GLOBAL STABILITY OF PHASE-LOCKED LOOPS

Nikolay V. Kuznetsov, Gennady A. Leonov, Svetlana S. Seledzhi
Saint-Petersburg State University, Universitetski pr.,Saint-Petersburg , Russia
leonov@math.spbu.ru

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Abstract: Survey of the application of PLL in computer architectures and microprocessors is given. Design of PLL in terms of phase relations is considered. Generalization of Viterbi theorem on the form of phase detector characteristic is made.

1 INTRODUCTION

Phase-locked loops (PLLs) are frequently encountered in radio engineering and communication. Phase-locked loops (PLLs) are frequently encountered in radio engineering and communication. Once they had been invented in the 1930s-1940s (De Bellescize, 1932; Wendt, Fredentall, 1943), intensive studies of the theory and practice of PLLs were carried out (Viterbi, 1966; Gardner, 1966; Lindsey, 1972; Lindsey and Chie, 1981; Leonov, Reitmann and Smirnova, 1992; Leonov, Ponomarenko and Smirnova, 1996; Leonov and Smirnova, 2000; Kroupa, 1973; Kroupa, 2003; Best, 2003, Razavi, 2003; Razavi, 2001; Egan, 2000; Egan, 1998; Abramovitch, 2002).

One of the first applications of phase-locked loop (PLL) is related to the problems of a data transfer by means of a radio signal. In the radio engineering the PLL is applied for a carrier synchronization, carrier recovery, demodulation, frequency division and multiplication.

With the occurrence of the architecture with the chips, operating on different frequencies, the phase-locked loop systems are applied to the generation of the internal frequencies of chips and the generation of frequencies and synchronization of the operating of different devices and a data buses. For example, the modern motherboards of computers contain different devices and data buses operating on different frequencies and often requiring synchronization.

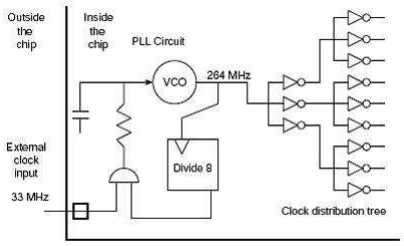

Figure 1: David Greaves. Lecture course. Structured Hardware Design

The actual problem for processors is the problem of energy saving. One of solutions of this problem in practice is a decreasing of kernel frequency with processor load. For example, for the processors of company VIA C7/C7-M it is known that in them the technology VIA TwinTurbo is realized which makes it possible that the processor changes over from the full-load conditions to a power-down mode in a processor tact due to the two blocks of phase-locked loops in processor. Les us consider the applying of a phase-locked loop system for the kernels control in the processors with multiple kernels on the example of the processors of company AMD K8L such that the separate PLL-generators provide the independent clock frequency for kernels. The independent phase-locked loops permit us to distribute more uniformly kernel loading to save the energy and to diminish

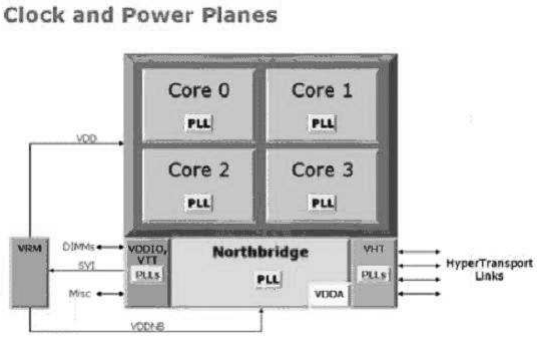

Figure 2: AMD K8L

a heat generation on account of that each kernel operates on its frequency. For comparison, many two-kernel processors, including Intel Core 2 Duo, maximize a frequency of both kernels even if really it is loaded one kernel only. In the processors of Intel Kentsfield are used two phase-locked loops for the four kernels: two kernels can operate on a maximal frequency while two another processors can be rest.

Subsequently the phase-locked loop systems gain sufficiently wide application for the solution of the problems of a clock skew and a synchronization and generation of frequencies for the sets of the chips of computer architectures and micro architectures of chips. For example, clock skew is very important characteristic of processors (see, for example: [Xanthopoulos, 2001; Bindal, 2003]).

