CONTROL OF SYNCHRONIZATION IN DELAY-COUPLED NEURAL HETEROGENEOUS NETWORKS

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Abstract

We study synchronization in delay-coupled neural networks of heterogeneous nodes. It is well known that heterogeneities in the nodes hinder synchronization when becoming too large. We show that an adaptive tuning of the coupling matrix can be used to counteract the effect of the heterogeneity. Our adaptive controller is demonstrated on ring networks of FitzHugh-Nagumo systems which are paradigmatic for excitable dynamics but can also – depending on the system parameters – exhibit self-sustained periodic firing. We show that the adaptively tuned time-delayed coupling enables synchronization even if parameter heterogeneities are so large that excitable nodes coexist with oscillatory ones.

Key words

Oscillation control, Speed-gradient algorithm, Neural networks

1 Introduction

The ability to control nonlinear dynamical systems has brought up a wide interdisciplinary area of research that has evolved rapidly in the past decades [Schöll and Schuster, 2008]. Besides the control of isolated systems, control of dynamics in spatiotemporal systems and on networks has recently gained much interest [Kehrt et al., 2009; Hövel, Dahlem and Schöll, 2010; Flunkert et al., 2010; Omelchenko et al., 2011]. Adaptive control methods are of particular interest in situations where parameters drift or are uncertain and have been successfully applied in the control of network dynamics [Selivanov et al., 2012; Lehnert et al., 2014]. Here, we show that they also can be used to counteract the effect of heterogeneous nodes in the synchronization of delay-coupled networks.

Synchronization in neural networks has gained a lot of attention lately [Lehnert et al., 2011] since it is involved in processes as diverse as learning and visual perception on the one hand [Fries, 2005; Uhlhaas et al., 2009; Singer, 1999] and the occurrence of Parkinson's disease and epilepsy on the other hand [Tass et al., 1998; Poeck and Hacke, 2001; Uhlhaas et al., 2009]. Control of synchronization has so far focused on networks of identical nodes [Zhou, Lu and Lu, 2008; Lu and Qin, 2009; Lu et al., 2012; Selivanov et al., 2012; Guzenko, Lehnert and Schöll, 2013; Lehnert et al., 2014]. However, in realistic networks the nodes will always be characterized by some diversity meaning that the parameters of the different nodes are not identical but drawn from a distribution. It is well known that such heterogeneities in the nodes can hinder or prevent synchronization and that the coupling strength is a crucial parameter in this context [Strogatz, 2000; Sun, Bollt and Nishikawa, 2009]. Here, we develop a method to adaptively control synchronization in networks of heterogeneous nodes.

Our method is based on the speed-gradient (SG) method, which was previously used in the control of delay-coupled networks [Selivanov et al., 2012; Guzenko, Lehnert and Schöll, 2013; Lehnert et al., 2014], however, not in the presence of node heterogeneities. In order to apply the SG method, we suggest a goal function which characterizes the quality of synchronization. Based on this measure an adaptive controller is developed which ensures synchronization even if the parameter heterogeneities become such large that some nodes – if uncoupled – undergo a Hopf bifurcation and behave distinctly different from the other nodes in the network. We demonstrate our algorithm on the FitzHugh-Nagumo (FHN) system [FitzHugh, 1961; Nagumo, Arimoto and Yoshizawa, 1962], a generic model for neural dynamics.

The paper is organized as follows: Section 2 is a recapitulation of the SG method, while Sec. 3 introduces the model. Section 4 discusses development of the adaptive control algorithm for two delay-coupled FHN systems. In Sec. 5, the method is generalized to larger ring networks. Finally, we conclude with Sec. 6.

2 Speed-Gradient Method

In this section, we briefly review the speed-gradient (SG) method [Fradkov, 2007]. Consider a general nonlinear dynamical system

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{g}, t) \tag{1}$$

with state vector $\mathbf{x} \in \mathbb{C}^n$, input (control) variables $\mathbf{g} \in \mathbb{C}^m$, and nonlinear function $\mathbf{F}$. Define a control goal

$$Q(\mathbf{x}(t), t) \leqslant \Delta, \tag{2}$$

for $t \geqslant t^*$, where $Q(\mathbf{x}, t) \geqslant 0$ is a smooth scalar goal function and Δ is the desired level of precision. For example, if we want to force the trajectory of system (1) to follow the desired trajectory $\mathbf{x}^*(t)$, we can use a goal function in the form $Q(\mathbf{x}(t)) = (\mathbf{x}(t) - \mathbf{x}^*(t))^2$.

