Multistability induced by delay and controlled by delay

C. Masoller

Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Colom 11, E-08222 Terrassa, Spain

Abstract

Feedback loops are relevant for the control of human balance, for generating persistent memory in neuronal systems, for cellular differentiation in genetic circuits, etc. Time-delays, arising from the fact that signal propagation realistically occurs at a finite velocity, are often a source of multistability, chaotic behavior and oscillation death. In this contribution I will discuss the interplay between the intrinsic nonlinearity of an oscillatory neuron and a weak time-delayed feedback loop representing a recurrent synaptic connection. The neuron dynamics is modeled based on a Hodgkin-Huxley type model of thermally sensitive neurons [1, 2] that presents a variety of firing patterns, including subthreshold oscillations, regular spike activity and spike trains, that is, narrow groups of spikes separated by long spike-less intervals. In the regime of subthreshold oscillations, due to the excitable character of the dynamics, it can be expected that even weak feedback can be a strong perturbation of the subthreshold oscillations, and can induce drastic changes. I will show how the feedback amplifies the oscillation amplitude, inducing threshold-crossings and giving rise to a firing activity that is self-regularized by the delay time of the feedback loop. Multistability of firing patters occurs, induced by the delayed feedback loop and controlled by the delay time of the loop. Negative feedback enhances the oscillations, but if the feedback weak, it does not induce firings for all delay times: there are feedback-induced spikes only in windows of the delay centered at $\tau \sim (n + 1/2)T_0$ with n integer and T_0 the intrinsic oscillation period. Moreover, in these windows the firing dynamics is regularized by the delay: for short delays the neuron fires tonic spikes, while for longer delays it fires spikes with skippings. These results will be interpreted in terms of a simple model of an oscillator with a weak delayed feedback loop. I will also analyze the dynamics of an ensemble of neurons mutually coupled through their delayed mean field. The neurons displays different behaviors depending on the delay of their coupling. Either all the neurons fire spikes, or they all display subthreshold oscillations, or some neurons display subthreshold oscillations while the others display spiking behavior (i.e., the ensemble divides into clusters). Again, the ensemble is self-regularized by the delay time and the neuronal oscillations are in-phase or out-of-phase depending on the delay. There is also multi-stability of solutions with the coexistence, for certain delay values, of inphase and out-of-phase behavior.

[1] H. A. Braun, M. T. Huber, M. Dewald, K. Schafer, and K. Voigt, "Computer simulations of neuronal signal transduction: The role of nonlinear dynamics and noise ", Int. J. Bif. Chaos **8**, 881 (1998).

[2] U. Feudel, A. Neiman, X. Pei, W. Wojtenek, H. A. Braun, M. T. Huber, and F. Moss, "Homoclinic bifurcation in a Hodgkin-Huxley model of thermally sensitive neurons", Chaos **10**, 231 (2000).