ONBOARD ALIGNMENT IDENTIFICATION OF SPACE TELESCOPE AND A STAR TRACKER CLUSTER

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Abstract

For high-accuracy pointing an orbital telescope (OT) it is required that the relative position of reference frames connected with the OT and the star trackers (STs) is precisely determined. The elaborated method for solving this problem is based on onboard processing of the information obtained at scanning the star sky simultaneously by the OT and the STs. We apply the multiply digital filtration of measurement, polynomial approximation and a vector spline extrapolation of the quaternion values for the OT and STs attitude. The accuracy estimates obtained at the alignment identification, are presented.

Key words

space telescope,star tracker, alignment calibration

1 Introduction

The characteristics of pointing an orbital telescope's (OT) line-of-sight onto observed objects and quality of observation information being obtained are strongly dependent on the accuracy of defining the relative position of reference frames (RFs) connected with an orbital telescope (OT) and with the main measuring devices, namely, the star trackers (STs) used in the spacecraft (SC) attitude control system. A special mode is organized for mutual binding these reference frames, when a telescope scans the star sky and simultaneously the optoelectronic STs' measurements are registered, see Fig. 1.

The obtained information is usually communicated on a terrestrial space center, where the relative position of RFs indicated and the real position of a telescope with respect to terrestrial objects during its course motion are defined more accurately. Along with this formation (digital "sewing together") of images, removal ("cutting off ") of superfluous information, and designing the space photos for their presentation to a customer

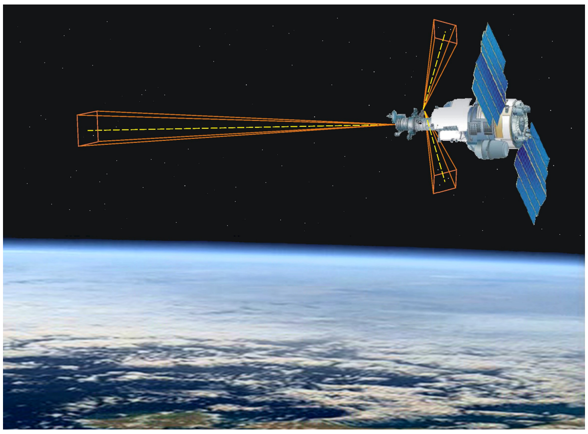

Figure 1. The mode of astronomical checking axes' concordance

are carried out. Such technology is in need of refinement when the space video-information enters the international market.

Many foreign space centers have software of their own to "sew together" electronic images and order only the preprocessed electronic video-information on a terrestrial part specified strictly that is completed by a service information about actual conditions of space survey directly from the SC board.

At progress of (Somov and Butyrin, 2008) and (Somov *et al.*, 2008), in this paper the elaborated innovation methods for a more accurate definition of actual position of the OT and the star tracker cluster (STC) by a posterior processing of the measurement information directly aboard spacecraft, are presented.

2 The Problems Statement

We introduce the bases composed from the units and the reference frames (RFs), namely:

- the inertial RF (IRF) $\mathbf{I}_{\oplus}$ $(\mathrm{O}_{\oplus}\mathrm{X}_{\mathrm{I}}^{\mathrm{e}}\mathrm{Y}_{\mathrm{I}}^{\mathrm{e}}\mathrm{Z}_{\mathrm{I}}^{\mathrm{e}})$ with the origin at the Earth center $\mathrm{O}_{\oplus}$;

