SYNTHESIS OF ROBUST DISCRETE-TIME SYSTEMS BASED ON COMPARISON WITH STOCHASTIC MODEL [1]

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Abstract: The paper considers a class of linear discrete-time systems with uncertain parameters. New approach to synthesis of robust stabilizing control is proposed. This approach consists of two steps. First the stochastic comparison system with multiplicative noises is constructed such that if this stochastic system is stable in the mean square then the original system with uncertain parameters is robustly stable. Second the stabilizing control problem for the comparison system is solved. To find the gain matrix of the stabilizing controller in the case of state feedback the LMI based algorithm is given and in the case of static output feedback new method and convergent iteration algorithm are obtained.

Keywords: Discrete-time system, uncertain parameters, stochastic system, comparison system, robust control, stabilizing control, state feedback, output feedback.

1. INTRODUCTION

The study of the systems with uncertain parameters is one from the main directions of the modern control theory called robust stability and control theory. In this theory there exist several approaches to describe the uncertainty models (Boyd *et al.*, 1994; Polyak and Shcherbakov, 2002). For the class of linear systems the affine and polytopic models have wide spreading. On the one hand these models allow effectively use the semidefinite programming technique in particular the linear matrix inequalities (LMI) technique (Boyd *et al.*, 1994; Polyak and Shcherbakov, 2002; Balandin and Kogan, 2007). On the other hand if the uncertainty vector is p - dimensional then this approach requires to solve 2^p m - dimensional linear matrix inequalities. It is clear that because such high dimension these models cannot be attractive especially in control engineering practice.

Intensive flow of researches based on semidefinite programming ideas has been reduced attention to other possible approaches (Bernstein, 1987; Barmish and Lagoa, 1997; Kan, 2000; Polyak and Shcherbakov, 2005). Bernstein (1987) proposed an interesting idea based on construction of stochastic Ito diffusion process, such that robust stability of a system with uncertain parameters follows from its stochastic stability. In this case the dimension of the problem is not dependent on the number of uncertain parameters. Unfortunately this idea was forgotten because it leads to nonstandard matrix quadratic equation for which the methods of solution were obtained only much years later (AitRami and ElGhaoui, 1996; AitRami and Zhou, 2000).

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These methods based on LMI optimization approach applicable if full state vector is available for controller but the complexity of the problem does not decrease if only observable output vector available for controller. The researches of the last years (Syrmos *et al.*, 1997; Garcia *et al.*, 2003; Polyak and Shcherbakov, 2005) show that this problem is not convex and its solution in principle can be not obtained by a simple way. Several attempts of some convex approximations usually lead to algorithms which do not guarantee convergence (Pakshin, 1997; Pakshin and Retinsky, 2005; Polyak and Shcherbakov, 2005). These facts attract attention to development of convergent iteration algorithms (Yu, 2004).

In this paper the new approach to synthesis of robust stabilizing control for linear discrete-time systems with uncertain parameters is proposed. This approach consists of two steps. First we construct the stochastic comparison system with the following property: if this system is stable in the mean square then considered system with uncertain parameters is robustly stable. Second the stabilizing control problem for the comparison system is solved. To find the gain matrix of the stabilizing controller in the case of state feedback an LMI based algorithm is given and in the case of static output feedback the new method and convergent iteration algorithm are obtained. All the proofs of theorems below are omitted because limited space.

2. STATEMENT OF THE PROBLEM

Consider the linear discrete-time uncertain system described by the following equations:

$$x_{n+1} = Ax_n + Bu_n + \sum_{i=0}^{p} \sigma_i(n)(A_i x_n + B_i u_n),\ y_n = Cx_n, \tag{1}$$

where x_n is m - dimensional state vector; u_n is k - dimensional control vector; y_n is r - dimensional output vector; $A, A_i \quad (i = 1, \dots, p)$ are $m \times m$ matrices; $B, B_i \quad (i = 1, \dots, p)$ is $m \times k$ matrices; C is $r \times m$ matrix; $\sigma_i(n)$ are variables which describe the uncertainties of a parameters, it is known only that these variables are bounded:

$$|\sigma_i(n)| \le \delta_i \qquad i = 1, \dots, p. \tag{2}$$

Consider the following problems:

- find the state feedback control

$$u_n = -Kx_n, \tag{3}$$

  providing exponential stability of closed loop system (1) under parameters uncertainties satisfying inequalities (2) (robust state feedback stabilization);

- find the output feedback control

$$u_n = -Fy_n, \tag{4}$$

  providing exponential stability of closed loop system (1) under parameters uncertainties satisfying inequalities (2) (robust output feedback stabilization). Here K and F are $k \times m$ and $k \times r$ gain matrices.

