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ADAPTIVE SYNCHRONIZATION IN NONIDENTICAL LURIE SYSTEMS WITH LIPSCHITZ NONLINEARITIES

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Abstract
For a network of interconnected Lurie systems with Lipschitz nonlinearities an adaptive leader-follower synchronization problem by output feedback is considered. The structure of decentralized controller and adaptation algorithm is proposed based on speed-gradient method. Sufficient conditions of synchronization are established. The main contribution of the paper is adaptive controller design and analysis of synchronization in network with nonidentical nodes under conditions of incomplete measurements, incomplete control and uncertainty.

Key words
Adaptive control, networks with nonidentical nodes, Lurie systems

1 Introduction

An enormous interest is observed recently in control of networks. The area is both relatively new and practically important since many physical systems can be considered as interconnected systems, arrays or networks. The list of such systems includes telecommunication networks, molecular ensembles, biological systems, trophic chains, embedded systems, vehicle or robots formations etc. Development of such systems is inspired by onrush of information and communication technologies, including wireless communications and wireless sensors. An interest is growing in modeling and control of biological, biochemical and social networks. However, coordinating controller design is getting more and more difficult due to complexity of spatially distributed networks. One of the most hard obstacles are restrictions caused by limited information exchange between subsystems. Though decentralized control problems are well studied [Siljak, 1990; Fradkov, 1990; Druzhinina and Fradkov, 1999], new settings arise creating more and more complex problems, e.g. control via communication channels of limited capacity. New problems require simultaneous consideration of control, communication and computing issues as well as application of physics (statistical mechanics) approaches.

Despite a great interest in control of network, only a restricted class of them are currently solved. E.g. in existing papers mainly linear models of subsystems are considered [Matveev and Savkin, 2009]. Besides in the literature on stabilization and synchronization availability of the whole state vector for measurement as well as appearance of control in all equations for all nodes is assumed [Lu and Chen, 2005; Yao, Hill, Guan and Wang, 2006; Zhou, Lu and Lu, 2006; Zhong, Dimirovski and Zhao, 2007].

In this paper we consider a network of nonidentical systems in Lurie form i.e. system models can be split into linear and nonlinear parts. Linearity of interconnections is not assumed i.e. links between subsystems can also be nonlinear. In the contrary to known works on adaptive synchronization of networks, see [Zhou, Lu and Lu, 2006; Zhong, Dimirovski and Zhao, 2007], only some output function is available and control appears only in a part of the system equations. It is also assumed that some plant parameters are unknown. The leading subsystem is assumed to be isolated and the control objective is to approach the trajectory of the leading subsystems by all other ones under conditions of uncertainty. All nonlinear functions as well as interconnection functions are assumed to be Lipschitz continuous.

To solve the posed problem the results of [Fradkov, 1990; Fradkov, Miroshnik and Nikiforov, 1999] and Yakubovich-Kalman lemma [Yakubovich, 1962] are employed. Adaptation algorithm is designed by the speed-gradient method. It is shown that the control goal is achieved under conditions of leader passivity and matching conditions, if the interconnection strengths satisfy some inequalities.

In paper [Junussov and Fradkov, 2009] case of identical nodes are studied. Synchronization in networks consisting of nonidenical nodes with other types of internal nonlinearity are studied in [Fradkov, Junussov and Ortega 2009].

The obtained results are illustrated by example: synchronization of an array of Chua circuits. The theoretical conclusions are confirmed by simulation results.

2 Problem statement

In this section the formal problem statement is given.

Consider a network of d interconnected nonidentical subsystems S_i, $i = 1, \dots, d, d \in \mathbb{N}$. Let subsystem S_i be described by the following equation

$$\dot{x}_i = A_i x_i + B_i u_i + \psi_0(x_i) + \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(x_i - x_j), \quad y_i = C^{\mathrm{T}} x_i, \qquad i = 1, \dots, d, \tag{1}$$

where $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^1, \alpha_{ij} \in \mathbb{R}^1, y_i \in \mathbb{R}^l$. Functions $\varphi_{ij}(\cdot)$, $i = 1, \dots, d$, $j = 1, \dots, d$, describe interconnections between subsystems. We assume $\varphi_{ii} = (0, 0, 0)^{\mathrm{T}}, i = 1, \dots, d$. Let matrices A_i, B_i and functions $\varphi_{ij}(\cdot)$, $i = 1, \dots, d, j = 1, \dots, d$, depend on the vector of unknown parameters $\xi \in \Xi$, where Ξ is known set.