- Greenwich geodesic RF (GRF) $\mathbf{E}_\mathrm{e}$ ($\mathrm{O}_\oplus \mathrm{X}^\mathrm{e}\mathrm{Y}^\mathrm{e}\mathrm{Z}^\mathrm{e}$) that is rotated with respect to the IRF with the angular rate vector $\boldsymbol{\omega}_\oplus \equiv \boldsymbol{\omega}_\mathrm{e}$;
- horizon RF (HRF) $\mathbf{E}_\mathrm{e}^\mathrm{h}$ ($C\,\mathrm{X}_c^\mathrm{h}\mathrm{Y}_c^\mathrm{h}\mathrm{Z}_c^\mathrm{h}$) with origin at point C and the ellipsoidal geodesic coordinates – altitude H_c, latitude B_c and longitude L_c;
- the SC body RF (BRF) $\mathbf{B}=\{\mathbf{b}_i\}(\mathrm{O}xyz)$ with origin in its mass center O;
- orbital RF (ORF) $\mathbf{O} = \{\mathbf{r}^\mathrm{o}, \boldsymbol{\tau}^\mathrm{o}, \mathbf{n}^\mathrm{o}\}$ $(\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o})$;
- the base $\mathbf{S} = \{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3\}$ and the sensor reference frame (SRF) of an optical telescope $\mathrm{S}\,x^sy^sz^s$;
- the image field reference frame (FRF) $\mathrm{O}_i\,x^iy^iz^i$ with the origin in the center O_i of the telescope focal plane $y^i\mathrm{O}_iz^i$;
- the visual RF (VRF) $\mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ $(\mathrm{O}_\mathrm{v}\,x^\mathrm{v}y^\mathrm{v}z^\mathrm{v})$ with the origin in the center O_v of a CCD matrix in the telescope focal plane $y^i\mathrm{O}_iz^i$, moreover, the points O_i and O_v are coincident, and the unit $\mathbf{s}_1$ of the base $\mathbf{S}$ and unit $\mathbf{v}_1$ of the base $\mathbf{V}$ are strictly contrary.

Images of these reference frames were presented in (Somov and Butyrin, 2008).

Let the RF of p's star tracker (RFS_p) $\mathbf{A}_p = \{\mathsf{a}_p, \mathsf{b}_p, \mathsf{c}_p\}$ $(\mathrm{O}x_p^\mathsf{a}y_p^\mathsf{a}z_p^\mathsf{a})$ be connected with the CCD matrix in its focal plane, $p = 1\div4$, moreover, the RFS_p angular position is fixed in the BRF, the units a_p of the STs' optical axes belong to the cone's surface with a semi-angle γ^a, see Fig. 2, but their actual positions in the BRF are not exactly known. At last, we introduce the virtual (calculated) reference frame of a star cluster (RFSC) $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ $(\mathrm{O}\,x^\mathsf{a}y^\mathsf{a}z^\mathsf{a})$ that is calculated on the basis of proceeding the accessible measuring information from any combination of the star trackers. For simplicity we will propose that the bases $\mathbf{B}$ and $\mathbf{S}$ (the BRF and the SRF) coincide. The BRF state relative to the IRF is defined by the quaternion $\boldsymbol{\Lambda}$ and the angular rate $\boldsymbol{\omega}$ vector. The following problems are solved at the known coordinates of the SC mass center orbital motion:

- definition of the angular position of the base $\mathbf{S}$ and fixed mutual angular position of the bases $\mathbf{V}$ and $\mathbf{S}$ during a mode of astronomical checking axes' concordance (ACAC), when the measuring information only from a telescope is applied;

- definition of a fixed mutual angular position of the bases $\mathbf{A}$ and $\mathbf{S}$ (the alignment identification), when the measuring information obtained in the ACAC mode both from a telescope and star trackers is applied;

- high-accuracy definition of the actual angular position (quaternion $\boldsymbol{\Lambda}(t)$) and angular rate vector $\boldsymbol{\omega}(t)$ of the base $\mathbf{S}$ with respect to inertial base $\mathbf{I} \equiv \mathbf{I}_\oplus$ for any time moment $t \in \mathrm{T}_n \equiv [t_\mathrm{i}, t_\mathrm{f}]$ at the given interval T_n with the duration $T_n = t_\mathrm{f} - t_\mathrm{i}$ at optoelectronic observation of the given part of the Earth surface by the OT when the measuring information only from the star trackers is applied.

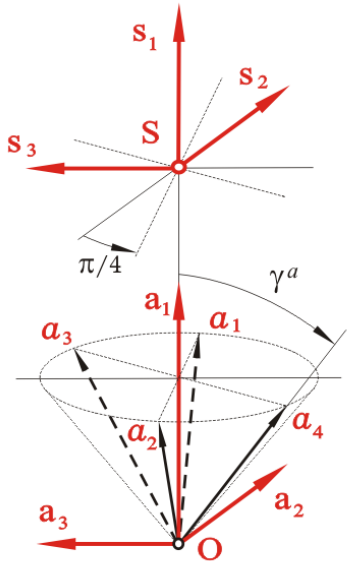

Figure 2. The bases $\mathbf{S}$ and $\mathbf{A}$

3 Smoothing the Discrete Measurements

The classical problem on polynomial approximation of the values $y_s = f(x_s), s = 1 \div n$ for the unknown scalar function $y = f(x)$ by the polynom $y = \sum_{i=0}^{m} a_i x^i$ with the degree $m < n$ using the method of least squares (MLS) consists in definition of the coefficients $a_i, i = 0 \div m$ from the condition