3. STOCHASTIC COMPARISON MODEL

Together with (1) consider the stochastic discrete-time system

$$x_{n+1} = A_{c\alpha} x_n + \sum_{i=1}^{p} \gamma_i A_{ci} x_n v_i(n),\ y_n = Cx_n, \tag{5}$$

where $A_{c\alpha} = (1+\alpha)^{1/2} A_c,\ A_c = (A - BG), \alpha > 0,\ A_{ci} = A_i - B_i G$; $v_i(n)$ are components of p - dimensional Gaussian white noise $v(n)$ with identity covariance matrix, γ_i are positive scalars, $i = 1, \dots, p$. Take the standard assumption that the noise $v(n)$ does not depend on the initial state of the system (5).

The following statements establish connection between stability of the system (5) and robust stabilization of the system (1). Denote $\mathbb{S}^m$ the space of real valued symmetric matrices.

Theorem 1. Let for some $\alpha > 0,\ \gamma > 0$ there exists positive definite solution $P \in S^m$ of the matrix equation

$$A_{c\alpha}^T P A_{c\alpha} - P + \sum_{i=1}^{p} \gamma_i^2 A_{ci}^T P A_{ci} + \gamma I = 0, \tag{6}$$

satisfying condition

$$(\alpha - \sum_{i=1}^{p} \frac{\delta_i^2}{\Gamma_i}) A_c^T P A_c + \gamma I > 0,$$

$$0 < \Gamma_i \le \gamma_i^2 - \delta_i (\sum_{j \ne i}^{p} \delta_j + \delta_i) \quad i = 1, \dots, p. \tag{7}$$

Then the control law

$$u_n = -Gx_n, \tag{8}$$

provides robust stabilization of the system (1).

Corollary 1. (Stochastic comparison model). Consider the stochastic system

$$x_{n+1} = A_\alpha x_n + B_\alpha u_n + \sum_{i=1}^{p} \gamma_i (A_i x_n + B_i u_n) v_i(n),\ \ y_n = Cx_n, \tag{9}$$

where $A_\alpha = (1+\alpha)^{1/2} A,\ \ B_\alpha = (1+\alpha)^{1/2} B$. Let

$$\alpha - \sum_{i=1}^{p} \frac{\delta_i^2}{\Gamma_i} > 0,$$

$$0 < \Gamma_i \leq \gamma_i^2 - \delta_i(\sum_{j \neq i}^{p} \delta_j + \delta_i) \quad i = 1, \ldots, p. \quad (10)$$

Then the control law (8) providing exponential stability in the mean square (ESMS) of the system (9) is the robust stabilizing control for the system (1).

So, the equations (9) plays the role of the comparison model in the robust stabilization problem of the system (1). This means that if we assign the noise intensity according to (10) and solve mean square stabilization problem with $G = K$ or with $G = FC$, then we obtain the robust stabilizing control for (1) with the state feedback or with the output feedback correspondingly.

4. ROBUST STABILIZATION VIA STATE FEEDBACK

It is known (Pakshin, 1994) that the state feedback control (3) provides exponential stability in the mean square of the system (9) if and only if there exists a matrix $P \in \mathbb{S}^m$, satisfying the inequalities

$$P > 0, \ (A_\alpha - B_\alpha K)^T P (A_\alpha - B_\alpha K) - P$$
$$+ \sum_{i=1}^{p} \gamma_i^2 (A_i - B_i K)^T P (A_i - B_i K) < 0. \quad (11)$$

The pair of matrices (P, K), is said to be state stabilizing pair or shortly stabilizing pair if it satisfies (11). The inequalities (11) are bilinear with respect to stabilizing pair. Using Schur complement theorem (Boyd *et al.*, 1994) it is possible to write equivalent linear matrix inequalities with respect to variables $X = P^{-1}$ and $Y = KP^{-1}$:

$$X > 0, \ \begin{bmatrix} X & Z \\ Z^T & D(X) \end{bmatrix} > 0, \quad (12)$$

where $Z = [(A_\alpha X - B_\alpha Y)^T \ \gamma_1 (A_1 X - B_1 Y)^T \ldots \gamma_p (A_p X - B_p Y)^T] \ D(X) = \mathrm{diag}[X]_1^{p+1}$. So we can formulate the following theorem.

