

# ANALYSIS OF MEASURING CIRCUITS WITH CAPACITIVE CONVERTERS

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## Abstract

There has been given solution of the technical problem of invariant measurement of the capacitive primary converter (PC) parameters on the alternating current without application of potential-current signals. As a measuring circuit there has been used the voltage divider circuit and as an output signal the angle of phase shift between the two output voltages of the measuring circuit has been used. To form the output signal, temporary separation of the measurement channel has been used. The advantages of the phase method are mostly due to possibilities of using microcontrollers. In the technical solutions of the problems under consideration the microcontroller regulates the measurement process and processes the measurement results.

## Key words.

capacitive primary converter, measuring circuit, phase method

## 1 Introduction

Capacitive sensors (CS) are widely used in information-measuring and controlling systems for conversion of mechanical quantities and many technological parameters. CS are irreplaceable in systems for measuring electro-physical properties of materials and media and their humidity in particular. The wide application of CS is due to the simplicity of their structure and low costs, the stability of their parameters in the wide range of changes in temperature the capacitive primary converter (PC).

In recent years when MEMS-technology and devices have emerged, which combine microelectronic and micromechanical components, the application of capacitive sensitive elements has immensely increased. Wide application of CS demands that simple, precise and reliable meters of PC be developed, which should be compatible with current microcontroller devices for processing the information and controlling the measurement process. The measuring circuit (MC), which along with the PC is a constituent part of CS, is to ensure that the measurement result of the PC informative parameter is invariant to both destabilizing factors, affecting the PC (e.g. voltage and frequency of the feeding generator), and its non-informative parameters.

A lot of recent research has been devoted to the problem of measurement of capacitive PC parameters [Batishchev and Melentiev, 2005; Schepetov, 2008; Arbutov, 2008]. In particular in [Arbutov, 2008] a detailed analysis of measuring circuits, which are used in CS, has been carried

out; and there have been developed measuring circuits of high accuracy with potential-current, time-frequency and code output signals intended for invariant measurement of the capacitive PC informative parameters. In these developments there have been used structural methods for increasing the accuracy of measurement of PC parameters on the basis of time and space separation of measurement channels. In most of these developments for measurement of the PC informative parameters its conversion into potential-current signals is generally used. Such conversion cannot provide exact invariant measurement without correcting circuits, since these signals are exposed to the impact of the measuring circuit feeding voltage changes, offset and drift voltage of operational amplifiers, inner noises and outer interference.

In the paper there have been described some technical solutions of the problem considered, which are suggested by the authors who used the phase method in combination with the method of temporary separation of the measurement channel.

## 2 The basic scheme of measurer

In problems of measurement of electro-physical properties of substances, put into the inter-electrode space of the capacitive PC, the PC equivalent circuit itself is the connection of capacity  $C_x$  and active resistance  $R_x$ , which is called equivalent parallel resistance (EPR) [Jezhora, 2007]. Herewith, depending on the problem being solved by the device the informative parameter can be  $C_x$  as well as  $R_x$ . For instance, while measuring humidity by the dielectric method the informative parameter is  $C_x$ , and EPR  $R_x$  is the parasitic parameter; it represents active losses of energy, occurring in the substance under the study. In its turn  $R_x$  is the informative parameter under control of the quality of insulating substances.

The basic scheme of invariant measurement of  $C_x$  and  $R_x$  is represented in fig. 1 [Mamikonyan B.M. and Mamikonyan Kh.B., 2015]. In the scheme there are indicated: 1- measuring circuit; 2 – the capacitive PC under the study; 3 – programmable generator of sinusoidal signals; 4 – electronic switch; 5 – programmable microcontroller; 6 – digital counting device (DCD); 7 – interface converter (UART-USB); 8 – computer.