$$\sum_{s=1}^{n}\{(\sum_{i=0}^{m} a_i x_s^i) - y_s\}^2 \Rightarrow \min\ .$$

Using the elegant Gauss notation $[u] \equiv \sum_{s=1}^{n} u_s$, one can obtain the system of $m+1$ normal scalar equations

$$\begin{gathered}\sum_{i=0}^{m} a_i[x^i] = [y];\quad \sum_{i=0}^{m} a_i[x^{i+1}] = [xy];\\ \ldots\ldots\ \sum_{i=0}^{m} a_i[x^{i+m}] = [x^m y].\end{gathered} \tag{1}$$

At introducing the vector-column $\mathbf{a} = \{a_0, a_1, ...a_m\}$ with the dimension $m+1$ by the trivial way, this system is presented in the vector-matrix form $\mathbf{Ca} = \mathbf{b}$. The non-singular matrix $\mathbf{C} = ||c_{ik}||$ is always symmetrical and "recursive" ($c_{ik} = c_{i-1,k+1}$), the required vector-column $\mathbf{a}$ is computed on the basis of standard algorithms (Lanczos, 1956).

The degree m for the polynomial approximated by the MLS must be chosen taking into account the length of access data $y_s = f(x_s), s = 1 \div n$, e.g. the value n. The solution of practical tasks demonstrates that it is rational to apply method (filter) of the Savitsky – Goley (Orfanidis, 1996) polynomial smoothing that is a modification of the MLS for large values n. Here the sequence of the discrete values $y_s = f(x_s)$ is approximated in a "moving" window (frame) with the length

$n_* << n$, where n_* is a whole odd number and also a "moving polynomial" with small degree m, for example $m = 3$. The first frame is formed by the values $y_s = f(x_s), s = 1 \div n_*$ beginning from the first measurement, and a polynomial with the given degree is constructed for it by the MLS. Then the frame is displaced on one value and the approximation is carried out again. Every time in the output sequence one can use only the single value of an approximation polynomial that corresponds to the center $(n_* - 1)/2$ of the current position of a "moving frame". Exceptions are the first and last frames, here the output values of the smoothing procedure are calculated at the polynomial points that correspond to the first and last halves of boundary frames. The values of 3-dimensional vector function $\mathbf{y}_s = \mathbf{f}(x_s)$, $s = 1 \div n$ of the scalar argument using the Savitsky – Goley filter are smoothed out by the standard application of this procedure for values of each component of the vector-column composed from mapping values of the vector function on the axes of some orthogonal basis.

The problem on definition of the mutual attitude of two orthogonal bases on the basis of the data about two sets of units that are arbitrarily placed in the bases, is more complex. Let a set of the units $\mathbf{b}_i$ be given that are measured in the SC body base $\mathbf{B}$, and a set of values of the units $\mathbf{r}_i$ corresponding to them specified in the inertial base $\mathbf{I}$. The classical problem of vector matching (the Wahba problem) is formulated as follows: let us define an orthogonal matrix $\mathbf{A}$ with a determinant equal to +1, which minimizes the quadratic index

$$L(\mathbf{A}) = \tfrac{1}{2}\Sigma a_i |\mathbf{b}_i - \mathbf{A}\mathbf{r}_i|^2,$$

where the non-negative numbers a_i are the weighing coefficients. It has been strictly proved that the solution of this problem is the optimal quaternion $\mathbf{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$, $\boldsymbol{\lambda} = \{\lambda_i, i = 1 \div 3\}$ that is equivalent to the required orthogonal matrix $\mathbf{A}$ and is defined as an eigenvector of the matrix $\mathbf{K}$ with the maximum eigenvalue $q_{\max}$, e. g. by relations