Introduce equations of an ith isolated subsystem $\dot{x}_i = A_i x_i + B_i u_i + \psi_0(x_i), y_i = C^{\mathrm{T}} x_i, i = 1, \dots, d$ and equations of the leading (master) system

$$\dot{\overline{x}} = A\overline{x} + B\overline{u} + \psi_0(\overline{x}), \quad \overline{y} = C^{\mathrm{T}} \overline{x}, \tag{2}$$

where $\overline{u}(t) \in \mathbb{R}^1$ is specified in advance. Let A, B, C and $\psi_0(\cdot)$ be known and not depending on the vector of unknown parameters $\xi \in \Xi$.

Let the control goal be specified as convergence of all subsystem trajectories to the trajectory of the leader:

$$\lim_{t \to +\infty} (x_i(t) - \overline{x}(t)) = 0, \qquad i = 1, \dots, d. \tag{3}$$

The adaptive synchronization problem is to find a decentralized controller $u_i = \mathcal{U}_i(y_i, t)$ ensuring the goal (3) for all values of unknown plant parameters.

3 Controller structure

Denote $\sigma_i(t) = \mathrm{col}(y_i(t), \overline{u}(t))$. Let the main loop of the adaptive system be specified as set of linear tunable local control laws:

$$u_i(t) = \tau_i(t)^{\mathrm{T}} \sigma_i(t), \quad i = 1, \dots, d, \tag{4}$$

where $\tau_i(t) \in \mathbb{R}^{l+1}, i = 1, \dots, d$ are tunable parameters. To design adaptation laws for $\tau_i(t), i = 1, \dots, d$, the speed-gradient (SG) method is used. Following adaptation algorithm is derived

$$\dot{\tau}_i = -g^{\mathrm{T}}(y_i - \overline{y})\Gamma_i \sigma_i(t), i = 1, \dots, d, \tag{5}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ – $(l+1) \times (l+1)$ matrices, $g \in \mathbb{R}^l$.

In the next section condition of achievement of the goal (synchronization) and boundedness of $\tau_i(t), i = 1, \dots, d$, will be given.

4 Synchronization conditions

For analysis of the system dynamics the following assumptions are made.

A1) The functions $\psi_0(\cdot)$, $\varphi_{ij}(\cdot)$,$i = 1, \dots, d$, $j = 1, \dots, d$ are globally Lipschitz [1]:

$$\|\psi_0(x) - \psi_0(x')\| \leq L\|x - x'\|, \quad L > 0,$$
$$\|\varphi_{ij}(x) - \varphi_{ij}(x')\| \leq L_{ij}\|x - x'\|, \quad L_{ij} > 0,$$
$$i = 1, \dots, d, \quad j = 1, \dots, d.$$

A2)(matching conditions) For each $\xi \in \Xi, i = 1, \dots, d$ there exist vectors $\nu_i = \nu_i(\xi) \in \mathbb{R}^l$ and numbers $\theta_i = \theta_i(\xi) > 0$ such that

$$A = A_i + B_i \nu_i^{\mathrm{T}} C^{\mathrm{T}}, B = \theta_i B_i. \tag{6}$$

Consider real matrices $H = H^{\mathrm{T}} > 0, g$ of size $n \times n, l \times 1$ correspondingly and a number $\rho > 0$ such that:

$$HA_* + A_*^{\mathrm{T}} H < -\rho H, \quad HB = Cg, \tag{7}$$

where $A_* = A + LI_n$

Let $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B, \chi^*(s) = C^{\mathrm{T}}(sI_n - A - LI_n)^{-1}B, z \in \mathbb{C}$. Denote by $\lambda_{min}(H), \lambda_{max}(H)$ minimum and maximum eigenvalues of matrix H. Let's introduce notation ρ_* for stability degree of the function's $g^{\mathrm{T}}\chi^*(s)$ denominator. Further, denote $\gamma = \rho_*/(4d\lambda_*)$ where $\lambda_* = \lambda_{max}(H)/\lambda_{min}(H)$ is condition number of matrix H.