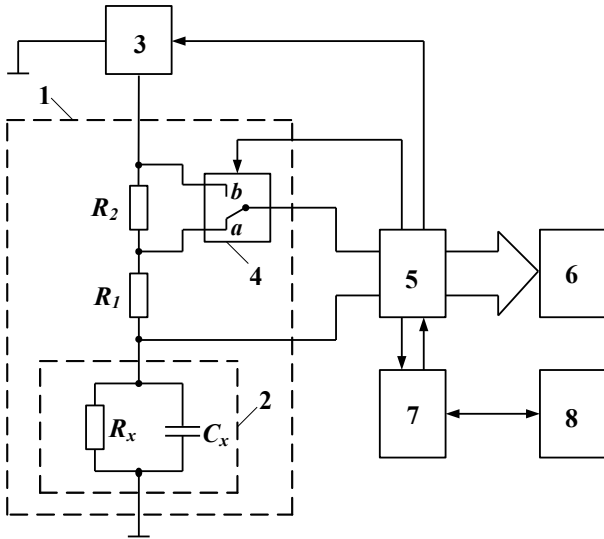


Fig.1. Simplified principal scheme of the meter of the capacitive PC parameters

In the MC two reference resistors are connected with the PC in series: basic  $R_1$  and additional  $R_2$ . The MC, obtained in the result of these connections as a voltage divider, is connected to the generator of sinusoidal signals. The MC has two output voltages relative to a common point; these voltages enter the microcontroller inlet: the total voltage  $u_s$  of the voltage divider, released off the general switch contact and the voltage  $u_x$  of the PC clamps. The output signal of the MC is the angle  $\varphi$  of phase shift between the voltages  $u_s$  and  $u_x$ .

### 3 Calculated formulas

We can find relation between PC parameters and the angle  $\varphi$ . In the starting switch position  $a$  for the value  $\varphi_1$  of the angle  $\varphi$  we can write:

$$\operatorname{tg} \varphi_1 = \frac{\operatorname{Im}(U_s/U_x)}{\operatorname{Re}(U_s/U_x)}. \quad (1)$$

It results in

$$\operatorname{tg} \varphi_1 = \frac{\omega R_1 R_x C_x}{R_1 + R_x}. \quad (2)$$

In the switch position  $b$

$$\operatorname{tg} \varphi_2 = \frac{\omega (R_1 + R_2) R_x C_x}{R_1 + R_2 + R_x}. \quad (3)$$

In these expressions  $\omega$  is the angle frequency of the generator,  $I$  is the MC current. Using formulas (2) and (3) we derive formulas for defining capacitive PC parameters.

**3.1 Formula for defining  $C_x$ .** From (2) and (3) we have:

$$\operatorname{ctg} \varphi_1 - \operatorname{ctg} \varphi_2 = \frac{R_2}{\omega R_1 (R_1 + R_2) C_x}, \text{ from which we obtain:}$$

$$C_x = \frac{R_2}{\omega R_1 (R_1 + R_2)} \cdot \frac{1}{\operatorname{ctg} \varphi_1 - \operatorname{ctg} \varphi_2}. \quad (4)$$

**3.2 Formula for defining  $R_x$ .** We divide formula (3) by

$$\text{formula (2): } \frac{\operatorname{tg} \varphi_2}{\operatorname{tg} \varphi_1} = \frac{(R_1 + R_2) \cdot (R_1 + R_x)}{R_1 [(R_1 + R_2) + R_x]}, \text{ from which it}$$

follows:

$$R_x = \frac{R_1 (R_1 + R_2) m}{R_2 - R_1 m}, \quad (5)$$

where the indication  $m = \frac{\operatorname{tg} \varphi_2}{\operatorname{tg} \varphi_1} - 1$  is accepted.

**3.3 Formula for defining  $\operatorname{tg} \delta$  (tangent of the angle of dielectric losses, occurring in the substance under the investigation).** ES are often used for quality control of the insulating materials, which are used in electro-technical equipment, for example, transformer oil. A sample of the material of the dielectric is put in the space between the capacitive PC covers. In this case resistance  $R_x$  in the substitution circuit of the capacitive PC reflects the thermal energy  $Q = (U_x^2/R_x) \cdot t$ , which is released by leakage current over time  $t$ . The following formula defines the power of these dielectric losses in case of a parallel circuit:

$$P_a = U_x^2 \omega C_x \cdot \operatorname{tg} \delta,$$

where  $\delta$  is the angle of dielectric losses. This expression shows that it is impractical to evaluate the insulation properties of the dielectric based on the value of the power of dielectric losses, as  $P_a$  also depends on the applied voltage. That's why the insulation properties of the dielectric are evaluated according to the value  $\operatorname{tg} \delta$ , which is equal to active-to-reactive ratio, and can also be expressed through parameters  $R_x$  and  $C_x$ . With parallel substitution circuit of the PC the angle  $\delta$  represents backlog of the total measuring current  $I$  from the current  $I_C$ , flowing through the branch  $C_x$ , and it is defined by the formula

$$\operatorname{tg} \delta = \frac{X_C}{R_x} = \frac{1}{\omega R_x C_x}.$$

We should mention, that unlike  $P_a$ , the value  $\operatorname{tg} \delta$  doesn't depend on the geometric sizes of dielectric.

To derive the formula  $\operatorname{tg} \delta$  we transform formulas (2) and (3) in the following way:

$$R_1 \cdot \operatorname{ctg} \varphi_1 = \frac{R_1 + R_x}{\omega R_x C_x}, \quad (R_1 + R_2) \cdot \operatorname{ctg} \varphi_2 = \frac{R_1 + R_2 + R_x}{\omega R_x C_x},$$

$$(R_1 + R_2) \cdot \operatorname{ctg} \varphi_2 - R_1 \cdot \operatorname{ctg} \varphi_1 = \frac{R_2}{\omega R_x C_x}.$$

Consequently

$$\operatorname{tg} \delta = \frac{1}{R_2} [(R_1 + R_2) \cdot \operatorname{ctg} \varphi_2 - R_1 \cdot \operatorname{ctg} \varphi_1]. \quad (6)$$

The obtained formulas (4), (5), (6) allow to perform separate measurement of capacitive PC parameters on the alternating current. This requires that only the angle of phase shift between two output voltages of the measuring two-pole be measured.

### 4 The process of measurement

In the process of measurement the microcontroller regulates the switch position and measures the values of the angles  $\varphi_1$  and  $\varphi_2$  in the corresponding positions of the switch. With the measured values of the angles the microcontroller computes the parameters  $R_x$ ,  $C_x$ ,  $\operatorname{tg} \delta$  by formulas (4), (5), (6) and outputs the measurement results to the digital display; as a digital display seven-

segment LED indicators are used. To increase the reliability of the measurement results the microcontroller performs 10 measurements in each point and displays the average result of these measurements on the indicator. When necessary, we can send digitized signals of the angles  $\varphi_1$  and  $\varphi_2$  from the microcontroller to the computer through the interface converter (e.g. AVR309), process them on the computer and display on the computer monitor.

Since in common case the measurement results of the PC parameters depend on the frequency of the current which feeds the measuring circuit, the problem of either stabilizing this frequency or its control in the measurement process occurs. With respect of this circumstance, as a feeding source of the measuring circuit a programmable generator of sinusoidal signals AD9833 has been used. Each time when measurement is performed the microcontroller sets the frequency of the generator and used this frequency value while calculating the PC parameters; as a result the generator frequency change does not affect the measurement accuracy. The generator voltage stability is not essential, since in formulas (4), (5), (6) the generator voltage does not appear.

Thus, the accuracy of defining the PC parameters only depends on the accuracy of measuring the angle  $\varphi$ , which is performed in the given device by the discrete account method; so the measurement accuracy appears to be higher, as compared to the methods, which use potential – current signals.

It should be noted that while choosing the MC parameters the condition  $R_2 = R_1 = R$  can be ensured without any restrictions, and instead of formulas (4), (5), (6) some simplified formulas can be used:

$$C_X = \frac{1}{2\omega R(ctg\varphi_1 - ctg\varphi_2)}, \quad R_X = \frac{2R(tg\varphi_2 - tg\varphi_1)}{2tg\varphi_1 - tg\varphi_2},$$

$$tg\delta = 2ctg\varphi_2 - ctg\varphi_1.$$

## 5 Other possible measuring circuits

Formulas (4) and (5) indicate, that in the circuit considered the measurement result  $C_X$  depends on the frequency  $\omega$  of the generator current, and  $R_X$  doesn't depend on it. If it is required that the measurement result  $C_X$  not depend on  $\omega$ , in the circuit of fig. 1 reference capacitors  $C_1$  and  $C_2$  can be used instead of resistors  $R_1$  and  $R_2$  (fig. 2a).