$$\begin{gathered}\mathbf{z} = \Sigma\, a_i \mathbf{b}_i \times \mathbf{r}_i; \quad \mathbf{B} = \Sigma\, a_i \mathbf{b}_i \mathbf{r}_i^{\mathrm{t}}; \quad \mathbf{S} = \mathbf{B} + \mathbf{B}^{\mathrm{t}}; \\ \mathbf{K} = \begin{bmatrix} \mathrm{tr}\,\mathbf{B} & \mathbf{z}^{\mathrm{t}} \\ \mathbf{z} & \mathbf{S} - \mathbf{I}_3 \mathrm{tr}\mathbf{B} \end{bmatrix}; \quad \mathbf{K}\,\mathbf{\Lambda} = q_{\max}\,\mathbf{\Lambda}. \end{gathered} \tag{2}$$

Relations (2) represent the QUEST algorithm (Markley and Mortar, 2000) for a quaternion's estimation, that is further applied for processing the measuring information obtained in the ACAC mode. The quaternion $\mathbf{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$ is an one-one related to the Rodrigues modified parameters' vector $\boldsymbol{\sigma}$ by the explicit analytic relations

$$\boldsymbol{\sigma} = \frac{\boldsymbol{\lambda}}{1 + \lambda_0};\ \lambda_0 = \frac{1 - \sigma^2}{1 + \sigma^2};\ \boldsymbol{\lambda} = \frac{2\boldsymbol{\sigma}}{1 + \sigma^2}\,. \tag{3}$$

These relations permit transforming a problem on smoothing the quaternion data to standard task on smoothing the vector measurements.

4 A Vector Extrapolation

To the well-known direct and backward quaternion kinematic equations there correspond the direct and backward kinematic equations for Rodrigues vector

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \tfrac{1}{4}\{\mathbf{I}_3(1 - \sigma^2) + 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^{\mathrm{t}}\}\boldsymbol{\omega}; \\ \boldsymbol{\omega} &= \tfrac{4}{(1+\sigma^2)^2}\{\mathbf{I}_3(1 - \sigma^2) - 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^{\mathrm{t}}\}\dot{\boldsymbol{\sigma}}\,. \end{aligned} \tag{4}$$

A problem on extrapolation of the SC angular rate vector at the time interval T_n by the values of a vector $\boldsymbol{\omega}_s \equiv \boldsymbol{\omega}(t_s)$ given at discrete time moments $t_s \in \mathrm{T}_n$ with a period $T_q = t_{s+1} - t_s$, where $s = 0, 1, 2...n_q \equiv 0 \div n_q$ and $n_q = T_n/T_q$, consists in calculation of a time vector function $\mathbf{p}(t)$, which defines approximately the angular rate vector $\boldsymbol{\omega}(t)\ \forall\ t \in \mathrm{T}_n$ at the condition $\mathbf{p}_s = \omega_s$. In the general case, the extrapolation of values $\boldsymbol{\omega}_k$ at the time moments $t_k \in \mathrm{T}_n$ with a step $T_a = t_{k+1} - t_k$ and the multiplicity of periods $k_q^a \equiv T_a/T_q \geq 1$ can be applied. The algorithm for solving this problem by 3-degree vector splines was presented in details [4].

Extrapolation of the quaternion values $\mathbf{\Lambda}_k$ given discretely by the quaternion $\mathbf{M}(t)\ \forall t \in \mathrm{T}_n$ is carried out as follows. At first, on the basis of biunique connection of the quaternion $\mathbf{\Lambda}$ with Rodrigues vector $\boldsymbol{\sigma}$, using explicit analytic relations (3), the sequence of values for the vector $\boldsymbol{\sigma}_k$ is calculated. Then the extrapolation procedure presented above is applied to this sequence. At last, the inverse transformation to the quaternion $\mathbf{M}(t)$ is carried out using explicit relations (3).