Theorem 1. Let for all $\xi \in \Xi$ assumptions A1, A2 hold. Let $B \neq 0$, matrix $A_* = A + LI_n$ be Hurwitz and for some $g \in \mathbb{R}^l$ the following frequency domain conditions hold:

$$\mathrm{Re}\, g^{\mathrm{T}}\chi^*(i\omega) > 0, \quad \lim_{\omega \to \infty} \omega^2\, \mathrm{Re}\, g^{\mathrm{T}}\chi^*(i\omega) > 0 \tag{8}$$

for all $\omega \in \mathbb{R}^1$. Then there exist $H = H^{\mathrm{T}} > 0, \rho > 0$ such that relations (7) hold.

In addition, if the inequalities

$$\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| < \gamma, i = 1, \dots, d \tag{9}$$

hold then adaptive controller (4), (5) ensures achievement of the goal

$$\lim_{t \to +\infty} (x_i(t) - \overline{x}(t)) = 0, \tag{10}$$

[1] Norms are Euclidean hereafter. Identity matrix of size n is denoted by I_n.

and boundedness of functions $\tau_i(t)$ on $[0,\infty)$ for all solutions of the closed-loop system (1), (2), (4), (5).

Proof. We need two auxiliary results. The first one is a version of Yakubovich-Kalman Lemma. It can be found in [Fradkov, Miroshnik and Nikiforov, 1999].

Lemma 1. *Let A, B, C be $n\times n, n\times m, n\times l$ real matrices and $u\in\mathbb{R}^m$, $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B, \operatorname{rank} B = m$. Then the following statements are equivalent:*
1) there exists matrix $H = H^{\mathrm{T}} > 0$ such that

$$HA + A^{\mathrm{T}}H < 0, HB = C; \tag{11}$$

2) polinomial $\det(sI_n - A)$ is Hurwitz and following frequency domain conditions hold

$$\operatorname{Re} u^{\mathrm{T}}\chi(i\omega)u > 0, \quad \lim_{\omega\to\infty}\omega^2 \operatorname{Re} u^{\mathrm{T}}\chi(i\omega)u > 0$$

for all $\omega\in\mathbb{R}^1$, $u\in\mathbb{R}^m, u\neq 0$.

The second result is related to connective stability of large scale interconnected systems. It is theorem 2.18 from [Fradkov, 1990] and can be can be derived from [Fradkov, Miroshnik and Nikiforov, 1999] (Theorem 7.6).

Consider a system $\mathcal{S}$ consisting of d interconnected subsystems $\mathcal{S}_i$, dynamics of each being described by the following equation:

$$\dot{x}_i = F_i(x_i,\tau_i,t) + h_i(x,\tau,t), \quad i = 1,\dots,d, \tag{12}$$

where $x_i \in \mathbb{R}^{n_i}$ – state vector, $\tau_i \in \mathbb{R}^{m_i}$ - vector of inputs (tunable parameters) of subsystem, $x = \operatorname{col}(x_1,\dots,x_d)\in\mathbb{R}^n$, $\tau = \operatorname{col}(\tau_1,\dots,\tau_d)\in\mathbb{R}^m$ - aggregate state and input vectors of system S, $n = \sum n_i$, $m = \sum m_i$. Vector-function $F_i(\cdot)$ describes local dynamics of subsystem S_i, and vectors $h_i(\cdot)$ describe interconnection between subsystems.