In order to design a control algorithm, the scalar function $\dot{Q} = \omega(\mathbf{x}, \mathbf{g}, t)$ is calculated, that is, the speed (rate) at which $Q(\mathbf{x}(t), t)$ is changing along the trajectories of Eq. (1):

$$\omega(\mathbf{x}, \mathbf{g}, t) = \frac{\partial Q(\mathbf{x}, t)}{\partial t} + [\nabla_{\mathbf{x}} Q(\mathbf{x}, t)]^{\mathrm{T}} \mathbf{F}(\mathbf{x}, \mathbf{g}, t). \tag{3}$$

Then the gradient of $\omega(\mathbf{x}, \mathbf{g}, t)$ with respect to the input variables is evaluated as

$$\nabla_{\mathbf{g}} \omega(\mathbf{x}, \mathbf{g}, t) = \nabla_{\mathbf{g}} [\nabla_{\mathbf{x}} Q(\mathbf{x}, t)]^{\mathrm{T}} \mathbf{F}(\mathbf{x}, \mathbf{g}, t). \tag{4}$$

Finally, we obtain the control function $\mathbf{g}$ from

$$\mathbf{g}(t) = \mathbf{g}^0 - \psi(\mathbf{x}, \mathbf{g}, t), \tag{5}$$

where the vector function $\psi(\mathbf{x}, \mathbf{g}, t) = \gamma \nabla_{\mathbf{g}} \omega(\mathbf{x}, \mathbf{g}, t)$ with some adaptation gain $\gamma > 0$, and $\mathbf{g}^0 = \mathrm{const}$ is an initial (reference) control value (often $\mathbf{g}^0 = 0$ is assumed). The algorithm (5) is called *speed-gradient* (SG) *algorithm in finite form* since it suggests to change $\mathbf{g}$ proportionally to the gradient of the speed of changing Q.

Several analytic conditions exist guaranteeing that the control goal (2) can be achieved in system (1) and (5). The main condition is the existence of a constant value of the parameter $\mathbf{g}^*$, ensuring attainability of the goal in the system $d\mathbf{x}/dt = \mathbf{F}(\mathbf{x}, \mathbf{g}^*, t)$. Details can be found in the control-related literature [Fradkov, 1979; Shiriaev and Fradkov, 2000].

The idea of this algorithm is the following: The term $-\nabla_{\mathbf{g}} \omega(\mathbf{x}, \mathbf{g}, t)$ points to the direction in which the value of $\dot{Q}$ decreases with the highest speed. Therefore, if one forces the control signal to "follow" this direction, the value of $\dot{Q}$ will decrease and finally be negative. When $\dot{Q} < 0$, then Q will decrease and, eventually, will tend to zero.

3 Model equation

The local dynamics of each node in the network is modeled by the FitzHugh-Nagumo (FHN) differential equations [FitzHugh, 1961; Nagumo, Arimoto and Yoshizawa, 1962]. The FHN model is paradigmatic for excitable dynamics close to a Hopf bifurcation [Lindner et al., 2004], which is not only characteristic for neurons but also occurs in the context of other systems ranging from electronic circuits [Heinrich et al., 2010] to cardiovascular tissues and the climate system [Murray, 1993; Izhikevich, 2000]. Each node of the network is described as follows:

$$\begin{aligned} \varepsilon \dot{u_i} &= u_i - \frac{u_i^3}{3} - v_i + \sum_{j=1}^{N} C_{ij} [u_j(t - \tau) - u_i(t)], \\ \dot{v_i} &= u_i + a_i, \quad i = 1, \dots, N, \end{aligned} \tag{6}$$

where u_i and v_i denote the activator and inhibitor variable of the nodes $i = 1, \dots, N$, respectively. τ is the delay, i.e., the time the signal needs to propagate between node i and j (here we will use $\tau = 1.5$). ε is a time-scale parameter and typically small (here we will use $\varepsilon = 0.1$), i.e., u_i is a fast variable, while v_i changes slowly. The coupling matrix $\mathbf{C} = \{C_{ij}\}$ defines the coupling strength between the nodes.

In the uncoupled system ($C_{ij} = 0$, $i, j = 1, \dots, N$), a_i is a threshold parameter: For $a_i > 1$ the ith node of the system is excitable, while for $a_i < 1$ it exhibits self-sustained periodic firing. This is due to a supercritical Hopf bifurcation at $a_i = 1$ with a locally stable equilibrium point for $a_i > 1$ and a stable limit cycle for $a_i < 1$. In previous publications, networks of homogeneous FHN systems were considered, i.e., $a_1 = a_2 = \ldots = a_N \equiv a$ [Brandstetter, Dahlem and Schöll, 2009; Schöll et al. 2008; Lehnert et al., 2011; Cakan, Lehnert and Schöll, 2009]. In particular, it was shown that for excitable systems, i.e., $a > 1$ and coupling matrices with positive entries zero-lag synchronization is always a stable solution independently of the coupling strength and delay time (as long as both are large enough to induce any spiking at all).

