PREFACE. Advances in Multistability and Control

Abstract

Multistability is one of the most exciting phenomena in dynamical systems. The coexistence of attractors is a common feature in different areas of science and in nature. Noise, inevitable in a real system can induce switches between coexisting states, thus leading to multistate intermittency. Some applications require control of multistability to avoid or predict these switches. The research results presented in this focus issue demonstrate how multistability arises in various dynamical systems, its use for different applications, and how we can control it.

Key words

Nonlinear dynamics, bistability, multistability, control, chaos, neuron, brain, perception, electronic circuit, EEG, complex network, artificial neural network, noise, intermittency, multiplexing, oscillators, synchronization.

Multistability is a widespread phenomenon occurring in dissipative systems when different stable solutions (or attractors) coexist for a given set of system parameters. Since real systems are dissipative, multistability is frequently revealed in nature. Among various examples, we highlight electronic circuits [Sevilla-Escoboza et al, 2015; Kamdoum Tamba and Bertrand Fotsin, 2017], lasers [Moskalenko et al., 2017], and neuron oscillators [Jaimes et al., 2017]. All multistable systems are highly sensitive to initial conditions, which means that a very small external perturbation causes a significant change in the final state. Moreover, these systems are aslo sensitive to a parameter value, so that a very small variation in a parameter can lead to a dramatic change in the system dynamics. Therefore, control of multistability is extremely important for applications where the system's final state must be predictable [Pisarchik and Ulrike, 2014; Sevilla-Escoboza et al, 2017].

It is known that noise in a multistable system can induce switches between coexisting states. This effect, referred to as *attractor hopping* [Huerta et al, 2008], *two-state intermittency* [Pisarchik and Pinto-Robledo, 2002], or *multistable intermittency* [Pisarchik et al., 2012; Hramov et al, 2016] was observed in many dynamical systems. The theory of noise-induced intermittency in bistable dynamical systems, proposed by Moskalenko et al [Moskalenko et al., 2017] states that the residence time distribution for each coexisting regime obeys a particular exponential law.

The most attractive multistable object is the brain. Since the appearance of the pioneering work of Atteneave in 1971 [Atteneave, 1971], where he used the term "multistability" to describe perception of ambiguous images for the first time, perception multistability has attracted attention of many scientists from different fields, including psychology, biology, physics and mathematics. Due to an extremely high research interest in this topic, several papers of this focus issue are devoted to studying perception with the help of bistable images [Grubov, Runnova et al., 1971; Grubov, Musatov et al., 2017; Bashkirtseva et al, 2017; Magallón-García et al., 2017]. The brain can be considered as a very complex dynamical system with a huge number of coexisting states, where each state can be associated with a particular thought. The thinking process results from switches between different thoughts. This process is initiated by external stimuli when the brain receives information from sensory organs. However, even without any external influence, i.e., when a person is completely isolated from the external environment, the mental process doesn't stop, because internal noise in the brain's neuronal network causes switches between coexisting brain states. Since this noise-induced multistate intermittency always exists, the brain cannot be considered a multistable system. In the mathematical sense, this is a metastable dynamical system which phase-space trajectory visits different coexisting states. The switching time between the states is determined by noise intensity and external stimuli.

Brain noise has different origins. To investigate the influence of various types of noise on perception, Bashkirtseva et al [Bashkirtseva et al, 2017] considered a simple perception energy model with multiplicative and additive noise. These two types of noise may be associated, respectively, with random physiological processes and random synaptic connections of brain neurons. The latter can be a consequence of interactions among different parts of brain neural networks functioning in a coordinated fashion, with associated energy exchange. They found that the sensitivity of the bistable perception model to noise depends on its the noise type. In particular, parametric noise leads to a total oscillation suppression when the system is in an unstable equilibrium, whereas in the presence of additive noise the oscillations still exist, but are strongly suppressed. Using catastrophe theory formalism and an approach based on stochastic sensitivity function, it is possible to predict the hysteresis squeezing caused by an increase in the additive noise intensity.

Ambiguous images are often used for studying decision-making process in the brain. Among a number of ambiguous images, the most popular one is the Necker cube. This is a flat 2D-image which due to optical illusion looks like a cube with transparent faces and visible ribs. Visual bistability consists in the fact that it is treated as a 3D-object oriented in two different ways, either left or right, especially if the contrast of different ribs of the Necker cube varies. Grubov et al [Grubov, Runnova et al., 1971] used this bistable image in their study of human perception. In particular, for the analysis of EEG data they proposed a new method based on the calculation of the maximum energy component in the continuous wavelet transform (skeletons). They found particular features of the EEG patterns associated with different ratios between delta, alpha, and betta rhythms before, during and after perception.

For classification of EEG patterns associated with different brain states, Grubov et al [Grubov, Musatov et al., 2017] proposed the use of artificial neural network (ANN). We developed experimental design and performed series of experiments on volunteers for real and imaginary movements of arms and legs. The both classification methods may have possible applications in biofeedback systems to design brain-computer interfaces.

Another interesting effect induced by noise in a neuronal network is the coherent resonance when an external stimulus approaches a neuron excitation threshold [Andreev et al., 2017]. This resonance was found not only with respect to noise, but also to the number of stimulated neurons and the network size. The increasing coherence in the complex network can be a result of enhancing burst synchronization.

Synchronization of multistable systems is also a very interesting research topic [Boccaletti et al, 2018]. The influence of multiplexing on synchronization between unstable chimera patterns was investigated by Florov et al [Frolov et al., 2017] in a multiplex network of non-locally coupled Kuramoto-Sakaguchi (KS) oscillators. They found that an increase in the inter-layer coupling strength improves stabilization of spatio-temporal chimera pattern. The effect of multiplexing can be used to control and localize unstable pattern formation in complex networks.