Let $Q_i(x_i,t)\geq 0, i = 1,\dots,d$ be local goal functions and let the control goal be:

$$\lim_{t\to\infty} Q_i(x_i,t) = 0, \qquad i = 1,\dots,d. \tag{13}$$

For all $i = 1,\dots,d$ we assume existence of smooth vector functions $x_i^*(t)$ such that $Q_i(x_i^*(t),t) \equiv 0$, i.e. $x_i^* = \operatorname{argmin}_{x_i} Q_i(x_i,t)$. Decentralized speed-gradient algorithm is introduced as follows:

$$\dot{\tau}_i = -\Gamma_i\nabla_{\tau_i}\omega_i(x_i,\tau_i,t), \qquad i = 1,\dots,d, \tag{14}$$

where

$$\omega_i(x_i,\tau_i,t) = \frac{\partial Q_i}{\partial t} + \nabla_{x_i}Q_i(x_i,t)^{\mathrm{T}}F_i(x_i,\tau_i,t),$$

$\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$, $m_i\times m_i$ - matrix.

Lemma 2. Suppose the following assumptions hold for the system $\mathcal{S}$:

1. Functions $F_i(\cdot)$ are continuous in x_i, t, continuously differentiable in τ_i and locally bounded in $t > 0$; functions $Q_i(x_i,t)$ are uniformly continious in second argument for all x_i in bounded set, functions $\omega_i(x_i,\tau_i,t)$ are convex in τ_i; there exist constant vectors $\tau_i^*\in\mathbb{R}^{m_i}$ and scalar monotonically increasing functions $\kappa_i(Q_i), \rho_i(Q_i)$ such that $\kappa_i(0) = \rho_i(0) = 0, \lim_{Q_i\to+\infty}\kappa_i(Q_i) = +\infty$
$$\omega_i(x_i,\tau_i^*,t) \leq -\rho_i(Q_i(x_i,t)), \tag{15}$$
and $Q_i(x_i,t)\geq\kappa_i(\|x_i - x_i^*(t)\|)$.
2. functions $h_i(x,\tau,t)$ are continuous and satisfy the following inequalities
$$|\nabla_{x_i}Q_i(x_i,t)^{\mathrm{T}}h_i(x,\tau,t)| \leq \sum_{j=1}^{d}\mu_{ij}\rho_j(x_j,t), \tag{16}$$
where matrix $M - I$ is Hurwitz, $M = \{\mu_{ij}\}$, $\mu_{ij} > 0$, I is identity matrix.

Then system (12),(14) is globally asymptotically stable in variables $x_i - x_i^*(t)$, all trajectories are bounded on $t\in[0,+\infty)$ and satisfy (13).

Denote $\tau_i^* = \operatorname{col}(\nu_i,\theta_i)$. Following Lyapunov function is used in the proof of Lemma 2:

$$V(x,\tau,t) = \sum_{i=1}^{d}\beta_i V_i,$$

where

$$V_i(x_i,\tau_i,t) = Q_i(x_i,t) + \frac{1}{2}(\tau_i - \tau_i^*)^{\mathrm{T}}\Gamma_i^{-1}(\tau_i - \tau_i^*).$$

The following goal function is suitable for prove of Theorem 1:

$$Q(z_i) = \frac{1}{2}\,z_i^{\mathrm{T}}Hz_i, \quad H = H^{\mathrm{T}} > 0,$$

where $z_i = x_i - \overline{x}$. Introduce auxiliary error subsystems:

$$\begin{aligned}\dot{z}_i =& A_i x_i + B_i u_i + \psi_0(x_i) + \sum_{j=1}^{d}\alpha_{ij}\varphi_{ij}(x_i - x_j) - \\ & (A\overline{x} + B\overline{u} + \psi_0(\overline{x}))\,,\\ \tilde{y}_i =& C^{\mathrm{T}}z_i, \qquad i = 1,\dots,d.\end{aligned} \tag{17}$$

Let's evaluate of the derivative of $Q(z_i)$ along trajectories of isolated subsystems:

$$\omega_i(x_i, \overline{x}, \tau_i) = z_i^{\mathrm{T}} H[A_i x_i + B_i \tau_i^{\mathrm{T}}(t)\tilde{y}_i + \psi_0(x_i) - A\overline{x} - B\overline{u} - \psi_0(\overline{x})]. \tag{18}$$

