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STABILITY OF SISO NONLINEAR SYSTEMS WITH PARAMETERS DISTURBANCES

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Abstract
Systems with piecewise-smooth parameter disturbances are considered. Switched system models describe dynamics of a control plants. System outputs are stabilized by the astatic nonlinear controllers. The system stability is studied with help of the common Lyapunov's function.

Key words
Stability, astatic controller, Lyapunov method, parameter disturbances.

1 Introduction

In this article SISO control systems are considered. The controlled plants have variable parameters disturbances. The disturbances and system variables have comparable rates. The system dynamics is stabilized by a astatic nonlinear controllers. The system stability is studied with help of the Lyapunov method. Many systems are exhibiting switchings between several subsystems that it is dependent on various environmental factors. The systems with switching have numerous applications in control of mechanical systems, automotive industry, aircraft control and many other fields. Some examples of such systems are discussed in [Wicks, Peleties and DeCarlo, 1998]. The systems with switching are usually described as special type of the models named switched system models. However the differential equations with piecewise-smooth parameters can be used to describe dynamics of some switched systems. So under certain conditions it is possible to study the stability of the systems with piecewise-smooth parameter disturbances by the methods gotten for the switched systems and on the contrary. A survey of basic problems on stability of switched systems has been proposed in [Davrazos and Koussoulas]. There have been many researches about robust stabilization [Wicks, Peleties and DeCarlo, 1998], [Zhai,Hai and Antsaklis, 2003], [Narendra and Balakrishnan,1994] and [Branicky, 1994]. The main approach to stability study presented in the literature on switched systems is Lyapunov's functions [Zhai,Hai and Antsaklis, 2003], [Narendra and Balakrishnan,1994] and [Branicky, 1994]. It is common practice if we choose Lyapunov's function approach then we use either the technique of common Lyapunov's function or the technique of multiple Lyapunov's functions [Davrazos and Koussoulas].

2 Problem statement

In this section we first describe the SISO control system with parameters disturbances, which has output derivatives in the feedback loop. And then we formulate the stability problem.

2.1 Plant description

For description of dynamics of the plant with the piecewise-smooth parameter disturbances a switched system model is used. So a controlled plant is given by

$$y^{(n)}(t)+\sum_{i=0}^{n-1} a_i^j(t) y^{(i)}(t)=\sum_{l=0}^{m} b_l^j(t) u^{(l)}(t), \qquad (1)$$

where $y(t)$ and $u(t)$ are the plant output and the control input accordingly, $y^{(r)} = d^r y/dt^r$; $b_0^j \neq 0$, $sign b_l^j b_l^{j+1} = 1$ for $j=\overline{1, J-1}$, $b_j(p)=\sum_{l=0}^{m} b_l^j p^{(l)}$ is Hurwitz's polynomial for $\forall j, l$; p is the differentiation operator; $n>m$. For the parameters a_i^j, b_l^j we enter a disturbance vector $\rho^j = (a_0^j,...,a_{n-1}^j,\ b_0^j,..., b_m^j)$, it's such that

$$\left|\rho_s^j\right| < \epsilon_s^j,\ \left|\frac{d\rho_s^j}{d\,t}\right| < \delta_s^j,\ \epsilon_s^j\ \delta_s^j = const.$$

Assume that ρ_s^j is described by

$$\rho_s^j(t) = \rho_{os} + \bar{\rho}_s^{\,j}(t),\ \bar{\rho}_s^{\,j} = d\,\tilde{\rho}_s^{\,j}, \qquad (2)$$

where $d = const > 0$, $\tilde{\rho}_s^j = \tilde{\rho}_s^j(t)$ is a smooth function for $t_j < t < t_{j+1}$, $t_{j+1} = t_j + \tau_j$, $t_s \le t_j \le t_f$, where

$$(\mathrm{t}_f - t_s) >> \tau_j > t_{tr}. \tag{3}$$

Here τ_j and t_{tr} are active time of the subsystem with jth disturbance vector and transition time conformably, t_s and t_f are initial and final time moments, t_j is a moment of jth switching. We suppose that ρ_s^0 is known, $s = \overline{0, S}$. The control purpose is the output regulation:

$$\lim_{t\to\infty} |r - y| < e_s,$$

where r is a constant reference signal, e_s is an allowable static error.