5 Telescope and the VRF Attitude Determination at the ACAC Mode

At scanning the star sky with a constant angular rate $\omega_z^{\mathrm{o}} \approx 0.015°/s$ and organizing "moving window" with telescope's field-of-view at a strictly fixed frequency of accumulating the electronic image charge packets along the columns of the CCD matrix, one can obtain the sequences of values both of the VRF attitude quaternion $\mathbf{\Lambda}_s^{\mathrm{v}}$ and the SRF attitude quaternion $\mathbf{\Lambda}_s^{\mathrm{s}}$ with respect to the IRF. The unit $\mathbf{s}_1$ of the base $\mathbf{S}$ and unit $\mathbf{v}_1$ of the base $\mathbf{V}$ are strictly contrary, therefore, at first, we define the sequence of attitude quaternion $\mathbf{\Lambda}_s^{\mathrm{s}}$ for the telescope optical base $\mathbf{S} = \{\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3\}$ with exact binding to the time moments t_s on the star sky photo based on the following technique. All sequence of N discerned stars onto the photo is divided into the groups (frames), so that each frame (window) would contain a desired odd number n of stars, according to the following rules:

- stars are arranged in the order of increasing the time moments t_s of their registration without omission;
- each next frame includes only one additional star.

Each i-th frame "is tied" to a time moment t_i^{m} according to its central star with the number i. Then two

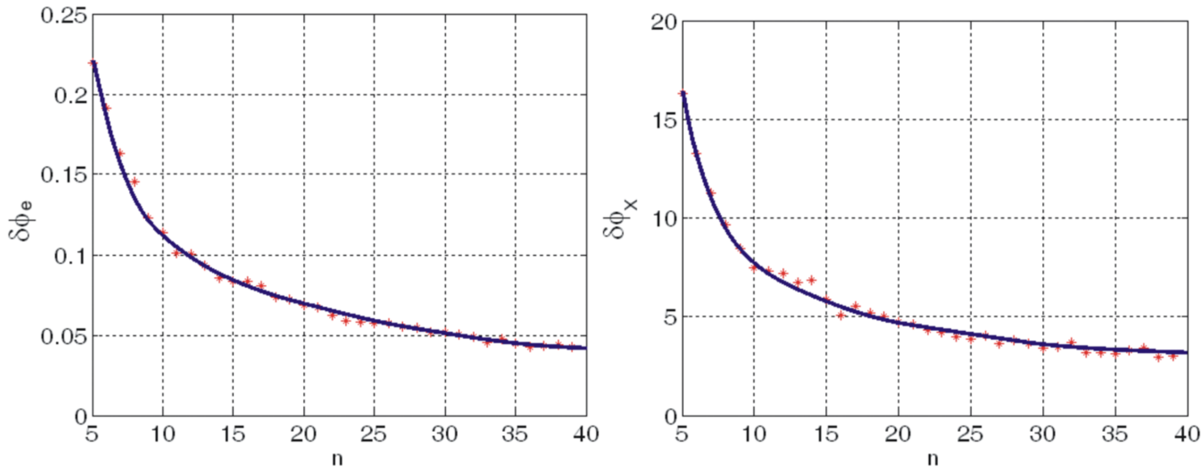

Figure 3. The QMD in determining the error $\delta\phi_{\rm e}$ and $\delta\phi_{\rm x}$ [arc sec] versus the star number n

sets of the unit directions on stars are defined for each frame: the set of units $\mathbf{r}^{\rm s}_{\nu}$ into the VRF by the stars' relative coordinates into the CCD matrix and the set of units $\mathbf{b}^{\rm s}_{\nu}$ into the IRF by direct ascents α_{ν} and inclinations δ_{ν}, $\nu = 1\div n$ of stars (from the star catalogue FK-5). In completion the QUEST procedure is called to determine the SRF attitude quaternion $\mathbf{\Lambda}^{\rm s}_{i}$ values with respect to the IRF at the time moments $t^{\rm m}_{i}$, $i = 1 \div N_k$, where N_k is the frame quantity in the electronic photo.

We have estimated the minimum stars' quantity in a frame necessary for determining the SRF attitude quaternion $\mathbf{\Lambda}^{\rm s}_{i}$ with an error of roughly some tenth parts of arc seconds. In the general case, the minimum number of stars in a frame equals $n = 3$, and the maximum number $n = N$ corresponds to the number of all stars discerned in the electronic photo. For describing deviation of the VRF from its required position in the IRF kinematic parameters were applied in the form of angle $\delta\phi_{\rm e}$ (deviation of the unit $\mathbf{v}_1 = -\mathbf{s}_1$ from its required position) and angle $\delta\phi_{\rm x}$ – a turn about a telescope's optical axis. The VRF's two positions in the IRF (absolute exact and restored according to photo data) are combined by two rotations on the angles $\delta\phi_{\rm e}$ and $\delta\phi_{\rm x}$, moreover the first rotation combines units on the OT optical axis. The results obtained with the frame of a fixed dimension $1^{\circ}.3 \times 1^{\circ}.3$ for the quadratic mean deviation (QMD) are presented in Fig. 3 . These results testify that to ensure a permissible error on determination of the OT axis' actual position into the IRF, it is enough ten stars being observed in a frame. The error $\delta\phi_{\rm x}$ on determination of turn around the telescope axis is dozen times worse even at larger star's quantity. That result is explained by the small telescope's field-of-view, e. g. by the insufficient measuring base.