Here, we investigate the case of heterogeneous nodes. In this case, perfect synchronization, i.e., $(u_1, v_1) = \ldots = (u_N, v_N) \equiv (u_s, v_s)$, is no longer a solution of Eq. (6) which can easily be seen by plugging

$(u_1, v_1) = \ldots = (u_N, v_N) \equiv (u_s, v_s)$ into Eq. (6). The node dynamics is then described by

$$\varepsilon \dot{u_s} = u_s - \frac{u_s^3}{3} - v_s + \sum_{j=1}^{N} C_{ij}[u_s(t-\tau) - u_s(t)],$$
$$\dot{v_s} = u_s + a_i, \quad i = 1, \ldots, N, \tag{7}$$

which is obviously not independent of i. This means that a perfectly synchronous solution does not exist in system (6) because the prerequisite for the existence of such a solution is that each node receives the same input if all nodes are in synchrony. However, solutions close to the synchronous solution might exist where the nodes spike at the same (or almost the same) time but with slightly different amplitudes. As we show, these solutions can be reached and stabilized by an adaptive tuning of the coupling matrix.

4 Two delay-coupled FitzHugh-Nagumo systems

This Section studies the most basic network motif consisting of two coupled systems without self-feedback. Consider two coupled FHN-systems with heterogeneous threshold parameters and bidirectional coupling

$$\begin{aligned} \varepsilon \dot{u_1} &= u_1 - \frac{u_1^3}{3} - v_1 + C_{12}[u_2(t-\tau) - u_1(t)], \\ \dot{v_1} &= u_1 + a_1, \\ \varepsilon \dot{u_2} &= u_2 - \frac{u_2^3}{3} - v_2 + C_{21}[u_1(t-\tau) - u_2(t)], \\ \dot{v_2} &= u_2 + a_2. \end{aligned} \tag{8}$$

We now want to apply the SG method to system (8) with the goal to synchronize the two heterogeneous nodes. As discussed above perfect synchronization in the form $(u_1, v_1) = (u_2, v_2)$ is not attainable in this case but the two systems will follow slightly different trajectories in the synchronized case. We, therefore, use as a goal function

$$Q(\mathbf{x}(t), t) = \frac{\varepsilon}{2}(u_1(t) - u_2(t) + a_1 - a_2)^2 + \frac{1}{2}(v_1(t) - v_2(t))^2. \tag{9}$$

The choice (9) ensures that the system follows trajectories for which

$$\begin{aligned} u_1(t) - u_2(t) &\approx -a_1 + a_2, \\ v_1(t) - v_2(t) &\approx 0, \end{aligned} \tag{10}$$

holds for $t \geqslant t^*$. Approximations (10) directly follow from the chosen goal function (9). Thus, the goal function (9) yields synchronization with a shift in the values of the activators and synchronization of the inhibitors of the two nodes.

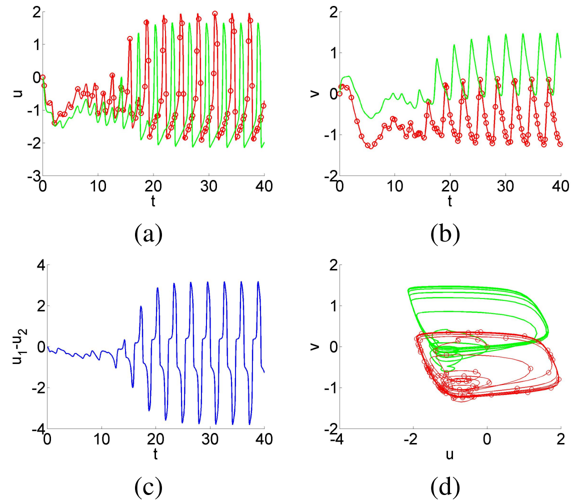

Figure 1. Dynamics of two coupled FitzHugh-Nagumo systems according to Eq. (8) with constant coupling matrix $\mathbf{C}$. Green solid line marks node one, red line with circles marks node two. (a) and (b): time series of the activator and the inhibitor, respectively; (c): difference $u_1 - u_2$ between the activator values, and (d): phase space. Parameters: $N = 2, \varepsilon = 0.1, \tau = 1.5, a_1 = 1.1, a_2 = 0.7, C_{12} = C_{21} = 1$. Initial conditions: $u_i(t) = v_i(t) = 0$, $i = 1, 2$, for $t \in [-\tau, 0]$.