By taking $\tau_i = \tau_i^*$ for $i = 1, \ldots, d$ (see (6)) we obtain

$$\begin{aligned}\omega_i(x_i, \overline{x}, \tau_i^*) = z_i^{\mathrm{T}} H[A_i x_i + B_i(\nu_i C^{\mathrm{T}} x_i + \theta_i \overline{u}) +\\ \psi_0(x_i) - A\overline{x} - B\overline{u} - \psi_0(\overline{x})] =\\ z_i^{\mathrm{T}} H[A z_i + (\psi_0(x_i) - \psi_0(\overline{x}))].\end{aligned}$$

Applying assumption A1 we obtain

$$\omega_i(z_i, \overline{x}, \tau_i^*) \le z_i^{\mathrm{T}} H(A + L I_n) z_i.$$

Let's apply Lemma 1 with A_* instead of A and Cg instead of C :

$$\omega_i(x_i, \overline{x}, \tau_i^*) \le \frac{1}{2} z_i^{\mathrm{T}} (H A_* + A_*^{\mathrm{T}} H) z_i.$$

Note that relation $HB = Cg$ is used for derivation of adaptive algorithm (5).

By taking $\rho_i(Q) = \rho \cdot Q$ we ensure that (15) holds for $i = 1, \ldots, d$. Other conditions from the first part of Theorem 2 hold, since the right hand side of the system (17) and function $Q_i(z_i)$ are continuous in z_i functions not depending in t for any $i = 1, \ldots, d$. Convexity condition is valid since the right hand side of (18) is linear in τ_i.

The interconnection condition (16) in our case reads:

$$|\nabla_{z_i} Q(z_i)^{\mathrm{T}} \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j)| \le \sum_{j=1}^{d} \mu_{ij} \rho \cdot Q(z_j), \tag{19}$$

where $i = 1, \ldots, d$, and matrix $M - I$ should be Hurwitz ($M = \{\mu_{ij}\}$, $\mu_{ij} > 0$). It can be derived that for $i = 1, \ldots, d$ inequality (19) holds if following inequality holds:

$$\frac{1}{2}\left(\sum_{j=1}^{d} |\alpha_{ij} L_{ij}|\right)\left(\sum_{j=1}^{d} 3\|z_i\|^2 + \|z_j\|^2\right) \le$$

$$\frac{\rho}{2\lambda_*} \sum_{j=1}^{d} \mu_{ij} \|z_j\|^2.$$

Without loss of generality we can take $\rho = \rho_*$. Denote $\zeta = 2d\gamma \left(\sum_{j=1}^{d} |\alpha_{ij} L_{ij}|\right)^{-1}$. Following choice of M ensures correctness of last inequality and Hurwitz property of $M - I$:

$$\mu_{ij} = \begin{cases} \dfrac{1}{2\zeta}(3d + 1), & i = j \\ \dfrac{1}{2\zeta}, & i \ne j \end{cases}$$

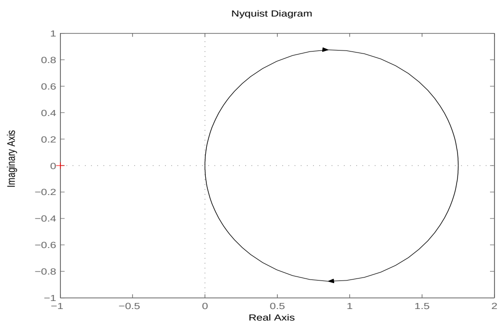

Figure 1. Nyquist plot of $\chi^*(i\omega), \omega \in \mathbb{R}^1$.

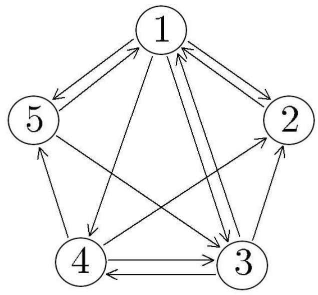

Figure 2. Connections graph

5 Example. Network of Chua circuits

Chua circuit is a well known example of simple nonlinear system possessing complex chaotic behavior [Wu and Chua, 1995]. Its trajectories are unstable at some values of parameters and it is represented in the Lurie form. Let us apply our results to synchronization with leading subsystem in the network of five interconnected nonidentical Chua systems.

Let $m_0 = -8/7, m_1 = -5/7, p = 2, q = 4, b = 1$ and $g = 1$.

