SURFACE CONTACT WITH FRICTION BETWEEN POLYHEDRIC DISCRETE ELEMENTS

Claude Bohatier
Laboratory LMGC
University of Montpellier 2
France
bohatier@lmgc.univ-montp2.fr

Ali Rafiee
Laboratory CMGD
Alès School of Mines
France
ali.rafiee@ema.fr

Marc Vinches
Laboratory CMGD
Alès School of Mines
France
marc.vinches@ema.fr

Abstract
In this communication we propose the modeling of the contacts between rigid solids on a principle of their localization at nodes. The results of the application of the Non-Smooth Contact Dynamics (NSCD) method to investigate the stability conditions of jointed rock slopes, and of several masonry structures in real scale, are presented in this paper.

The theoretical basis of a new approach which makes it possible to simultaneously consider a better localized stress analysis and a saving of computation time, is discussed.

Key words
NSCD method, surface contact, discrete element, masonry structure, seismic loading.

1 Introduction

The first use of discrete elements as polyhedron in numerical simulation was developed by Cundall and Strack (1979).

Here, the algorithm of the general resolution used in the Non Smooth Contact Dynamics method, developed in our laboratory according to the works of J.J. Moreau and M. Jean (1999) is based on a node by node resolution of the contact. The laws of Signorini for unilateral contact and of Coulomb for friction are not regularized. In some applications the friction and the number of contact nodes lead to an indetermination of the system of equations, particularly in collections of polyhedron, where the plane contacts are numerous [Chetouane, Dubois, Vinches and Bohatier, 2005]. However, by taking into account the history of a realistic activation of the contacts during the preparation and loading of the sample, one can generally avoid this indetermination. We present results of simulations comprising several complex geometries and pre-stressed structures, such as the cupola of Junas or comprising a great number of blocks for the example of Nîmes amphitheatre.

A new approach is proposed for a new determination of the local contact actions making it possible to better deal with the tilting of blocks against one another. It consists firstly of the proposition of a modeling of the contact surface that is statically acceptable, allowing the local resolution of the unknown nodal forces and, secondly, of a resolution algorithm using the history of activation of the various connections to avoid the global indetermination.

2 Numerical results using the NSCD method

The formulation of the NSCD method relies on a special formulation of the equation of motion. The term "non smooth" refers firstly to all the mathematical and mechanical background allowing us to deal with some extended kinds of laws. For the non-smoothness in time, the occurrence of velocity jumps is a well known feature of the second order dynamics with unilateral constraints on the position even with continuous media.

Additionally, the contact forces between two bodies are bound by the principle of mutual actions. The calculation of contact forces in the NSCD method is performed in two steps. First, the result of the interaction of the antagonist body B_a on the candidate body B_c can be considered equal to the force r_α acting at the contact point between these two bodies. At the contact point, we can define a local frame composed of three vectors (in a 3D model) including a normal vector n_α pointing from B_a to B_b and two tangential vectors and , which define the tangential space by respecting this convention $s_\alpha \times t_\alpha = n_\alpha$. On the other hand, we denote the gap distance between bodies along the normal direction. This value will be negative if there is interpenetration between the bodies.

In the second step, by the definition of a linear mapping H_α that relates the local forces to the global forces, verifying the following equation:

$$R_\alpha = H_\alpha(q) r_\alpha \qquad (1)$$

where $H_\alpha(q)$ is a mapping which contains the local information about contactors. Finally the global contact forces can be obtained by the relation

$$R = \sum_\alpha R_\alpha \qquad (2)$$

The same procedure is employed for the velocity calculation and the velocity of the bodies can be expressed in the local frame.

The contact conditions are solved at the local level, the impulse force is R and the relative velocity U. The dynamic equations are solved at the global level. The global impulse force is noted r and the global velocity vector is noted. H and H* are the mapping operators (Fig.1). See in [Jean, 1999; Moreau, 1988] for detailed explanations about the NSCD method.

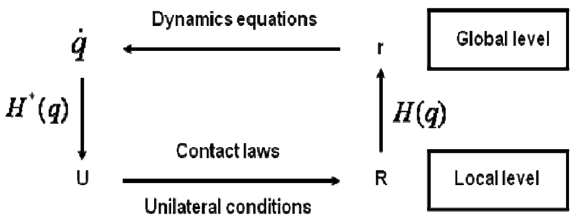


Figure 1. Algorithm of the NSCD Method.

In section (3) we propose a way to define stress field and deformation field in order to explain the evolution of the structure [Chetouane, Dubois, Vinches and Bohatier, 2005].