Theorem 2. The state feedback control (3) provides ESMS of the system (9) if and only if the system of LMIs (12) is feasible. The stabilizing pair is given by

$$P = X^{-1} \ \ K = YX^{-1}. \quad (13)$$

If the relations (10) hold, then this theorem gives sufficient conditions of robust stabilization of the system (1) via state feedback (3). The gain matrix of the robust stabilizing control is given by (13). Feasibility of the LMIs (12) can be checked and the stabilizing pair can be founded using YALMIP parser and SeDuMi solver for MATLAB.

We say that the system (9) is stabilizable in the mean square if there exists a state feedback control providing ESMS of this system. In this case it is possible to formulate the following optimal stabilization problem. Consider the cost functional

$$J = \sum_{n=0}^{\infty} x_n^T Q x_n + 2 x_n^T N u_n + u_n^T R u_n, \quad (14)$$

where $Q = Q^T \geq 0, R = R^T > 0$ and N are given matrices. Find state feedback control in the form (3), which stabilizes in the mean square the system (9) and minimizes (14).

Theorem 3. Let the system (9) be stabilizable in the mean square and let the pair $(Q^{1/2}, A)$ be observable. Then the solution of the optimization problem

$$\mathrm{trace} P \longrightarrow \max, \ P \in S^m \quad (15)$$

with LMI constrains

$$P > 0, \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix} \geq 0, \quad (16)$$

where

$$L_{11} = A_\alpha^T P A_\alpha - P + \sum_{i=1}^{p} \gamma_i^2 A_i^T P A_i + Q,$$
$$L_{12} = A_\alpha^T P B_\alpha + \sum_{i=1}^{p} \gamma_i^2 A_i^T P B_i + N,$$
$$L_{22} = B_\alpha^T P B_\alpha + \sum_{i=1}^{p} \gamma_i^2 B_i^T P B_i + R,$$

is equal to positive definite solution $P \in S^m$ of the matrix quadratic equation

$$A_\alpha^T P A_\alpha - P + \sum_{i=1}^{p} \gamma_i^2 A_i^T P A_i + Q -$$
$$(A_\alpha^T P B_\alpha + \sum_{i=1}^{p} \gamma_i^2 A_i^T P B_i + N)(B_\alpha^T P B_\alpha +$$
$$\sum_{i=1}^{p} \gamma_i^2 B_i^T P B_i + R)^{-1} (A_\alpha^T P B_\alpha +$$
$$\sum_{i=1}^{p} \gamma_i^2 A_i^T P B_i + N)^T = 0. \quad (17)$$

The control law (3) with the gain matrix

$$K = [B_\alpha^T P B_\alpha + \sum_{i=1}^{p} \gamma_i^2 B_i^T P B_i +$$
$$R]^{-1} [A_\alpha^T P B_\alpha + \sum_{i=1}^{p} \gamma_i^2 A_i^T P B_i + N]^T \quad (18)$$

stabilizes in the mean square the system (9) and minimizes the cost functional (14).

5. ROBUST STABILIZATION VIA OUTPUT FEEDBACK

According to (Pakshin, 1994) the output feedback control (4) provides ESMS of the system (9) if and only if there exists a matrix $P \in \mathbb{S}^m$, satisfying the inequalities

$$P > 0,\ \ (A_\alpha - B_\alpha FC)^T P(A_\alpha - B_\alpha FC) - P + \sum_{i=1}^{p} \gamma_i^2 (A_i - B_i FC)^T P(A_i - B_i FC) < 0. \quad (19)$$

The pair of matrices (P, F), is said to be output stabilizing pair if it satisfies (19). It is clear that if (P, F) is an output stabilizing pair then (P, FC) is a state stabilizing pair. The solution of the bilinear inequality (19) is connected with essential difficulties. This inequality can be reduced to two LMIs with respect to mutually inverse matrices but it does not simplify the problem. These difficulties attract attention to alternative approaches.