The elaborated technique for a more accurate definition of the base $\mathbf{S}$ position with respect to the inertial base $\mathbf{I}$ is based on widening the measuring astronomical basis at the expense of long-time SC scanning motion with an angular rate $\omega^{\rm o}_{z} \approx 0.015^{\circ}/s$ on the pitch channel, perhaps with the time technology breaks of star observation by a telescope: it is assumed the possibility of the telescope motion with the closed cover. For example, at the general time interval of scanning with duration 1000 sec, it is enough to obtain the star images only on three time parts with the duration 100 sec – at the beginning, middle and end of the general time interval, Fig. 4. As a result at an angular rate $\omega^{\rm o}_{z} \approx 0.015^{\circ}/s$ of removing the telescope's optical axis into the "scanning plane", one can obtain the measuring astronomical basis with an angular dimension 15°. The numerical calculations were carried out with applying filtration of the telescope attitude quaternion estimations (precisely, estimations of the Rodrigues vector) by the Savitsky – Goley method. The obtained results indicated that such a measuring basis is quite sufficient for restoring the base $\mathbf{S}$ actual position with respect to the base $\mathbf{I}$ at the QMD on determination of a turn angle $\delta\phi_{\rm x}$ about the OT optical axis no more than $1''$, see Fig. 5.

When measuring information only from telescope is applied, determination of the fixed mutual angular positions for the bases $\mathbf{V}$ and $\mathbf{S}$ (their fixed mutual turn about the unit $\mathbf{s}_1 = -\mathbf{v}_1$) is carried out on the base of additional analyzing the numbers of the CCD matrix's columns, which have outputs with appearing accumulated star images, in periphery parts of the CCD lines. Moreover, it is a success at definition of constant technology errors on arrangement of the CCD matrix into focal plane of telescope. Then these errors are taken into consideration at planning observations of given parts at the Earth surface.

6 The RFSC Attitude Determination at the ACAC Mode

The virtual reference frame of the star tracker cluster (RFSC) $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$, see Fig. 2, is calculated by processing an accessible measuring information, obtained at the ACAC mode from any combination of the star trackers. The CCD matrix in the focal plane of each ST is fixed in the BRF, therefore, the "summary" field-of-view for the STC, based on any combination from no smaller then two star trackers, puts together a large measuring base. This measuring base is quite

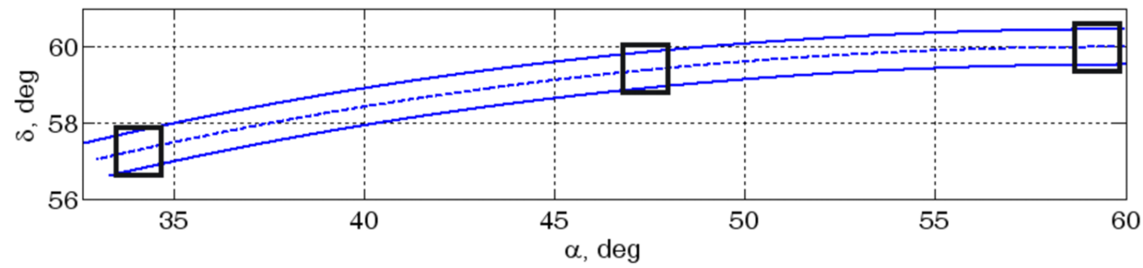

Figure 4. The trace of the CCD boundaries at the sky sphere during a long-time SC scanning motion