In the microprocessors there are often used the symmetrically distributed in the form of H-trees phase-locked loop systems (see, for example, [Martin Saint-Laurent et al., 2001]).

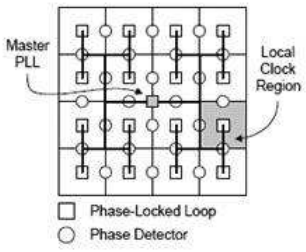

Figure 3: Distributed PLL

In the last ten years, PLLs have widely been used in array processors and other devices of digital information processing (Ugrumov, 2000; Lapsley et al., 1997; Smith, 1999; Solonina et al., 2000; Leonov and Seledzhi, 2002; DSP 56000 (DSP 56 K FAMUM/AD), 1992; Simpson, 1994). For examples, there are such PLLs in the processors DSP 56000 and DSP 56 K (Motorola) (DSP 56000 (DSP 56 K FAMUM/AD), 1992; Simpson, 1994). The PLLs showed their high efficiency as synthesizers of clock rates and as devices correcting a clock skew (Ugrumov, 2000; Lapsley et al., 1997; Smith, 1999; Solonina et al., 2000; Leonov and Seledzhi, 2002; Simpson, 1994). These properties of PLLs determine their specific features and their difference from the standard PLLs used in radio engineering.

The main requirement to PLLs for array processors is that they must be floating in phase. This means that the system must eliminate the clock skew completely. The elimination of the clock skew is one of the most important problems in parallel computing and information processing (as well as in the design of array processors (Kung, 1988).Several approaches to solving the problem of eliminating the clock skew have been devised for the last thirty years.

In developing the design of multiprocessor systems, a way was suggested (Kung, 1988) for joining the processors in the form of an H-tree, in which (Fig. 3,4) the lengths of the paths from the clock to every processor are the same. However, in this case the clock skew is not eliminated

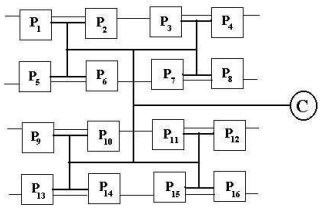

Figure 4: H-tree

completely because of heterogeneity of the wires (Kung, 1988). Moreover, for a great number of processors, the configuration of communication wires is very complicated. This leads to difficult technological problems.

Solving the clock skew problem at a hard- and software level has resulted in the invention of asynchronous communication protocols, which can correct the asynchronism of operations by waiting modes . In other words, the creation of this protocols enables one not to distort the final results by delaying information at some stages of the execution of a parallel algorithm. As an advantage of this approach, we may mention the fact that we need not develop a special complicated hardware support system. Among the disadvantages we note the deceleration of performance of parallel algorithms. In addition to the problem of eliminating the clock skew,

one more important problem arose. The increase in the number of processors in multiprocessor systems required an increase in the power of the clock. But the powerful clock came to produce significant electromagnetic noise. About ten years ago, a new method for eliminating the clock skew and reducing the generator's power was suggested. It consists of introducing a special distributed system of clocks controlled by PLLs. An advantage of this method, in comparison with asynchronous communication protocols, is the lack of special delays in the performance of parallel algorithms. This approach enables one to reduce significantly the power of clocks. Consider the general scheme of a distributed system of oscillators (Fig. 5)

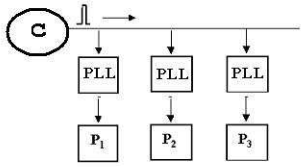

Figure 5: Distributed system of clocks controlled by PLLs

In this paper new type of floating PLL for processors Pk working in parallel is designed.

2 BLOCK DIAGRAM AND MATHEMATICAL MODEL OF PLL

A classical approach to the design of PLL is preliminary consideration of the two signals $f_1(t)$ and $f_2(t)$, passing through multiplier and filter with the transfer function $K(p)$ and pulse transient function $\gamma(t)$ (Fig. 6). It is often considered filter with the transfer function

$$K(p) = \frac{\beta}{p+\alpha}. \qquad (1)$$

Here α and β are positive constants, $g(t)$ is the output of filter.