2.2 Feedback law

First we are determining a control law. Accordingly (2) the system (1) is transformed to the kind

$$y^{(n)} = -\sum_{i=0}^{n-1} a_{0i}\, y^{(i)} + \sum_{l=0}^{L} b_{0l}\, u^{(l)} + m^j, \tag{4}$$

where b_{0l}, $l = \overline{0, L-1}$, are given constant parameters such that $b_0(p) = \sum_{l=0}^{L} b_{0l}\, p^l$ is a Hurwitz's polynomial; $m^j = \left[\bar{\rho}^j\right]^T \theta$ is a new unknown disturbance vector with the bounded norm and bounded first derivative for $t_j < t < t_j$,

$$\left|m^j\right| < \delta_{1m}^j, \left|\tfrac{d\, m^j}{d\, t}\right| < \delta_{2m}^j. \tag{5}$$

Here $\delta_{rm}^j = const$, $r = 1, 2$; θ is a measurements vector, $\theta^T = (-y, ..., -y^{(n-1)}, u, ..., u^{(m)})$. Let a reference dynamics is described as

$$y^{(n)} = F = -\sum_{i=0}^{n-1} a_i^*\, y^{(i)} + a_0^*\, r. \tag{6}$$

The coefficients of the equation (6) are received according to the given quality performance of the transient. Equate right parts (4), (6), then replace unknown disturbance m^j on a function k_m and perform the gotten equation relatively of the control

$$\sum_{l=0}^{L} b_{0l}\, u^{(l)} = \sum_{i=0}^{n-1} (-a_{0i}^* + a_{0i})\, y^{(i)} - k_m + a_0^*\, r. \tag{7}$$

So the equation (7) describes a feedback law. The disturbance m^j is changing relative to the reference dynamics fastly enough. Therefore for k_m we choose the fast algorithm

$$\frac{d\, k_m}{d\, t} = \gamma_m\, l_m \left(y^{(n)} - F\right), \tag{8}$$

where $\gamma_m = const$ is a gain, l_m is an auxiliary function. In particular case we have

$$\frac{d\, k_m}{d\, t} = \gamma_m\, l_{m1}\, sign\, \left(l_{m2}(y^{(n)} - F)\right).$$

The system has the feedback on the output derivatives that can be estimated by the low-inertia dynamic system [Vostrikov and Shpilevaya, 2005]. The astatic nonlinear controller structure is given (7) and (8). It is necessary to prove the stability of the system (4), (7) and (8) in which (3) and (5) are satisfied.

3 Stability study

Enter new variables for the system (4), (7) and (8). A deviation between the adjusted parameter k_m and the uncontrolled disturbance m^j is estimated according to expression

$$e_m^j = k_m - m^j$$

and a deviation of a system trajectory from a reference trajectory

$$\epsilon^j = y^{(n)} - F.$$

Substituting control law (7) into (4), we have

$$y^{(n)} = -\sum_{i=0}^{n-1} a_i^*\, y^{(i)} + b_0^*\, r - k_m + m^j. \tag{9}$$

From (9) we can see that $\epsilon^j = -e_m^j$. Consider the stability of the each subsystem. Prove the convergence $\epsilon^j \to 0$ or $e_m^j \to 0$ with the help of function $V = \left|e_m^j\right|$. The studied function derivative equals

$$\frac{d\, V}{d\, t} = -\gamma_m\, l_m\, \left|e_m^j\right| - \dot{m}^j\, sign\, e_m^j.$$

If we choose the auxiliary function and the gain as

$$l_m > 0, \gamma_m > \delta_{2m}^j$$

then the negative definiteness condition of the function derivative is carried out, $\frac{d\, V}{d\, t} < 0$. Let's notice the function l_m can be

$$l_m = l_0 + y^{-h}\ or\ l_m = l_0 + e_m^h.$$

Here $l_0 = const \ge 0$, $h = 0, 2, 4$. The kind of l_m influences on the properties of the closed loop system.

Proposition 1: The jth subsystem (4), (7) and (8) is a globally asymptotically stable if the conditions $l_m > 0, \gamma_m > \delta_{2m}^j$ are held.

According to Proposition 1 and the condition (3) we suppose that all possible mating commutations of the vector fields determining subsystems are stable. Using theorem given in [Mancilla-Aguilar, 2000] for a switching systems and choosing γ_m as

$$\gamma_m > \delta_{max},\ \delta_{max} = \max_{1 \leq\ j\ \leq\ J} \left|\delta_{2m}^{j}\right| \neq \infty, \qquad (10)$$

we formulate the following proposition.

Proposition 2: If (3) and (10) are held the closed system consisting of the globally asymptotically stable subsystems (4), (7) and (8) is globally asymptotically stable.

If the conditions (2) are correct the plant (4) is similar to system (1). Therefore the closed loop system (1), (7) and (8) is stable too.

4 Conclusion

We considered the systems with piecewise-smooth parameter disturbances. In the system the replacement of the parameters disturbances is made by the fast hybrid disturbance which depends on the system variables and the parameters disturbances. It has allowed obtaining of stationary and non-stationary parts of the plant model. The system output has been stabilized with the help of the astatic nonlinear controller. In result the system has the feedback on the output derivatives that can be estimated by the low-inertia dynamic system. The stability is studied using common Lyapunov's function.

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