One possible reasons for multistability in neuronal networks is the asymmetry in coupling strengths between neurons. This mechanism was studied in two electrically coupled Rulkov neurons [Jaimes et al., 2017], where the coexistence of two attractors was found at certain coupling strengths.

As it was already mentioned above, multistability control is very important for many applications [Pisarchik and Ulrike, 2014]. In this topic issue, Kamdoum Tamba and Bertrand Fotsin [Kamdoum Tamba and Bertrand Fotsin, 2017] demonstrate how coexisting attractors can be destroyed by using a linear augmentation technique. This type of control was experimentally implemented in a simple chaotic electronic circuit.

To conclude, I believe that the different manifestations of multistability described in this issue, as well as the methods for its characterization and control will be helpful for better understanding this exciting phenomenon and will stimulate future research and applications.

I would like to acknowledge support from the Ministry of Economy and Competitiveness (Spain) (project SAF2016-80240).

Guest Editor

Alexander N. Pisarchik
Center for Biomedical Technology
Technical University of Madrid
Spain
alexander.pisarchik@ctb.upm.es

References

Andreev, A., Makarov, V., Runnova, A., and Hramov, A. (2017). Coherent resonance in neuron ensemble with electrical couplings. *Cybernetics and Physics* **6**, pp. 141–144.

Atteneave, F. (1971). Multistability in perception. *Sci. Am.* **225**, pp. 63–71.

Bashkirtseva, I. A., Ryashko, L. B., and Pisarchik, A. N. (2017). Curp catastrophe in a bistable perception energy model with additive and parametric noise. *Cybernetics and Physics* **6**, pp. 131-134.

Boccaletti, S., Pisarchik, A. N., del Genio, C. I., and Amann, A. (2018). Synchronization: From Coupled Systems to Complex Networks (Cambridge University Press). ISBN: 9781107056268.

Grubov, V., Runnova, A., Zhuravlev, M., Maksimenko, V.,Pchelintseva, S., and Pisarchik, A. (2017). Perception of multistable images: EEG studies. *Cybernetics and Physics* **6**, pp. 103–107.

Grubov,V., Musatov,V., Maksimenko, V., Pisarchik, A., Runnova,A., and Hramov, A. (2017). Development of intelligent system for classification of multiple human brain states corresponding to different real and imaginary movements. *Cybernetics and Physics* **6**, pp. 108–113.

Frolov, N., Koronovskii, A., Makarov, V., Maksimenko, V., Goremyko, M., and and Hramov, A. (2017). Control of pattern formation in complex networks by multiplexity. *Cybernetics and Physics* **6**, pp. 121–125.

Jaimes-Reátegui, R., García-Vellisca, M. A., Pisarchik, A. N., and del Pozo-Guerrero, F. (2017). Bistability in Hindmarsh-Rose oscillators induced by asymmetric electrical coupling. *Cybernetics and Physics* **6**, pp. 126–130.

Hramov, A. E., Koronovskii, A. A., Moskalenko, O. I., Zhuravlev, M. O., Jaimes-Reátegui, R, and Pisarchik, A. N.

(2016). Separation of coexisting dynamical regimes in multistate intermittency based on wavelet spectrum energies in an erbium-doped fiber laser. *Phys. Rev. E* **93**, 052218.

Huerta-Cuellar, G., Pisarchik, A. N., and Barmenkov, Yu. O. (2008). Experimental characterization of hopping dynamics in a multistable fiber laser. *Phys. Rev. E* **78**, 035202R.

Kamdoum Tamba, V. and Bertrand Fotsin, H. (2017). Multistability and its control in a simple chaotic circuit with a pair of light-emitting diodes. *Cybernetics and Physics* **6**, pp. 114–120.

Magallón-García, D. A., Jaimes-Reátegui, R., Huerta-Cuellar, G., Gallegos-Infante, L. A., Soria-Fregoso, C., and García-López, J. H. (2017). Study of multistable visual perception using a synergetic model. *Cybernetics and Physics* **6**, pp. 120–126.

Moskalenko, O. I., Koronovskii, A., Hramov, A. E., Zhuravlev, M. O., and Jaimes-Reátegui, R. (2017). Residence time distributions for coexisting regimes of bistable dynamical systems subjected to noise influence. *Cybernetics and Physics* **6**, pp. 97–102.

Pisarchik, A. N., Jaimes-Reátegui, R., Sevilla-Escoboza, J. R., and Huerta-Cuellar, G. (2012). Multistate intermittency and extreme pulses in a fiber laser. *Phys. Rev. E* **86**, 056219.

Pisarchik, A. N. and Pinto-Robledo, V. J. (2002). Experimental observation of two-state on-off intermittency. *Phys. Rev. E* **66**, 027203.

Pisarchik, A. N. and Feudel, U. (2014). Control of multistability. *Physics Reports* **540**, pp. 167–218.

Sevilla-Escoboza, R., Huerta-Cuellar, G., Jaimes-Reátegui, R., García-López, J. H., Medel-Ruiz, C. I., Castañeda, C. E., López-Mancilla, D., and Pisarchik, A. N. (2017).Error-feedback control of multistability. *Journal of the Franklin Institute* **354**, pp. 7346–7358.

Sevilla-Escoboza, R., Pisarchik, A. N., Jaimes-Reátegui, R., and Huerta-Cuellar, G. (2015). Selective monostability in multi-stable systems. *Proceedings Royal Soc. A* **471**, 20150005.