Consider further the two type of signal $f_1(t)$ and $f_2(t)$:

$$f_j(t) = A_j \sin(\omega_j(t)t + \psi_j) \qquad (2)$$

$$f_j(t) = A_j \mathrm{sign}(\sin(\omega_j(t)t + \psi_j)) \qquad (3)$$

Here A_j,ψ_j are some numbers, $\omega_j(t)$ are differentiable functions, satisfying the following condition.

For any number $\tau \geq 0$ and any positive small number δ the following inequalities

$$|\gamma(t) - \gamma(\tau)| \ll 1, \quad |\omega_j(t) - \omega_j(\tau)| \ll 1, \quad \forall t \in [\tau, \tau+\delta] \qquad (4)$$

$$(\omega_1(\tau) - \omega_2(\tau))\delta \ll 1, \quad \omega_j(\tau)\delta \gg 1, \qquad (5)$$

where $j = 1, 2$, are satisfied.

Conditions (4) and (5) means that on the small intervals $[\tau, \tau+\delta]$ the functions $\omega_j(t)$ and $\gamma(t)$ are "almost constants", and the functions $f_j(t)$ are fast oscillating.

The oscillations of the type (2) under the conditions (4) and (5) turn out to be near the harmonic ones and are representative for many radiotechnology generators (Viterbi, 1966; Lindsey, 1972). The oscillations of the type (3) are representative for the pulse sequences of clock (Ugrumov, 2000; Lapsley et al., 1997; Smith, 1999; Solonina et al., 2000; Leonov and Seledzhi, 2002), centered with respect to zero voltage.

Now we introduce the functions $\theta_j(t) = \omega_j(t)t + \psi_j$, under conditions (4) and (5), which we shall call the phases of oscillations $f_j(t)$.

Consider a block diagram in Fig. 7.

Here PD is a nonlinear block with the characteristic $\varphi(\theta)$, which is called a phase detector. At the inputs of a block of PD there are the phases $\theta_j(t)$, the output is the function $\varphi(\theta_1(t) - \theta_2(t))$. Then the signal $\varphi(\theta_1(t) - \theta_2(t))$ acts on the filter with the transfer function $K(p)$. The function $G(t)$ is an output of filter.

The classical design of PLL is based on the following well-known result (Viterbi, 1966; Lindsey, 1972).

Theorem 1 *If conditions (2), (4), and (5) are valid and* $\varphi(\theta) = \frac{1}{2}A_1A_2\cos\theta$, *then for the same initial states of filter we have*

$$|g(t) - G(t)| \ll 1, \quad \forall t \geq 0, \quad t\delta \ll 1. \qquad (6)$$

Consider now 2π-periodic function $\varphi(\theta)$ of the form

$$\varphi(\theta) = \begin{cases} A_1A_2(1 + 2\theta/\pi) & \text{for} \quad \theta \in [-\pi, 0] \\ A_1A_2(1 - 2\theta/\pi) & \text{for} \quad \theta \in [0, \pi]. \end{cases} \qquad (7)$$

Theorem 2 *If the functions* $f_j(t)$ *have the form (3), the function* $\varphi(\theta)$ *the form (7) and conditions (4), (5) are valid, then for the same initial states of filter we have inequality (6).*

Remark that the block diagram in Fig. 6 can be realized by using standard electronic elements, namely multipliers and filters (Aleksenko, 2004). The block diagram in Fig. 4 is asymptotically (in the sense of relation (6)) equivalent to the block diagram in Fig. 3.