The results obtained for Nîmes amphitheatre subjected to a sinusoidal loading are presented in Fig. 2. The model of the amphitheatre is created in the AutoCAD® software (Fig.3), and then this geometry is converted into standard input file of the LMGC90 code. Fig.4 shows the distribution of the vertical forces due to the weight of the blocks in the structure. The behavior of this structure is investigated for a dynamic load, applying an imposed velocity on the supporting element of the model. A three second long sinusoidal velocity is applied in three directions (Fig.5): the supporting element acts as a shaking table.

Figure 2. Nîmes amphitheatre.

The states of the structure are illustrated in Fig. 6. The upper arch structure of the pushes out the surrounding wall. This type of displacement is visible in the building.

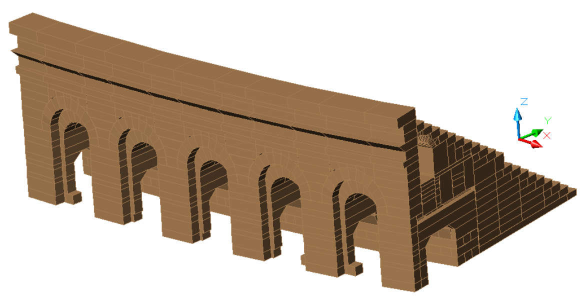


Figure 3. 3D model generated with AutoCAD software including 2670 blocks for five arches on the first floor.

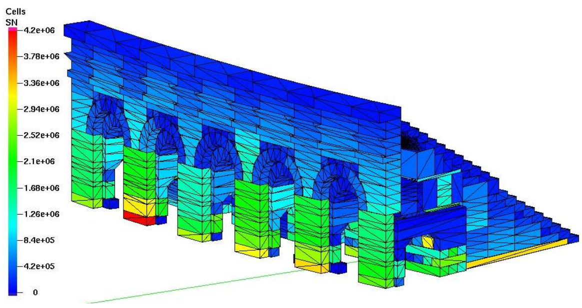
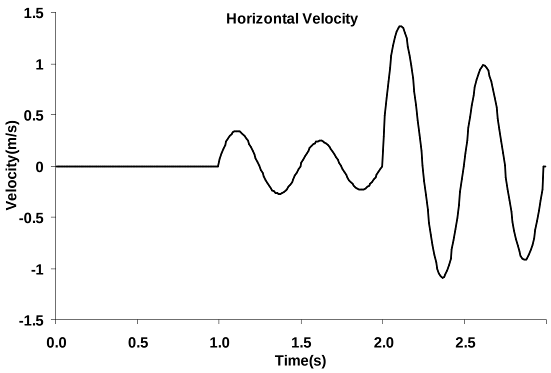


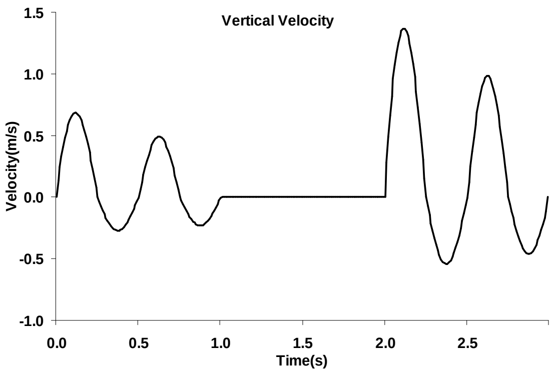
Figure 4. Static vertical contact forces field

As another example in a masonry structure, the mechanical behavior of the cupola of Junas is studied (Fig. 7). The structure is a stone arch included in a 4m radius half-sphere. It has a 1m diameter circular hole at its top, and rests on the five 2m high pillars. In this study, the possibility of realization of this structure is investigated. The stability of the cupola is studied for a dry friction coefficient of 0.7. The structure remains stable even with a 0.5 friction coefficient between the stone blocks.

The NSCD method is also used for rock mass studies. They are considered as an assembly of rigid bodies generated by a fracture system. The models of rock mass are generated in the AutoCAD® software by the codes developed for this purpose [Rafiee, 2008]. The information measured in situ, such as the orientation and the spacing between fractures, is given to the code in order to automatically generate a stochastic model of the jointed rock mass. Fig. 8 illustrates the results obtained in 2D and 3D. A model is presented with 6 fracture families measured in a stone quarry in the southwest of France.



(a)



(b)

Figure 5. Time history of input velocity in m/s, (a) vertical velocity, (b) horizontal velocity.