From Eq. (5) with $\mathbf{g} = \mathbf{C}$, system (8), goal function (9), and $\psi(\mathbf{x}, \mathbf{C}, t) = \gamma \nabla_{\mathbf{C}} \omega(\mathbf{x}, \mathbf{C}, t)$ an adaptive law is straightforwardly derived:

$$\begin{aligned} C_{12}(t) &= C_{12}^0 - \gamma(u_1(t) - u_2(t) + a_1 - a_2) \\ &\qquad \times (u_2(t-\tau) - u_1(t)), \\ C_{21}(t) &= C_{21}^0 - \gamma(u_2(t) - u_1(t) + a_2 - a_1) \\ &\qquad \times (u_1(t-\tau) - u_2(t)), \end{aligned} \tag{11}$$

where $\gamma > 0$ is the gain and C_{12}^0, C_{21}^0 are the initial values of the control parameter. The appropriate value of γ has to be determined by numerical simulations. Note that a similar approach has been used to tune the coupling strength in a network of Rössler systems in the paper [Guzenko, Lehnert and Schöll, 2013].

For constant coupling strength, i.e., $\gamma = 0$, the two coupled FHN systems do not synchronize in-phase, but approach an anti-phase synchronized state: Figure 1 shows in panels (a) and (b) the time series of the activators and the inhibitors, respectively, and in panel (d) the phase portrait. Though the first node is in the excitable regime ($a_1 = 1.1 > 1$) both nodes oscillate due to the nonzero coupling matrix $\mathbf{C}$. However, they do not synchronize as can clearly be seen in panel (c) which depicts the difference $u_1 - u_2$ between the activator values. Instead, they phase lock with a phase shift of approximately π which corresponds to an anti-phase synchronized state.

We now adapt the coupling strength according to Eq. (11) in order to synchronize the two systems where

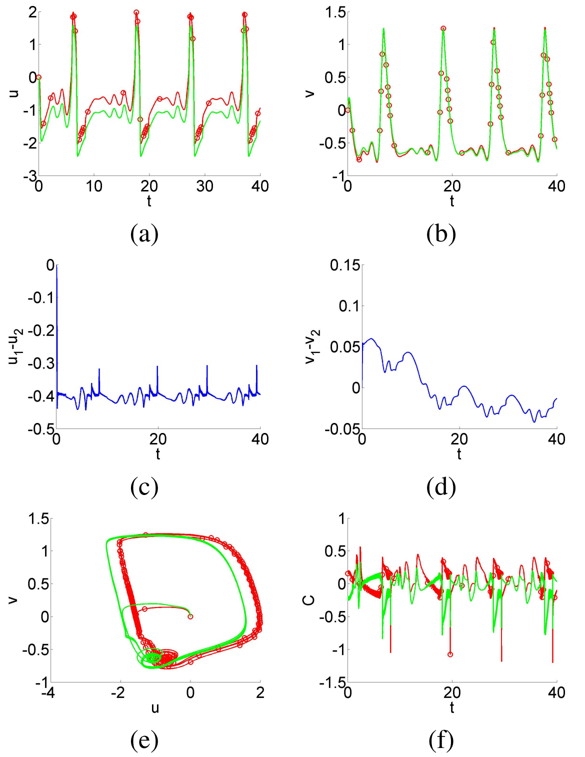

Figure 2. Adaptive control of two coupled FitzHugh-Nagumo systems (Eq. (8)). (a) and (b): time series of the activator and the inhibitor, respectively; (c) and (d): differences between the activator and the inhibitor values, respectively; (e): phase space, (f): time series of the coupling strength adapted according to Eq. (11). Parameters: $\gamma = 20$, $C^0_{12} = C^0_{21} = 0$. Other parameters and initial conditions as in Fig 1.

the result is shown in Fig. 2. The two systems reach the desired synchronized state (see the time series of the activators and the inhibitors in Fig. 2(a),(b), respectively, and the difference between their values in Fig. 2(c),(d)). Thus, the control is successful.