Introducing the equivalent block diagram (Fig. 7) we can consider the control problem in the context of synchronization theory (Lindsey, 1972; Leonov

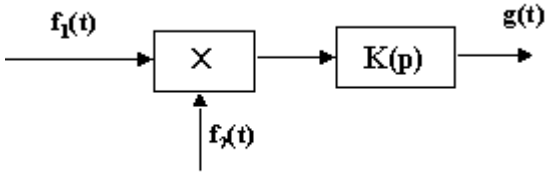

Figure 6: Multiplier and filter

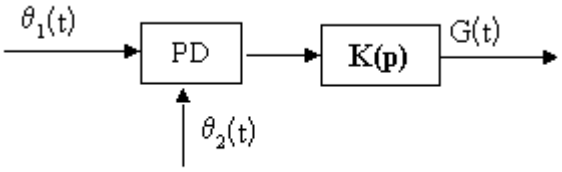

Figure 7: Phase detector and filter

and Seledzhi, 2002), a universal principle of which is a transformation of the difference of phases of two oscillations into the control action on the frequency of slave oscillator.

On the other hand, the block diagram (Fig. 6) is electronic realization of this general principle of synchronization theory.

Using Theorem 2, we can do the design of the block diagram of floating PLL, which plays a role of the function of frequency synthesizer and the function of correction of the clock-skew.

Such a block diagram is shown in Fig. 8.

Here C is a master ocillator, D is a delay, IF is a filter with transfer function (1), SO is a slave oscillator, PD1 and PD2 are programmable dividers of frequencies, P is a processor.

The relay element R plays a role of floating correcting block. The introducing of it allow us to null a residual clock skew, which arises for the nonnull initial difference of frequencies of master and slave oscillators.

Note, the electronic realization of clock and delay can be found in (Ugrumov, 2000) and that of multipliers, filters, and relays in (Aleksenko, 2004). The description of dividers of frequency can be found in (Solonina et al., 2000).

Assume, as usual, that the frequency of master oscillator is constant, namely $\omega_1(t) \equiv \omega_1 = const$. The parameter of delay line T is chosen in such a way that $\omega_1(T+\tau) = 2\pi k + 3\pi/2$. Here k is a certain natural number, $\omega_1\tau$ is a clock skew.

By Theorem 2 and the choice of T the block diagram, shown in Fig. 8, can be changed by the close (in the sense of condition (6)) block diagram, shown

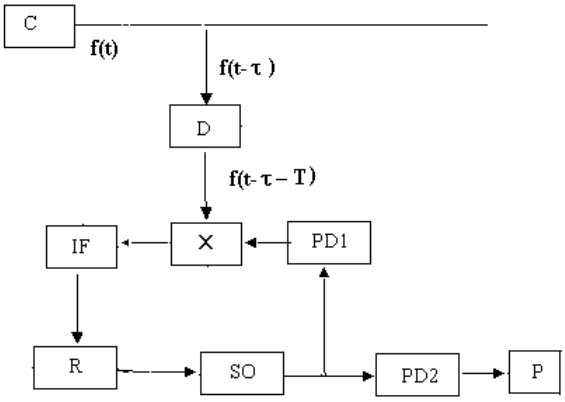

Figure 8: Block diagram of PLL

in Fig. 9.

Here 2π is a periodic characteristic of phase detector. It has the form

$$\varphi(\theta) = \begin{cases} 2A_1A_2\theta/\pi & \text{for} \quad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ 2A_1A_2(1-\theta/\pi) & \text{for} \quad \theta \in [\frac{\pi}{2}, \frac{3\pi}{2}], \end{cases} \tag{8}$$

$\theta_2(t) = \theta_3(t)/M$, $\theta_4(t) = \theta_3(t)/N$, where the natural numbers M and N are the parameters of programmable divisions PD1 and PD2.

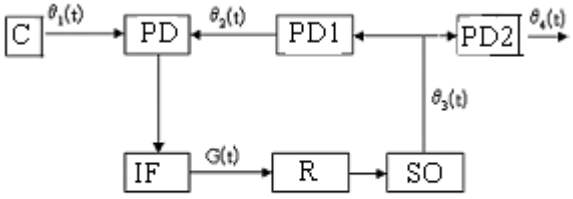

Figure 9: Equivalent block diagram of PLL

For transient process (capture mode) the following conditions

$$\lim_{t\to+\infty}(\theta_4(t) - \frac{M}{N}\theta_1(t)) = \frac{2\pi kM}{N} \tag{9}$$

(phase capture)

$$\lim_{t\to+\infty}(\dot\theta_4(t) - \frac{M}{N}\dot\theta_1(t)) = 0 \tag{10}$$

(frequency capture) must be satisfied.