Suppose that the control (3) stabilizes in the mean square the system (9). If the equation

$$FC = K, \quad (20)$$

has exact solution with respect to matrix F, then this matrix is the gain matrix of output stabilizing control (4) and it can be easily found from this equation. Unfortunately it is possible only with a special structure of the matrix K. To find exact solution of (20) we try impose the structural constrains for the matrix K. Follows (Yu, 2004) write singular value decomposition for the matrix C :

$$C = USV^T,\ U^T U = I,\ V^T V = I, \quad (21)$$

where U and V are orthogonal matrices, S is rectangular matrix which diagonal elements represent singular values of C, and other elements are zeroes. Define

$$F = KC^+, \quad (22)$$

where superscript $+$ denotes Moore-Penrose inverse. Denoting $\hat{A} = V^T AV$, $\hat{B} = V^T B$, $\hat{K} = KV = [\hat{K}_1 \quad \hat{K}_2]$, where $\hat{K}_1 = KV_1$, $V_1 \in \mathbb{R}^{m\times r}$, $\hat{K}_2 = KV_2$, $V_2 \in \mathbb{R}^{m\times(m-r)}$ and taking into account (21) we have

$$A - BFC = V\left(\hat{A} - \hat{B}[\hat{K}_1 \quad \hat{K}_2]\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}\right)V^T, \quad (23)$$

similarly

$$A_i - B_i FC = V\left(\hat{A}_i - \hat{B}_i[\hat{K}_1 \quad \hat{K}_2]\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}\right)V^T. \quad (24)$$

Using these relations we can write

$$KC^+C = KV_1V_1^T = K(I - V_2V_2^T) = K - KV_2V_2^T. \quad (25)$$

The relations (23), (24) do not depend on a specific value of $\hat{K}_2$. At the same time if

$$\hat{K}_2 = KV_2 = 0 \quad (26)$$

then from (25) we obtain that equation (20) holds. So if K is the gain matrix of state feedback stabilizing control (3) satisfying (26), then (22) is the gain matrix of output feedback stabilizing control (4).

The gain matrix K nonlinearly depends on variables X and Y from (12) and attempt to solve the LMIs (12) together with the constraint (26) is not effective. For this reason we take into consideration this constraint using another approach.

Let us consider the cost functional

$$J = \mathrm{E}[\sum_{n=0}^{\infty}(x_n^T Qx_n + u_n^T Ru_n)], \quad (27)$$

where E is expectation operator, $Q = Q^T \geq 0,\ R = R^T > 0$ are given matrices. Suppose that there exists a solution of the following optimal stabilization problem. Find the control in the form (3), which stabilizes in the mean square the system (9) and minimizes the functional (27) along solutions of the system (9) with constrains (26). Solving these problem via Lagrange multiplier method we obtain the following result.

Theorem 4. Let there exists a solution of the equations

$$(A_\alpha - B_\alpha K)Y(A_\alpha - B_\alpha K)^T - Y + \sum_{i=1}^{p} \gamma_i^2 (A_i - B_i K)Y(A_i - B_i K)^T + X = 0,$$

$$(A_\alpha - B_\alpha K)^T P(A_\alpha - B_\alpha K) - P + \sum_{i=1}^{p} \gamma_i^2 (A_i - B_i K)^T P(A_i - B_i K) + K^T RK + Q = 0, \quad (28)$$

$$K = [R + B_\alpha^T PB_\alpha + \sum_{i=1}^{p} \gamma_i^2 B_i^T PB_i]^{-1}[B_\alpha^T PA_\alpha + \sum_{i=1}^{p} \gamma_i^2 B_i^T PA_i][I - V_2(V_2^T Y^{-1}V_2)^{-1}V_2^T Y^{-1}],$$

$$KV_2 = 0,$$

satisfying conditions

$$Y > 0,\ \ (A_\alpha - B_\alpha K)Y(A_\alpha - B_\alpha K)^T - Y + \sum_{i=1}^{p} \gamma_i^2 (A_i - B_i K)Y(A_i - B_i K)^T < 0. \quad (29)$$

Then the control law (4) with the gain matrix $F = KC^+$ stabilizes in the mean square the system (9).

To solve the equations (28) for finding of the gain matrix K of the stabilizing control we propose the following iterative algorithm, which provides that for all steps the condition (29) holds.