5 Adaptive Synchronization in ring networks

We now want to apply our method to larger networks. To this end, we consider a ring network of N nodes where the coupling matrix $\mathbf{C}$ has the following form

$$\mathbf{C} = \begin{pmatrix} 0 & C_{12} & 0 & \cdots & 0 \\ 0 & 0 & C_{23} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & C_{(N-1)N} \\ C_{N1} & 0 & 0 & \cdots & 0 \end{pmatrix}. \quad (12)$$

As in the case of two nodes, from Eq. (5) with $\mathbf{g} = \mathbf{C}$, goal function

$$Q(\mathbf{x}(t), t) = \frac{1}{2}\sum_{i=1}^{N} \big[\varepsilon(u_i(t) - u_{(i+1) \bmod N}(t) + a_i - a_{(i+1) \bmod N})^2 + (v_i(t) - v_{(i+1) \bmod N}(t))^2\big], \quad (13)$$

and

$$\psi(\mathbf{x}, \mathbf{C}, t) = \gamma \nabla_{\mathbf{C}} \omega(\mathbf{x}, \mathbf{C}, t), \quad (14)$$

we derive the following adaption law

$$C_{i(i+1) \bmod N}(t) = C^0_{i(i+1) \bmod N} - \gamma\big[2u_i(t) - u_{(i-1) \bmod N}(t) - u_{(i+1) \bmod N}(t) + 2a_i - a_{(i-1) \bmod N}(t) - a_{(i+1) \bmod N}(t)\big] \times (u_i(t) - u_{(i+1) \bmod N}(t - \tau)), \quad (15)$$

where γ is the gain and $C^0_{i(i+1) \bmod N}$, $i = 1, \ldots, N$ is the initial value of the corresponding control parameter.

Figure 3 presents the results of a simulation of the plant (6) with adaption law (15). This method provides a synchronization of the activators with the shift in the values (see the time series of the activators in Fig. (3)(a)) and synchronization of the inhibitors (see the time series of the inhibitors in Fig. (3)(b)) for all nodes.

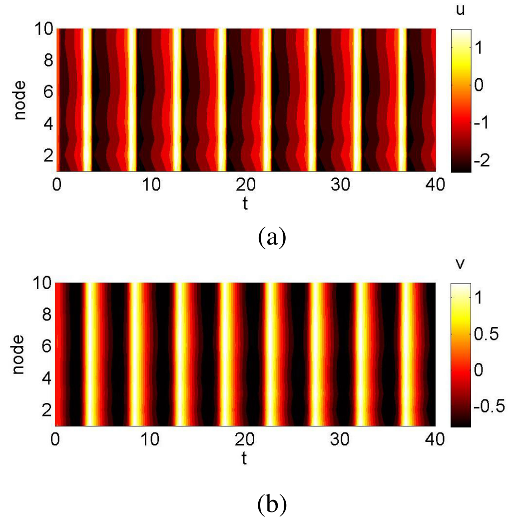

Figure 3. Adaptive control of synchronization of a ring of ten FitzHugh-Nagumo systems according to Eq. (6) with coupling matrix (12). (a) and (b): time series of the activator and the inhibitor of all nodes, respectively. Parameters: $N = 10, \varepsilon = 0.1, \tau = 1.5$, $\gamma = 10$, $C^0_{i(i+1) \bmod N} = 0$, $i = 1, \ldots, N$. Initial conditions: $u_i(t) = 0, v_i(t) = 0, i = 1, \ldots, N$, for $t \in [-\tau, 0]$.

6 Conclusion

We have proposed a novel adaptive method for controlling synchrony in heterogeneous networks. It is well known that networks with heterogeneous nodes are much less likely to synchronize than networks of identical nodes. Furthermore, synchrony will take place in a state where the trajectories of the different nodes are not identical but small deviations can be observed. We have suggested a goal function to characterize this type of synchrony. Based on this goal function and the speed-gradient (SG) method, we have derived an adaptive controller which tunes the coupling matrix such that synchrony is stable despite the node heterogeneities.

We have demonstrated our method on networks of FitzHugh-Nagumo systems, a neural model which is considered to be generic for excitable systems close to a Hopf bifurcation. We have started our consideration from the simple motif of two delay-coupled, heterogeneous nodes. After that, we have generalized our method to larger networks and applied it to ring networks. It has been shown that our method enables synchronization even if the node parameters are chosen such diverse that one of the systems would exhibit self-sustained oscillations without coupling, while the other one would remain in a stable equilibrium point, i.e., one of the uncoupled systems is above, and the other is below the Hopf bifurcation.

Given the paradigmatic nature of the FitzHugh-Nagumo system, we expect our method to be applicable in a wide range of excitable systems. Furthermore, the application of the SG method to the control of networks with heterogeneous nodes suggests that other adaptive controllers that are based on the SG method [Selivanov et al., 2012; E. Schöll et al., 2012; Guzenko, Lehnert and Schöll, 2013; Lehnert et al., 2014] are also robust towards heterogeneities.

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