In the starting switch position, shown in fig. 2a, when using analogical formula (1), for the angle  $\varphi_1$  we obtain:

$$tg\varphi_1 = -1/\omega R_X (C_X + C_1), \quad (7)$$

in which the minus sign means that the voltage  $u_X$  is ahead of  $u_S$  by phase.

In the second position of the switch we will have:

$$tg\varphi_2 = -1/\omega R_X (C_X + C_{12}), \quad (8)$$

where  $C_{12} = C_1 \cdot C_2 / (C_1 + C_2)$ .

We divide (8) by (7);  $\frac{tg\varphi_2}{tg\varphi_1} = \frac{C_X + C_1}{C_X + C_{12}}$ , consequently,

$$C_X = \frac{C_1 \cdot tg\varphi_1 - C_{12} \cdot tg\varphi_2}{tg\varphi_2 - tg\varphi_1},$$

from which it is obvious that in

this case the measurement result  $C_X$  depends neither on  $R_X$ , nor on the frequency  $\omega$ .

The formula for defining  $R_X$  is  $R_X = \frac{ctg\varphi_1 - ctg\varphi_2}{\omega(C_1 - C_{12})}$ ,

here the separate measurement of  $R_X$  is provided, but the measurement result depends on the frequency  $\omega$ .

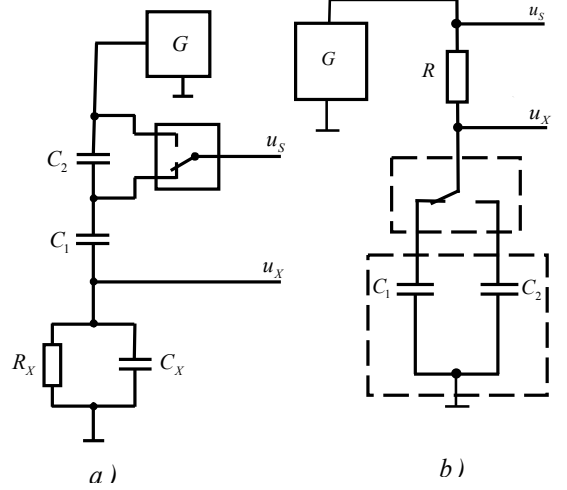


Fig. 2. The schemes of measuring circuits of the informative parameter meter: a) - of the capacitive PC with capacitive voltage divider; b) - of the differential capacitive PC.

To measure the informative parameter of the differential capacitive PC displacements by the phase method the MC is used, the scheme of which is given in fig. 2b. Here the reference resistor  $R$  is switched on through an electronic switch in series with PC. In the switch starting position, shown in fig. 2b, when analogical formula (1) is used, we obtain  $tg\varphi_1 = \omega RC_1$  for the angle  $\varphi_1$ . In the second position of the switch we obtain an analogical formula for the angle  $\varphi_2$ :  $tg\varphi_2 = \omega RC_2$ . For the differential capacitive PC the informative parameter is the increment  $\Delta C$  of the capacities, occurring as an impact of the displacement being measured:  $C_1 = C_0 + \Delta C$ ,  $C_2 = C_0 - \Delta C$ . Here  $C_0$  is the initial value of capacities, it is constant and known (is indicated in the passport data of the PC). We can obtain the formula for defining the increment  $\Delta C$  in the following way:

$$tg\varphi_1 - tg\varphi_2 = \omega R(C_1 - C_2) = 2\omega R\Delta C,$$

consequently,

$$\Delta C = \frac{tg\varphi_1 - tg\varphi_2}{2\omega R}. \quad (9)$$

It is obvious that the generator frequency  $\omega$  affects the result of the PC informative parameter conversion. The wide application of differential capacitive PC is due to the stability of their parameters in the wide range of temperature changes. The presence of two identical halves of the PC in one body allows to perform relative conversion of working capacity. However, the advantages laid down in the differential structure are only revealed completely when the informative parameter is the relative increment of the capacity  $K(C) = \Delta C/C_0$  [Arbuzov et al., 2011]. Such conversion automatically provides logometric