Relations (9) and (10) are the main requirements of PLL for array processors. Time of transient processors depend on initial data and be sufficiently large for multiprocessors system (see Fig. 2) (Leonov and Seledzhi, 2002; Kung, 1988). Here difference between beginning of transient process and beginning of performance of parallel algorithm (see Fig. 5) can

be some minutes. This difference is very large for electronic systems.

Assuming that the characteristic of relay is of the form $\Psi(G) = \mathrm{sign}G$ and the actuating element of slave oscillator is linear, we have

$$\dot{\theta}_3(t) = R\mathrm{sign}G(t) + \omega_3(0), \qquad (11)$$

where R is a certain number, $\omega_3(0)$ is initial frequency, $\theta_3(t)$ is a phase of slave oscillator.

Taking into account relations (11), (1), (8), and the block diagram in Fig. 9, we have the following differential equations of PLL

$$\begin{aligned} &\dot{G} + \alpha G = \beta\varphi(\theta) \\ &\dot{\theta} = -\tfrac{R}{M}\mathrm{sign}G + (\omega_1 - \tfrac{\omega_3(0)}{M}). \end{aligned} \qquad (12)$$

Here $\theta(t) = \theta_1(t) - \theta_2(t)$.

3 CRITERION OF GLOBAL STABILITY OF PLL

System (12) can be written as

$$\begin{aligned} &\dot{G} = -\alpha G + \beta\varphi(\theta) \\ &\dot{\theta} = -F(G), \end{aligned} \qquad (13)$$

where

$$F(G) = \frac{R}{M}\mathrm{sign}G - (\omega_1 - \frac{\omega_3(0)}{M}).$$

Theorem 3 *If the inequality*

$$|R| > |M\omega_1 - \omega_3(0)| \qquad (14)$$

is valid, then any solution of system (13) as $t \to +\infty$ tends to a certain state of equilibrium.

If the inequality holds

$$|R| < |M\omega_1 - \omega_3(0)|, \qquad (15)$$

then all the solutions of system (13) tends to infinity as $t \to +\infty$.

Consider the states of equilibrium for system (13). For any equilibrium we have

$$\dot{\theta}(t) \equiv 0, \quad G(t) \equiv 0, \quad \theta(t) \equiv \pi k.$$

Theorem 4 *Let relation (14) be valid. In this case, if $R > 0$, then the following equilibria*

$$G(t) \equiv 0, \quad \theta(t) \equiv 2k\pi \qquad (16)$$

are locally asymptotically stable and the following equilibria

$$G(t) \equiv 0, \quad \theta(t) \equiv (2k+1)\pi \qquad (17)$$

are locally unstable. If $R < 0$, then equilibria (17) are locally asymptotically stable and equilibria (16) are locally unstable.

Thus, for relations (9) and (10) to be satisfied it is necessary to choice the parameters of system in such a way that the inequality holds

$$R > |M\omega_1 - \omega_3(0)|. \qquad (18)$$

4 PROOFS OF THEOREMS

Proof Theorem 2. It is well know that, for a filter with an impulse transition function $\gamma(t)$, input $\varepsilon(t)$, output $\sigma(t)$, and eigenoscillation $\alpha(t)$, the following relation holds:

$$\sigma(t) = \alpha(t) + \int_0^t \gamma(t-s)\xi(s)\,ds.$$

Therefore, the formula

$$\begin{aligned} &g(t) - G(t) = \int_0^t \gamma(t-s)[A_1A_2\mathrm{sign}[\sin(\omega_1(s)s + \\ &+\psi_1)\sin(\omega_2(s)s + \psi_2)] - \\ &-\varphi(\omega_1(s)s - \omega_2(s)s + \psi_1 - \psi_2)]\,ds \end{aligned}$$

is valid.