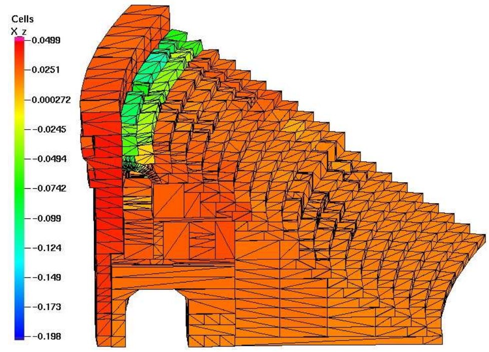
In addition we propose to improve the calculation of the contact force between the polyhedrons in order to obtain a better prediction of the tilting between the polyhedrons, and reduce computation time.

3 Modeling of plane contact

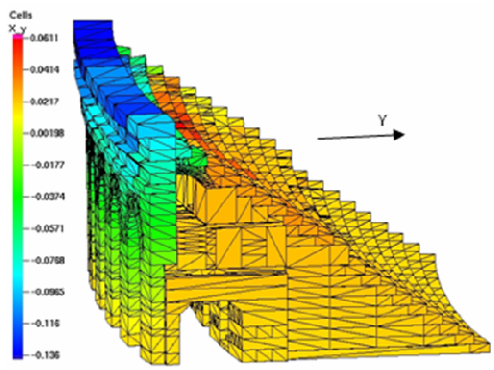
Figure 7 shows the static results for a pre-stressed structure “the cupola of Junas“. Figure 8 deals with an application to rock mechanics.

In 2D, the contact of convex polygons is considered to happen on a segment in the plane (figure 9). The actions of contact are considered localized at points A and B. In the case of a parallelepiped, assumed to always remain in dry contact with friction, on a tilted plane and subjected to its weight P applied to the centre of inertia G, one notes that we have 3 equations and 4 unknown nodal forces. We then make the additional assumption that the tangential actions are proportional to the normal actions. This assumption is compatible with the apparition of the slip threshold. Assuming that the solids are rigid, the sliding can only take place simultaneously at the ends A and B of the segment. We then have a nonlinear problem to be solved, of four equations for four unknown variables. This method of modeling can be applied in 3D to the case of the contact of an edge of a polyhedron with a plane surface.

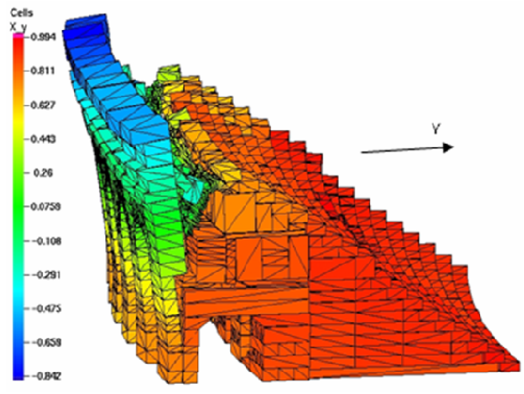
In the case of a 3D contact, the contact of convex polyhedrons is carried out on points or lines or surfaces. The case of the linear contact can be reduced to the 2D case, in the plane of contact.



(state 1)



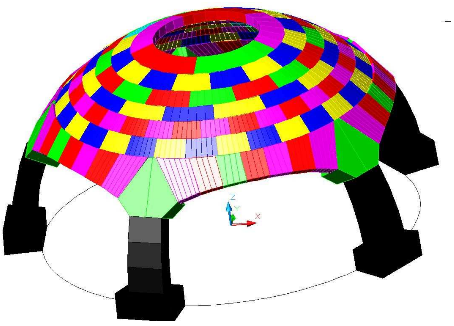
(state 2)



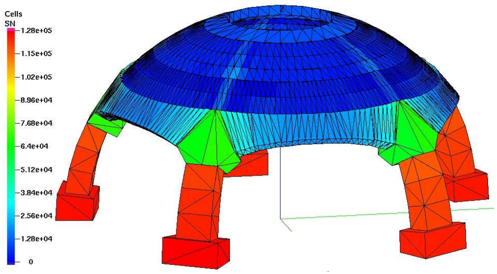
(state 3)

Figure 6. Three states of the amphitheatre structure submitted to a dynamic loading.

The contact surface is considered on a plane and the contour of the zone of contact is a convex polygon. We will locate the actions of contact on the vertices of these polygons which we will call contact nodes.