Let the leading subsystem be described by the equation

$$\dot{\overline{x}} = A\overline{x} + B\overline{u} + \psi_0(\overline{x}), \quad \overline{y} = C^{\mathrm{T}}\overline{x},$$

where $\overline{x} \in \mathbb{R}^3$ is state vector of the system, $\overline{y} \in \mathbb{R}^1$ is output available for measurement, $\overline{u}$ is scalar control variable, $\psi_0(\overline{x}) = \mathrm{col}(pv(\overline{x}_1), 0, 0)$, where $v(x) =$

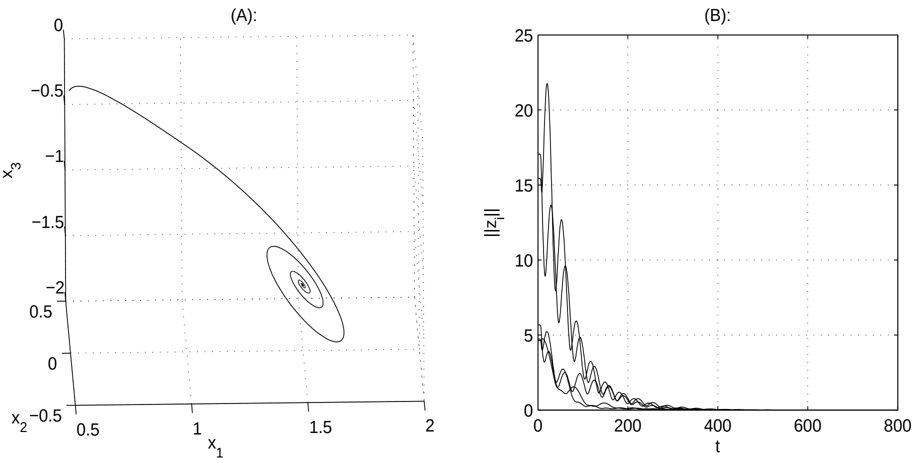

Figure 3. (A): Phase portrait of leading subsystem, (B): $\|z_i\|$

$-0.5(m_0 - m_1)(|x+1| - |x-1| - 2x)$. Further, let

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{pmatrix},$$

$B = \operatorname{col}(b, 0, 0), C = \operatorname{col}(1, 0, 0)$. Apparently Lipschitz constant $L = p|\frac{m_0 - m_1}{2}|$.

Matrix A_* is Hurwitz. Transfer function $\chi^*(s) = C^{\mathrm{T}}(sI - A - LI_n)^{-1}B_L \approx (s^2 + 0.14s + 3.76)/(s^3 + 0.71s^2 + 3.84s + 2.15)$. It is seen from the Nyquist plot of $\chi(i\omega), \forall\omega \in \mathbb{R}^1$, presented on Fig. 1, that first frequency domain inequality of (8) holds. The second frequency domain inequality of (8) also holds since relative degree of $\chi(s)$ is equal to one and highest coefficient of its numerator is positive.

Let subsystem S_i for $i = 1, \ldots, 5$ be described by (1) with $u_i, \alpha_{ij} \in \mathbb{R}^1$. By choosing $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = (2, 7, 1, 8, 2), \theta_i = 1/i, i = 1, \ldots, 5$ and using (6) we obtain matrices A_i, B_i for $i = 1, \ldots, 5$, which are not equal, i.e. nodes are nonidentical. Denote $\varphi_{ij} = \varphi_{ij}(x_i - x_j), i = 1, \ldots, 5, j = 1, \ldots, 5$. Let $\varphi_{14}, \varphi_{25}, \varphi_{32}, \varphi_{42}, \varphi_{45}, \varphi_{52}, \varphi_{53}$, be equal to $(0, 0, 0)^{\mathrm{T}}$. Further, let