Partitioning the interval $[0,t]$ into intervals $[k\delta,(k+1)\delta]$ and using assumptions (4) and (5), we replase the above integral with the following sum:

$$\begin{aligned} &\sum_{k=0}^{m} \gamma(t-k\delta)[\int_{k\delta}^{(k+1)\delta} A_1A_2\mathrm{sign}[\cos((\omega_1(k\delta) - \\ &\omega_2(k\delta))k\delta + \psi_1 - \psi_2) - \cos((\omega_1(k\delta) + \\ &+\omega_2(k\delta))s + \psi_1 + \psi_2)]\,ds - \varphi((\omega_1(k\delta) - \\ &-\omega_2(k\delta))k\delta + \psi_1 - \psi_2)\delta]. \end{aligned}$$

The number m is chosen in such a way that $t \in [m\delta,(m+1)\delta]$. Since $(\omega_1(k\delta) + \omega_2(k\delta))\delta \gg 1$ the relation

$$\begin{aligned} &\int_{k\delta}^{(k+1)\delta} A_1A_2\mathrm{sign}[\cos((\omega_1(k\delta) - \omega_2(k\delta))k\delta + \psi_1 - \psi_2) - \\ &-\cos((\omega_1(k\delta) + \omega_2(k\delta))s + \psi_1 + \psi_2)]\,ds \approx \\ &\approx \varphi((\omega_1(k\delta) - \omega_2(k\delta))k\delta + \psi_1 - \psi_2)\delta, \end{aligned} \qquad (19)$$

holds. Here, we used the relation

$$A_1A_2 \int_{k\delta}^{(k+1)\delta} \mathrm{sign}[\cos\alpha - \cos(\omega s + \psi_0)]\,ds \approx \varphi(\alpha)\delta$$

for $\omega\delta \gg 1$, $\alpha \in [-\pi,\pi]$, $\psi_0 \in R^1$.

Formula (19) implies inequality (6).

To prove Theorem 3, we formulate an extension of the Barbashin–Krasovskii theorem to dynamical

systems with a cylindrical phase space. Consider the differential inclusion

$$\frac{dx}{dt} \in f(x),\ x \in R^n,\ t \in R^1, \qquad (20)$$

where $f(x)$ is a semicontinuous vector function whose values are bounded closed convex sets $f(x) \subset R^n$. Here, R^n is an n-dimensional Euclidean space. Recall the basic definitions of the theory of differential inclusions.

Definition 1 *We say that $U_\varepsilon(\Omega)$ is an ε-neighborhood of set Ω if*

$$U_\varepsilon(\Omega) = \{x | \inf_{y\in\Omega} |x-y| < \varepsilon\}$$

where $|\cdot|$ is the Euclidean norm in R^n.

Definition 2 *A function $f(x)$ is called semicontinuous at a point x if, for any $\varepsilon > 0$, there exists a number $\delta(x,\varepsilon) > 0$ such that the following containment holds:*

$$f(y) \in U_\varepsilon(f(x)),\ \forall y \in U_\delta(x).$$

Definition 3 *A vector function $x(t)$ is called a solution of the differential inclusion if it is absolutely continuous and, for the values of t at which the derivative $\dot{x}(t)$ exists, the inclusion*

$$\dot{x}(t) \in f(x(t))$$

holds.

Under the above assumptions on the function $f(x)$, the theorem on the existence and continuability of solution of the differential inclusion (20) holds (Yakubovich et al., 2004). Now, assume that linearly independent vectors $d_1,\dots,d_m$ satisfy the following relations:

$$f(x+d_j) = f(x),\ \forall x \in R^n. \qquad (21)$$

Usually, $d_j^* x$ is called the phase or angular coordinate of system (20). Since property (21) allows us to introduce the cylindrical phase space (Yakubovich et al., 2004), system (20) with property (21) is often called a system with cylindrical phase space.

The following theorem is an extension of the well–known Barbashin–Krasovskii theorem to differential inclusions with a cylindrical phase space.