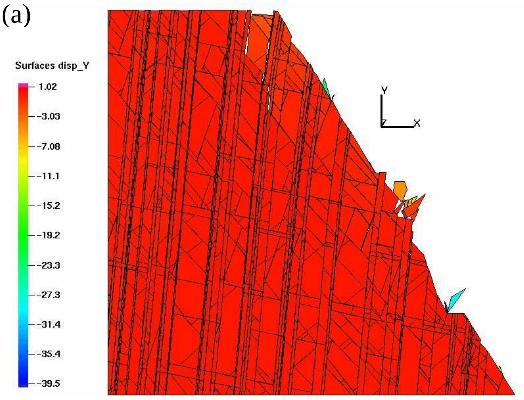
(a)



(b)

Figure 7. cupola of Junas : (a) 3D geometry of the cupola in AutoCAD format, (b) Static calculations of the vertical force distribution.

(a)



(b)

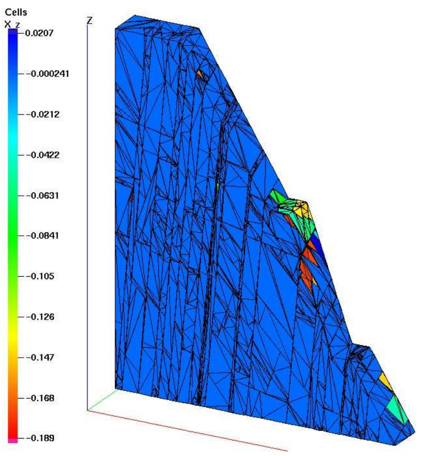


Figure 8. Jointed rock mass modeling: (a) 2D model of a rock mass subjected to seism vibration, (b) 3D model under rock mass weight, for rock slope stability analysis purpose.

4 Modeling of the tangential actions on a plane surface

The assumptions about normal and tangential actions providing the possibility to avoid the indetermination in the case of plane contact are comprehensively explained in [Bohatier, Perales, Vinches and Nemoz-Gaillard, 2007]. The contact force in each node is composed of normal and tangential forces.

Here the hypothesis of a linear distribution of the tangential actions is presented, with its mechanical consequences. For a 2D problem, we can write for the dependency of the tangential forces at vertices A and B in a polygonal contact surface with regard to the normal forces:

$$\frac{\vec{T}_A \bullet \overrightarrow{AB}}{N_A} = \frac{\vec{T}_B \bullet \overrightarrow{AB}}{N_B} \qquad (3)$$

$\vec{T}_A$ and $\vec{T}_B$ are the tangential forces N_A and N_B are the positive normal forces.

For generalization to 3D problems, as a first step, a plane contact on a triangular surface can be considered, with the actions of contact located at its vertices [Perales, 2007].

After writing the three equations (3) for each edge of the triangle, it is possible to verify that these relations are compatible with the various situations:

- The three nodes slide: the relative movement between the surfaces is slipping; the unknown node variables are speeds along the direction given by the sliding hypothesis direction.

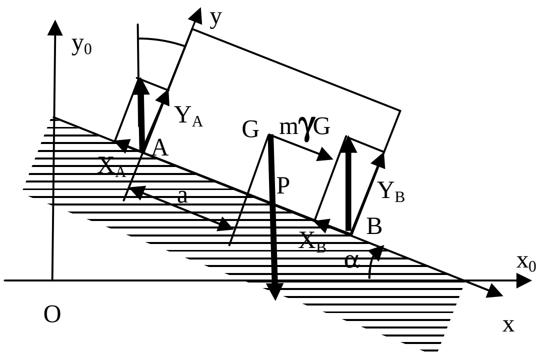


Figure 9. Hypothesis of the distribution of the tangential actions in a direction

- A node is fixed with conditions of adherence; the relative movement of the solid is then a rotation around this node.
- Two nodes do not slide in which case the third node necessarily does not slide:

There is no possibility of a relative movement.

The proposed assumption (3) is compatible with the sliding conditions with regard to the Coulomb friction laws (5) and (7) and the equiprojectivity property of the velocity field onto the contact surface (4).

Equicprojectivity of velocity $\vec{V}_A$ and $\vec{V}_B$ at A and B:

$$\vec{V}_A \bullet \overrightarrow{AB} = \vec{V}_B \bullet \overrightarrow{AB} \quad (4)$$

First Coulomb law:

$$\left\| \vec{T}_A \right\| \leq f\, N_A \quad (5)$$

f is the friction coefficient

When the sliding occurs along $\overrightarrow{AB}$

$$\left\| \vec{T}_A \right\| = f\, N_A \quad \text{and} \quad \left\| \vec{T}_B \right\| = f\, N_B \quad (6)$$

then the second Coulomb law at point A and point B:

$$\vec{T}_A = -\lambda_A \vec{V}_A \quad \text{and} \quad \vec{T}_B = -\lambda_B \vec{V}_B \quad (7)$$