$$\begin{aligned}
&\varphi_{12} = (\sin(x_{11} - x_{21}), 0, 0)^{\mathrm{T}}, \\
&\varphi_{13} = (0, x_{12} - x_{32}, 0)^{\mathrm{T}}, \\
&\varphi_{15} = (0, 0, \sin(x_{13} - x_{53}))^{\mathrm{T}}, \\
&\varphi_{21} = (x_{21} - x_{11}, 0, x_{23} - x_{13})^{\mathrm{T}}, \\
&\varphi_{23} = (0, \sin(x_{22} - x_{32}), 0)^{\mathrm{T}}, \\
&\varphi_{24} = (0, x_{22} - x_{42}, 0)^{\mathrm{T}}, \\
&\varphi_{31} = (\sin(x_{31} - x_{11}), 0, 0)^{\mathrm{T}}, \\
&\varphi_{34} = (\sin(x_{31} - x_{41}), 0, 0)^{\mathrm{T}}, \\
&\varphi_{35} = (x_{31} - x_{51}, x_{32} - x_{52}, x_{33} - x_{53})^{\mathrm{T}}, \\
&\varphi_{41} = (0, \sin(x_{42} - x_{12}), 0)^{\mathrm{T}}, \\
&\varphi_{43} = (\sin(x_{41} - x_{31}), 0, 0)^{\mathrm{T}}, \\
&\varphi_{51} = (x_{51} - x_{11}, 0, x_{53} - x_{13})^{\mathrm{T}}, \\
&\varphi_{54} = (0, x_{52} - x_{42}, 0)^{\mathrm{T}}.
\end{aligned}$$

Lipschitz constants of all φ_{ij} are equal to 1. Connections graph is shown on Fig. 2.

It follows from Theorem 1 that decentralized adaptive control (4) provides synchronization goal (3) if for all $i = 1, \ldots, 5$ inequality $\sum_{j=1}^{5} |\alpha_{ij}| < \gamma$ holds, i.e. if interconnections are sufficiently weak.

Consider following control of leading subsystem $\overline{u} = \frac{1}{b}\left[(-(1 + m_0)p + 1)\overline{x}_1 + p\overline{x}_2\right]$. Let us put $\Gamma_i = I, i = 1, \ldots, d$, where I – identity matrix, and

$$\begin{aligned}
&\overline{x}_1(0) = 0.5, \quad \overline{x}_2(0) = 0, \quad \overline{x}_3(0) = 0, \\
&x_1(0) = (7, 14, 0.4)^{\mathrm{T}}, \quad x_2(0) = (0, 4, 4)^{\mathrm{T}} \\
&x_3(0) = (1, -1, 4.5)^{\mathrm{T}}, \quad x_4(0) = (3, -4, 0.2)^{\mathrm{T}} \\
&x_5(0) = (2, 8, 15).
\end{aligned}$$

Denote by α 5×5 matrix with element α_{ij} lying in the i-th row and the j-th column, $i, j = 1, \ldots, 5$, and let's take

$$\alpha = \begin{pmatrix} 0 & 0.0051 & 0.1395 & 0 & 0.1676 \\ 0.0662 & 0 & 0.0921 & 0.0065 & 0 \\ 0.2013 & 0 & 0 & 0.2271 & 0.1430 \\ 0.0907 & 0 & 0.0675 & 0 & 0 \\ 0.0663 & 0 & 0 & 0.2773 & 0 \end{pmatrix}.$$

Let us choose adaptive control $u_i, i = 1, \ldots, 5$ as in (4) and apply Theorem 1.

Phase portrait of the leading system and norms of errors $\|z_i\|, i = 1, \ldots, 5$ found by 40 sec. simulation are shown on Fig. 3. Simulation results demonstrate reasonable convergence (synchronization) between subsystems.

6 Conclusions

In contrast to a large number of previous results, we obtained synchronization conditions for nonlinear dynamical networks with nonidentical nodes, incomplete measurement, incomplete control, incomplete information about system parameters and coupling. The design of the control algorithm providing synchronization property is based upon speed-gradient method [Fradkov, 1990; Fradkov, Miroshnik and Nikiforov, 1999], while derivation of synchronizability conditions is based on Yakubovich-Kalman lemma [Yakubovich, 1962]. Note that g-monotonicity condition seems more suitable for ensuring chaotic behaviour of the leading subsystem, see [Junussov and Fradkov, 2009], [Fradkov, Junussov and Ortega 2009].

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