0.018	2.50
3.35	0.6920	0.6749	0.017	2.53
3.40	0.6590	0.6513	0.008	1.18
3.45	0.6410	0.6287	0.012	1.96
3.50	0.6100	0.6069	0.003	0.51
3.55	0.5900	0.5860	0.004	0.69
3.60	0.5700	0.5658	0.004	0.74
3.65	0.5650	0.5464	0.019	3.41
3.70	0.5340	0.5277	0.006	1.20
3.75	0.5190	0.5096	0.009	1.84
3.80	0.5040	0.4922	0.012	2.41
3.85	0.4880	0.4753	0.013	2.67
3.90	0.4730	0.4590	0.014	3.05

Table 6. Experimental and LFEM – Set 2

Length (m)	Frequencies (Hz)		Differences	
	Experimental results	EFM Nonlinear	Hz	%
1.00	8.1790	8.2290	-0.050	0.61
1.05	7.4770	7.5029	-0.026	0.34
1.10	6.6220	6.8633	-0.241	3.52
1.15	6.3170	6.2981	0.019	0.30
1.20	5.7680	5.7969	-0.029	0.50
1.25	5.1880	5.3510	-0.163	3.05
1.30	4.8830	4.9528	-0.070	1.41
1.35	4.5170	4.5960	-0.079	1.72
1.40	4.3030	4.2752	0.028	0.65
1.45	3.9060	3.9859	-0.080	2.00
1.50	3.6930	3.7242	-0.031	0.84
1.55	3.5100	3.4867	0.023	0.67
1.60	3.2960	3.2705	0.025	0.78
1.65	3.0820	3.0733	0.009	0.28
1.70	2.8990	2.8928	0.006	0.21
1.75	2.7470	2.7273	0.020	0.72
1.80	2.5940	2.5751	0.019	0.74
1.85	2.4720	2.4348	0.037	1.53
1.90	2.2990	2.3053	-0.006	0.27
1.95	2.1970	2.1854	0.012	0.53
2.00	2.1060	2.0743	0.032	1.53
2.05	1.9840	1.9710	0.013	0.66
2.10	1.8920	1.8756	0.016	0.88
2.15	1.8010	1.7860	0.015	0.84
2.20	1.7240	1.7023	0.022	1.27
2.25	1.6480	1.6240	0.024	1.48
2.30	1.5560	1.5507	0.005	0.34
2.35	1.4950	1.4819	0.013	0.88
2.40	1.4340	1.4173	0.017	1.18
2.45	1.3730	1.3565	0.017	1.22
2.50	1.3120	1.2992	0.013	0.99
2.55	1.2510	1.2451	0.006	0.47
2.60	1.2210	1.1941	0.027	2.25
2.65	1.1600	1.1458	0.014	1.24
2.70	1.1290	1.1001	0.029	2.63
2.75	1.0680	1.0568	0.011	1.06
2.80	1.0380	1.0157	0.022	2.20
2.85	0.9920	0.9766	0.015	1.57
2.90	0.9770	0.9395	0.037	3.99
2.95	0.9160	0.9042	0.012	1.31
3.00	0.8850	0.8718	0.013	1.51
3.05	0.8540	0.8397	0.014	1.70
3.10	0.8240	0.8091	0.015	1.84
3.15	0.7930	0.7799	0.013	1.68
3.20	0.7630	0.7519	0.011	1.48
3.25	0.7320	0.7251	0.007	0.95
3.30	0.7170	0.6995	0.018	2.50
3.35	0.6920	0.6749	0.017	2.53
3.40	0.6590	0.6513	0.008	1.18
3.45	0.6410	0.6287	0.012	1.96
3.50	0.6100	0.6069	0.003	0.51
3.55	0.5900	0.5860	0.004	0.69
3.60	0.5700	0.5658	0.004	0.74
3.65	0.5650	0.5464	0.019	3.41
3.70	0.5340	0.5277	0.006	1.20
3.75	0.5190	0.5096	0.009	1.84

3.80	0.5040	0.4922	0.012	2.41
3.85	0.4880	0.4753	0.013	2.67
3.90	0.4730	0.4590	0.014	3.05

6 Conclusions

To study the buckling phenomenon in slender structures under nonlinear effects, taking into account the axial forces influence on the structures natural frequencies, this work has employed a simplified analytical solution banded on the Rayleigh's Method, which has been used as a reference for the FEM application. Two sets of tests in reduced models, have been carried out in the Laboratory of Nonlinear Dynamics of the Polytechnic School of the University of São Paulo.

For the first test, the average difference between the experimental results and the Rayleigh solution is 8.69%. It is important to point out that the models with longer length have presented a static equilibrium configuration rather than the expected deformed configuration with respect to the bar axis original straight position, and have oscillated about his initial configuration, indicating that the beam has buckled It is also important to point out that in a few tests there has been some plastic deformation of the material. Both situations have not been contemplated by the analytical hypothesis. If the experimental results from these cases had been discarded, the average differences between the experimental results and the Rayleigh solution for the compression force and structure self-weight would have been reduced to 2.94%.

Numerical simulations by finite elements of the same models, considering linear and nonlinear behavior under P-Delta effect, have verified that the difference between the linear analysis and the nonlinear analysis results exponentially grows; reaching 44%, with the average differences exceeding 17%. If the comparison had been done between the LFEM solution and the experimental results of frequency would have reached 57%, and the average differences would have been 22.91%. Similar behavior has been observed in the second set of tests, with average differences of 5.02% and 5.04% between the experimental results and the LFEM and NLFEM, respectively. With respect to the Rayleigh's Method, a difference of 2.06% has been observed. This result has revealed a less geometric stiffness effect on the natural frequencies of vibration, due to the fact that the tests have been conducted until the possible length of 3.90m of the structure, with buckling length of 5.80m. The first set of tests has revealed a distinct situation, with a 1.00m expected buckling length, the structure has reached only the length of 0.85m. It is important to remember that both real structures and laboratory models possess tolerable imperfections.

The structure natural frequency can suffer significant alteration if calculated with the inclusion of the negative part of the geometric stiffness due to the compression force. The experimental results have confirmed the nonlinear analytical and numerical straight bar buckling results.

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