(1) Assign the matrices $Q \geq 0,\ R > 0,\ X > 0$ and obtain the initial value of the gain matrix K from (12), (13). This matrix provides stabilization in the mean square of the system (9)

(2) Solve the Sylvester equations with respect to Y_i and P_i :

$$(A_\alpha - B_\alpha K_i)Y_i(A_\alpha - B_\alpha K_i)^T + \sum_{j=1}^{p} \gamma_j^2 (A_j - B_j K_i)Y_i(A_j - B_j K_i)^T + X - Y_i = 0,$$
$$(A_\alpha - B_\alpha K_i)^T P_i (A_\alpha - B_\alpha K_i) - P_i + K_i^T R K_i + Q + \sum_{j=1}^{p} \gamma_j^2 (A_j - B_j K_i)^T P_i (A_j - B_j K_i) = 0.$$

Evaluate the gain increment

$$\Delta K_i = [B_\alpha^T P_i B_\alpha + \sum_{j=1}^{p} \gamma_j^2 B_j^T P_i B_j + R]^{-1} [B_\alpha^T P_i A_\alpha + \sum_{i=1}^{p} \gamma_i^2 B_i^T P A_i][I - V_2 (V_2^T Y_i^{-1} V_2)^{-1} V_2^T Y_i^{-1}] - K_i$$

and update the gain $K_{i+1} = K_i + \beta_i \Delta K_i$, where $0 < \beta_i < 2$ and β_i is chosen so that the system (9) with updated gain is exponentially stable in the mean square. Set $i = i + 1$.

(3) If $\parallel K_i V_2 \parallel < \epsilon$, then stop the procedure and let $F = KC^+$, else go to step 2.

The following theorem gives a method of obtaining the parameter β, which provides both stability in the mean square of the system (9) for all steps of the algorithm and convergence of this algorithm. Denote

$$M_1 = A_\alpha - B_\alpha K_i, \quad W = Y_i, M_2 = -B_\alpha \Delta K_i,$$
$$N_j = A_j - B_j K_i, \quad \widetilde{N_j} = -B_i \Delta K_i,$$
$$a = \parallel X^{-1/2}(M_2 W M_2^T + \sum_{j=1}^{p} \gamma_j^2 \widetilde{N_j} W \widetilde{N_j}^T) X^{-1/2} \parallel_2$$
$$b = 2 \parallel X^{-1/2}(M_1 W M_2^T + \sum_{j=1}^{p} \gamma_j^2 N_j W \widetilde{N_j}^T) X^{-1/2} \parallel_2, \quad j = 1, \ldots, p.$$

Theorem 5. (Convergence of the algorithm). Let the parameter β_i is choused on each step from the condition

$$\beta_i < \min\{\beta_+, 2\},$$

where β_+ is positive root of the quadratic equation

$$a\beta^2 + b\beta - 1.$$

Then the considered algorithm converges and the control law (3) with the gain matrix $K = K_i,\ i = 1, 2, \ldots$ provides ESMS of the system (9).

6. EXAMPLE

Consider the problem of stabilization of the angular longitudinal aircraft motion under given flight parameters uncertainty. The linearized model of this motion is given by the following equations:

$$\begin{aligned} \dot{\vartheta} &= \omega_z, \\ \dot{\omega}_z &= -a_{mz}^{\alpha}\vartheta - a_{mz}^{\omega z}\omega_z + a_{mz}^{\alpha}\Theta + a_{mz}^{\delta}\delta, \\ \dot{\Theta} &= -a_y^{\alpha}\vartheta + a_y^{\alpha}\Theta, \end{aligned} \quad (30)$$

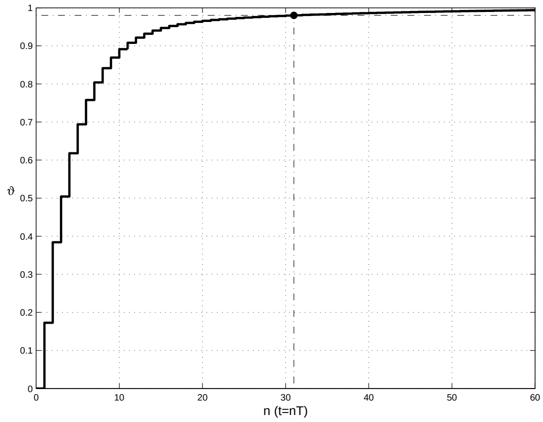

Fig. 1. Typical step response (ϑ)