Theorem 5 *Suppose that there exists a continuous function $V(x) : R^n \to R^1$ such that the following conditions hold:*

1) $V(x+d_j) = V(x),\ \forall x \in R^n,\ \forall j = 1,\dots,m$;

2) $V(x) + \sum_{j=1}^{m} (d_j^* x)^2 \to \infty$ as $|x| \to \infty$;

3) for any solution $x(t)$ of inclusion (20) the function $V(x(t))$ is nonincreasing;

4) if $V(x(t)) \equiv V(x(0))$, then $x(t)$ is an equilibrium state.

Then, any solution to inclusion (20) tends to the stationary set as $t \to +\infty$.

Recall that the tendency of the solution to the stationary set Λ as t means that

$$\lim_{t\to+\infty} \inf_{z\in\Lambda} |z - x(t)| = 0.$$

A proof of Theorem 5 can be found in (Yakubovich et al., 2004).

Proofs Theorems 3 and 4. Let $R > |M\omega_1 - \omega_2(0)|$. Consider the Lyapunov function

$$V(G,\theta) = \int_0^G \Phi(u)du + \beta \int_0^\theta \varphi(u)du,$$

where $\Phi(G)$ is a single-valued function coinciding with $F(G)$ for $G \neq 0$. At $G = 0$,function $\Phi(G)$ can be defined arbitrary. At points t such that $G(t) \neq 0$, we have

$$\frac{dV(G(t),\theta(t))}{dt} = -\alpha G(t) F(G(t)). \qquad (22)$$

Note that, for $G(t) = 0$, the first equation of system (12) implies

$$\dot{G}(t) \neq 0 \text{ for } \theta(t) \neq k\pi.$$

This implies that there are no sliding solutions of system (12). Then, relation (22) and the inequality $F(G)G > 0,\ \forall G \neq 0$, imply that conditions (3) and (4) of Theorem 5 are satisfied. Moreover, $V(G,\theta+2\pi) \equiv V(G,\theta)$ and $V(G,\theta) \to +\infty$ as $G \to +\infty$. Therefore , conditions (1) and (2) of Theorem 5 are satisfied. Hence, any solution of system (12) tends to the stationary set as $t \to +\infty$. Since the stationary set of system (12) consists of isolated points, any solution to system (12) tends to are equilibrium state as $t \to +\infty$.

If the inequality

$$-R > |M\omega_1 - \omega_3(0)|, \qquad (23)$$

is valid, then, instead of the function $V(G,\theta)$, one should consider the Lyapunov function $W(G,\theta) = -V(G,\theta)$ and repeat the above considerations.

Under inequality (15), we have the relation $F(G) \neq 0,\ \forall G \in R^1$. Together with the second equation of system (12), this implies that

$$\lim_{t\to+\infty} \theta(t) = \infty.$$

Thus, Theorem 3 is completely proved.

To prove Theorem 4, one should note thus, if condition (18) holds in a neighborhood of points $G = 0$, $\theta = 2\pi k$, then the function $V(G,\theta)$ has the property

$$V(G,\theta) > 0 \quad \text{for} \quad |G| + |\theta - 2k\pi| \neq 0.$$

Together with equality (22), this implies the asymptotic stability of these equilibrium states.

In a neighborhood of points $G = 0$, $\theta = (2k+1)\pi k$, the function $V(G,\theta)$ has the property

$$V(0,\theta) < 0 \quad \text{for} \quad \theta \neq (2k+1)\pi.$$

Together with equality (22), this implies the instability of these equilibrium states.

If inequality (23) holds, then, instead of the function $V(G,\theta)$, one should consider the function $W(G,\theta) = -V(G,\theta)$ and repeat the considerations.

5 CONCLUSIONS

Here the classical A.J.Viterbi ideas (Viterbi, 1966) are developed and generalized for design of PLL with pulse modulation. Introduction of relay element in the block diagram after filter is essentialy new construction for floating PLL with respect to previous design of floating PLL for radio engineering (Viterbi, 1966). In this paper is showed that main requirements to PLL for multiprocessors systems is global stability. Necessary and sufficient conditions of global stability for floating PLL are obtained. For proof of this results direct Lyapunov method is developed.

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