correction of the PC error, allowing to reduce significantly the temperature error and exclude the influence of dielectric permeability of inter-electrode medium on the conversion result. In the investigated case we take into consideration the following relation so as to obtain the conversion function of this informative parameter

$$tg\varphi_1 + tg\varphi_2 = \omega R(C_1 + C_2) = 2\omega RC_0,$$

consequently

$$C_0 = \frac{tg\varphi_1 + tg\varphi_2}{2\omega R}. \quad (10)$$

With respect of expressions (9) and (10) we obtain the formula for  $K(C)$ :

$$K(C) = \frac{tg\varphi_1 - tg\varphi_2}{tg\varphi_1 + tg\varphi_2} = \frac{\sin(\varphi_1 - \varphi_2)}{\sin(\varphi_1 + \varphi_2)}. \quad (11)$$

As it is evident from (11) in this case both the frequency of the generator and the value of the resistor resistance  $R$  (consequently also transition resistance of the switch contacts) do not affect the measurement result.

Measuring circuit parameters are chosen so that at the starting point of the meter when  $C_1 = C_2 = C_0$ , could be the condition  $\varphi_1 = \varphi_2 \approx 45^\circ$ , where maximum sensitivity of conversion is provided. Consequently, from the condition  $tg\varphi = R_0/X_{C_0} = 1$  the value of  $R_0$  should be equal to  $X_{C_0} = 1/\omega C_0$ . For example, when  $C_0 = 50$  pF and  $f = 50$  kHz we should have

$$R_0 = \frac{1}{\omega C_0} = \frac{1}{2\pi \cdot 50 \cdot 10^3 \cdot 50 \cdot 10^{-12}} = 63,7 \text{ kOhm}.$$

From the (11) we obtain an formula for calculating the relative error of the  $K(C)$  measurement:

$$\delta K(C) = \frac{f}{f_0} \cdot (\varphi_1 - \varphi_2) \cdot ctg(\varphi_1 - \varphi_2). \quad (12)$$

From formula (12) it is obvious, that conversion accuracy depends on the clock frequency  $f_0$  of MC generator and the frequency  $f$  of the measurement circuit supply generator. In prototype device  $f_0 = 64$  MHz,  $f = 50$  kHz, therefore the ratio

$$f/f_0 = 50 \cdot 10^3 / 64 \cdot 10^6 = 0,78125 \cdot 10^{-3},$$

i.e. conversion accuracy (table 1) will be estimated by formula

$$\delta K(C) = 0,78125 \cdot 10^{-3} \cdot (\varphi_1 - \varphi_2) \cdot ctg(\varphi_1 - \varphi_2). \quad (13)$$

## 6 Conclusion

Analysis of the MC with capacitive PC, considered here, shows that the proposed method of separate measurement of PC parameters on the alternating current, based on the phase method application, combined with temporary division of the measurement channel, provides digital invariant measurement of the capacitive PC informative

parameters. The proposed technical solution is simple in practical realization and can ensure high measurement accuracy with the limit of permissible basic relative error, which doesn't exceed 0,1%. These advantages are due to the application of phase signals instead of potential - current signals.

Table 1. The results of  $\delta K(C)$  accuracy calculation by formula (13)

$\Delta C$ pF	$C_1$ , pF	$C_2$ , pF	$\varphi_1$ grad	$\varphi_2$ grad	$ctg(\varphi_1 - \varphi_2)$	$\delta K(C)$ %
0	50	50	45.00	45.00	$\infty$	0,078
1	51	49	45.58	44.44	49.99	0.078
2	52	48	46.13	43.85	24.98	0.0781
3	53	47	46.68	43.24	16.63	0.078
4	54	46	47.22	42.63	12.46	0.078
6	56	44	48.25	41.36	8.27	0.078
8	58	42	49.25	40.05	6.17	0.077
10	60	40	50.21	38.67	4.90	0.077
15	65	35	52.45	35.01	3.18	0.076
20	70	30	54.48	30.98	2.30	0.074
25	75	25	56.32	26.59	1.75	0.071
30	80	20	58.01	21.81	1.37	0.067

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