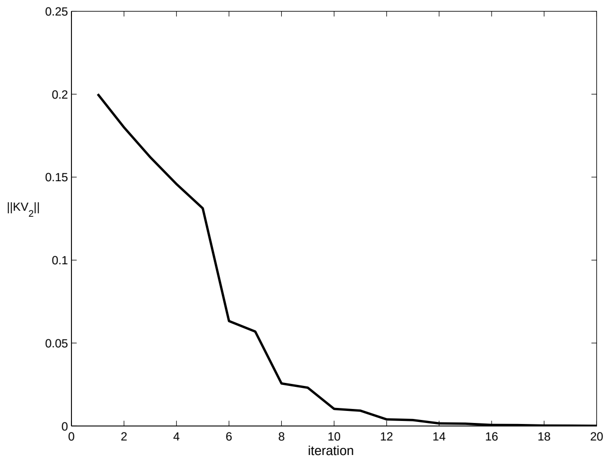

Fig. 2. Trajectory of norm of $K_i V_2$

where ϑ is the pitch angle, ω_z is the angular velocity, $\Theta = \vartheta - \alpha$, α is the angle of attack, δ is the elevator angle. The state and control vectors of the considered system are

$$x(t) = [\vartheta\ \omega_z\ \Theta]^T, \ \ u(t) = \delta,$$

Usually only ϑ and ω_z are available for direct measurement and we have

$$y(t) = [\vartheta\ \omega_z]^T.$$

In the considered flight mode the aircraft has the following parameters uncertainties:

$$a_{mz}^{\alpha} \in a_{mz0}^{\alpha} \pm \Delta a_{mz}^{\alpha},\ \Delta a_{mz}^{\alpha} = 0.3a_{mz0}^{\alpha},$$
$$a_{y}^{\alpha} \in a_{y0}^{\alpha} \pm \Delta a_{y}^{\alpha},\ \Delta a_{y}^{\alpha} = 0.3a_{y0}^{\alpha},$$
$$a_{mz}^{\omega z} \in a_{mz0}^{\omega z} \pm \Delta a_{mz}^{\omega z}\ \Delta a_{mz}^{\omega z} = 0.3a_{mz0}^{\omega z},$$
$$a_{mz}^{\delta} \in a_{mz0}^{\delta} \pm \Delta a_{mz}^{\delta},\ \Delta a_{mz}^{\delta} = 0.3a_{mz0}^{\delta}.$$

The nominal numerical values take from (Krasovskii, 1973): $a_{mz0}^{\alpha} = 78$, $a_{y0}^{\alpha} = -2.8$, $a_{mz0}^{\omega z} = 4.1$, $a_{mz0}^{\delta} = -57$. Suppose that the control law is formed using on-board computer such that

$$u(t) = u(nT) = u_n,$$
$$nT \le t < (n+1)T,\ n = 0, 1, \ldots,$$

where T is sample period.

The problem is to stabilize the system (30) against the given uncertainties by means of constant static output feedback control law (3).

To find the gain matrix of the control taw (3) we use the algorithm from previous section. In this algorithm we obtain the initial value of the gain matrix K_0 by the formula (18), where the matrix P is solution of the optimization problem (15) with LMI constrains (16).

To find the weight matrices Q, N, R first we obtain the cost functional for the continuous time system using Johnson's algorithm (Johnson, 1988) then calculate the parameters of equivalent discrete time model and equivalent cost functional (14).

As a result of computing with sample period $T = 0.015$s. we obtain the gain matrix $F = [-27.5\ -1.8]$. Figure 1 shows a typical step response of the closed-loop system with the computed gain matrix; figures 2,3 show the norm K_iV_2 and scaling factor β in dependence on the number of iterations. All LMI/LME

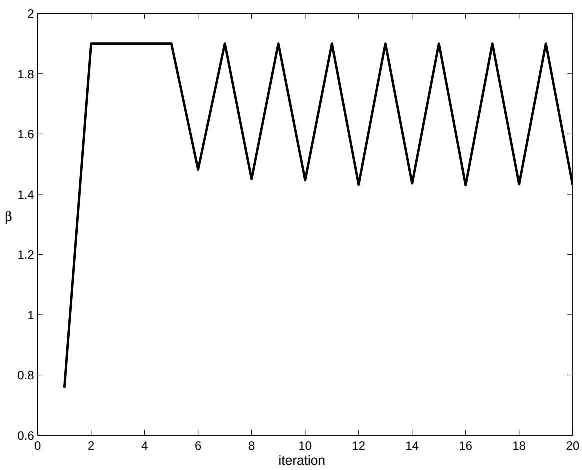

Fig. 3. Trajectory of scaling factor β_i

programming was done within the framework of the YALMIP parser with the SeDuMi solver for MATLAB.

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