λ_A and λ_B are scalar parameters, (7) and (4) give

$$\frac{\vec{T}_A}{\lambda_A} \bullet \overrightarrow{AB} = \frac{\vec{T}_B}{\lambda_B} \bullet \overrightarrow{AB} \quad (8)$$

(8) is compatible with the assumption (3) when

$$\frac{\lambda_B}{\lambda_A} = \frac{N_B}{N_A} \quad (9)$$

We propose to extend (3) that is valid for a discrete force field to a continuous tangential surface force field

$$\frac{d\vec{T} \bullet \overrightarrow{IM}}{p dS} = Cte \quad (10)$$

p is the contact pressure.

When the motion is a translation then

$$Cte = f \left\| \overrightarrow{IM} \right\| \cos(\overrightarrow{dT}, \overrightarrow{IM}) \quad (11)$$

For any other motion $Cte = 0$ if I is the instantaneous rotation centre.

Finally the new contact algorithm starts by the valuation of a sticking contact where the assumption (3) is taken into account. When a nodal contact force is greater than the authorized value by Coulomb's law it is limited, then its direction is used to determine the sliding velocity. When the solid slides on a triangular surface the set of 6 dynamic equations for one solid is consistent to determine the normal nodal contact force with three nodal contact points.

When there are more than three nodal contact points the spatial linear dependency of normal forces and two tangential equations given by the supplementary nodes provides the extra equations for the solution.

4 Conclusion

Several computations of real structures are presented in the first part of this paper. The obtained results are improved from what was available, after using a new contact detection algorithm based on a modified Cundall's algorithm [Perales, 2007]. We are now working on some ways to reduce computation times and improve the study of the tilting of blocks. In this paper we propose a new modeling of the plane contact, based on the spatial dependency of the tangential forces with regards to the normal forces. The computation tests with this new modeling show that the unknown nodal forces have the same resultant force but not the same nodal values that are given by the classical NSCD method. With this new modeling the statically admissible nodal values are more realistic. It can therefore be expected that these results provide a better prediction for the relative tilting between blocs. Furthermore, we are working on the assumption that a better valuation of the contact forces will lead to a reduction of computation time.

References

Bohatier C., Perales R., Vinches M., Nemoz-Gaillard M. (2007) *New development on distinct elements modelling using the NSCD method*, Computational methods in structural dynamics and earthquake engineering COMPDYN proceeding, Crete,Island, Greece, June 2007.

Chetouane B., Dubois F., Vinches M., Bohatier C. (2005) *NSCD discrete element method for modeling masonry structures*, Int. Journal for Numerical Methods in Engineering, vol. 64, p. 65-94.

Cundall PA, Strack ODL (1979) A discrete numerical model for granular assemblies, Geotechnics, 29(1), 47-65.

Cundall PA, Strack ODL (1979) *The distinct element method as a tool for research in granular media*, Part I and II, Report to National Sc. Foundation, Eng., Department of Civil and Mineral Eng., University of Minnesota, Minneapolis, MN, vol.1, 76-20711.

Jean M. (1999) *The non smooth contact dynamics method.* Comput. Methodes Appl. Mech. Eng., 177: 235-257, Special issue on computational modelling of contact and friction, J.A.C. Martins and A. Kalbring, editors.

Moreau J.J. (1988) *Unilateral contact and dry friction in finite freedom dynamics*. In Morreau JJ and Panagiotopoulos PD, editors, Non smooth mechanics and applications, Springer Verlag.

Moreau J.J. (1997), *Numerical investigation of tangential zones in granular materials*, Proc. HLRZ-workshop one friction, Arching, Dynamics Contact, World Scientific, Singapore, Grassberger, P. and Wolf, D. eds, 233-247.

Moreau J.J. (1999) *Some basics of unilateral dynamics*, In: Pfeifer F., Glocker Ch. (Eds.) Unilateral Multibody Contacts. Kluwer, Dordrecht Boston London, 1-14.

Moreau J.J. (2005) *Indetermination in the numerical simulation of granular systems*, Powders and grains, vol. 1, Balkema, Leiden, 2005, pp.109-112.

Perales R. (2007), *Modelling by discrete elements of the mechanical behavior of the three-dimensional built works. Contribution to the definition of plane contact elements*. PhD thesis, University of Montpellier, 27 Sept. 241 pages.

Rafiee A., Vinches M., (2008) *Application of geostatistical characteristics of rock mass fracture systems in 3D model generation*, Int. Journal of Rock Mechanics and Mining Sciences, vol.45, p. 644-652.