Estimation of dynamic buckling loads in approximate shell models via frequency-domain methods

V. Reitmann
St. Petersburg State University, Russia
E-mail: VReitmann@aol.com

We study the stability and dissipativity of viscoelastic plates with constant thickness under dynamic loads given by the equations ([1, 7])

$$\frac{D}{h}(1-R^*)\nabla^4 w = L(w,F) - \rho\,\frac{\partial^2 w}{\partial t^2}\;,$$
$$\frac{1}{E}\nabla^4 F = -\frac{1}{2}\,(1-R^*)L(w,w)\;. \tag{1}$$

In system (1) w is the deflection, F is the stress function, D is the plate's boundary stiffness, h is the thickness, ρ is the material's density and E is the elastic modulus. R^* denotes an integral operator with the relaxation kernel $R(t) = kt^{\alpha-1}e^{-\varepsilon t}$, where $k > 0$, $\varepsilon > 0$ and $\alpha \in [0,1]$ are parameters. The expression $L(w,w)$ is defined by

$$L(w,w) = 2\,\frac{\partial^2 w}{\partial x_1^2}\,\frac{\partial^2 w}{\partial x_2^2} - 2\left(\frac{\partial^2 w}{\partial x_1\,\partial x_2}\right)^2\;. \tag{2}$$

The boundary conditions may be expressed as:

$$w = \frac{\partial^2 w}{\partial x_1^2} + \mu\,\frac{\partial^2 w}{\partial x_2^2} = 0 \quad \text{at} \quad x_1 = 0 \quad \text{and} \quad x_1 = a\,; \tag{3}$$
$$w = \frac{\partial^2 w}{\partial x_2^2} + \mu\,\frac{\partial^2 w}{\partial x_1^2} = 0 \quad \text{at} \quad x_2 = 0 \quad \text{and} \quad x_2 = b\,; \tag{4}$$
$$\frac{\partial^2 F}{\partial x_2^2} = -\frac{1}{h}\,[k_1 + k_2\cos\theta t] \quad \text{and} \quad \frac{\partial^2 F}{\partial x_1 \partial x_2} = 0 \quad \text{at} \quad x_1 = 0 \quad \text{and} \quad x_1 = a\;; \tag{5}$$
$$\frac{\partial^2 F}{\partial x_1^2} = 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial x_1 \partial x_2} = 0 \quad \text{at} \quad x_2 = 0 \quad \text{and} \quad x_2 = b\;. \tag{6}$$

In (3) – (6) μ is Poisson's ratio ; $a > 0$ and $b > 0$ are the length and the width of the plate ; k_1, k_2 and θ are certain real parameters.
If we assume that the plate is hinged at its boundaries, the solution w of the boundary-value problem (1) – (6) can be expressed as

$$w(x_1,x_2,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}(t)\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}\;. \tag{7}$$

Using the Bubnov-Galerkin method one obtains ([1, 7]) for $a = b = l$ a system of nonlinear integro-differential equations which can be written for $z := w_{mn}$ as

$$\ddot z + \Omega^2(1 - q_0\cos\theta t)z - \omega^2 R^* z + q_1 z(1-R^*)z^2 = 0\,, \tag{8}$$

where $(R^*f)(t) = k_1\int_0^t k\tau^{\alpha-1}e^{-\varepsilon\tau}f(\tau)d\tau$ and $\Omega, \omega, q_0, q_1, k_1$ and θ are also parameters. For certain cases of these parameters we get the linear Mathieu equation

$$\ddot z + \Omega^2(1 - q_0\cos\theta t)z = 0 \tag{9}$$

and, with a further parameter q_4, the nonlinear Mathieu-Duffing equation

$$\ddot{z} + \Omega^2(1 - q_0 \cos\theta t)z + q_4\, z^3 = 0\ . \tag{10}$$

Exploring frequency-domain methods developed in [10, 5] we derive sufficient conditions for the boundedness of solutions on $\mathbb{R}_+$ and the dissipativity of equations of the type (8) for nonsingular kernels ($\alpha = 1$). Under these conditions dynamic buckling is impossible. In particular we investigate the class of systems with periodic in time linear and nonlinear parts with the help of the frequency theorems for nonautonomous periodic in time equations ([9, 3]). We also consider the case of a singular kernel ($\alpha \in (0, 1)$). This leads to ordinary differential equations with fractional derivatives ([2]). The functional-analytic investigation of such singular differential equations can be done in certain interpolation spaces ([8]). For the characterization of boundedness properties of the solutions and their regularity we use the solvability of Riccati equations for plate problems ([6]).

References

[1] Badalov, F. B., Kh. Eshmatov, and U. I. Akbarov: *Stability of viscoelastic plate under dynamic loading.* Soviet Appl. Mech. (Transl. from Prikl. Mekh.), **27** 9, 892 – 899, 1991.

[2] Bagley, R. L. and P. Torvik: *On the fractional calculus model of viscoelastic behavior.* J. Rheol., **30**, 233 – 255, 1986.

[3] Fabbri, R., R. Johnson, and C. Núñez: *On the Yakubovich frequency theorem for linear non-autonomous control processes.* Discrete Contin. Dynam. Systems, **9**, 677 – 704, 2003.

[4] Kantz, H. and V. Reitmann: *Determining functionals for bifurcations on a finite-time interval in variational inequalities.* In: Proc. EQUADIFF 2003, World Scientific, Hasselt, Belgium, 1098 – 1100, 2003.

[5] Koryakin, Yu. A., G. A. Leonov, and B. L. Neroslavskii: *On the boundedness of solutions of a linear nonautonomous system of differntial equations. Problems in the theory of periodic motions.* Ischevsk, **3**, 29 – 34, 1979. (Russian)

[6] Lasiecka, I., D. Lukas, and L. Pandolfi: *Input dynamics and nonstandard Riccati equations with applications to boundary control of damped wave and plate equations.* J. of Optimiz. Theory and Appl., **84** 3, 549 - 574, 1995.

[7] Sun, Y. X. and S. Y. Zhang: *Chaotic dynamic analysis of viscoelastic plates.* Int. J. of Mech. Sc., **43**, 1195 – 1208, 2001.

[8] Triebel, H.: Interpolation Theory, Function Spaces, and Differential Operators. North-Holland, Amsterdam, 1978.

[9] Yakubovich, V. A.: *Linear-quadratic optimization problem and the frequency theorem for periodic systems I.* Siber. Math. Jour., **27**, 614 – 630, 1986.

[10] Yakubovich, V. A., G. A. Leonov, and A. Kh. Gelig: Stability of Systems with Discontinuous Nonlinearities. Singapure, World Scientific, 2004.