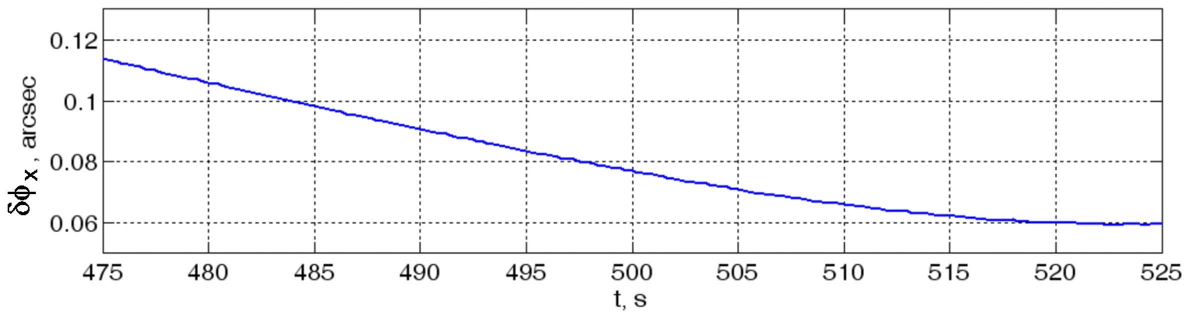

Figure 5. The QMD in determining the error $\delta\phi_{\mathrm{x}}$ during a long-time SC scanning motion

sufficient for high-accuracy determination of the RFSC position with respect to the same inertial base **I**. Naturally, the best results are obtained if it is possible to obtain measuring information from all forth star trackers and to fulfill next onboard processing, first by the QUEST algorithm and then with filtering by the Savitsky – Goley method. In the other limited calculated case, the virtual RFSC is constructed based on the information about angular positions of the optical axes units a_p for any two star trackers.

It is clear, that if we have estimations for the VRF attitude quaternion and for the RFSC attitude quaternion with respect to the same inertial base **I**, then it is simple to obtain a constant correction quaternion for taking into account their reciprocal position. Such a correction quaternion is applied in the SC attitude control system at observing the Earth surface.

7 The VRF Attitude More Accurate Definition at Observing the Earth

A more accurate definition of the attitude quaternion $\mathbf{\Lambda}(t)$ actual values and the angular rate vector $\boldsymbol{\omega}(t)$ of the base **S** relative to the inertial base **I** and for any time moment $t \in \mathrm{T}_n$ at optoelectronic observing a given part of the Earth surface, when the measuring information is applied only from the star trackers, is a very complex problem.

The mutual alignment calibration of the VRF and the RFSC carried out at the ACAC mode is checked first at the mode of observing the Earth polygons with some well-known (strong) points. The necessity of such additional calibration is accounted by different conditions of observing the ”cold” space and the ”warm” Earth. At this identification mode the SC attitude control system fulfills a programmed angular motion of the telescope in the IRF, given by a set of the vector splines. These splines are calculated from the conditions of observing a terrestrial polygon with the given azimuth of scanning. The orientation quaternion $\mathbf{\Lambda}(t)$ and vector $\boldsymbol{\omega}(t)$ as explicit time functions at a given time interval are calculated on the basis of the vector composition of all elemental motions in the GRF taking into account a current observation perspective, initial coordinates of on-earth object and a needed scanning azimuth.

Let the vector-columns $\boldsymbol{\omega}^{\mathrm{s}}_{\mathrm{e}} = \{\omega^s_{ei}\}$ and $\mathbf{v}^{\mathrm{s}}_{\mathrm{e}} = \{v^s_{ei}\}$ accordingly present in the SRF its angular rate and velocity of the SC mass center motion with respect to GRF, the matrix $\tilde{\mathbf{C}} = \| \tilde c_{ij} \|$ defines the SRF attitude with respect to the HRF and the scalar function $D(t)$ presents the observation oblique range along the line-of view. Then for any point at the telescope focal plane the components $\tilde V^i_y(\tilde y^i, \tilde z^i) \equiv \dot{\tilde y}^i$ and $\tilde V^i_z(\tilde y^i, \tilde z^i) \equiv \dot{\tilde z}^i$ of the normed image motion velocity vector are computed by the vector-matrix relation

$$\begin{bmatrix}\dot{\tilde y}^i\\ \dot{\tilde z}^i\end{bmatrix} = \begin{bmatrix}\tilde y^i & 1 & 0\\ \tilde z^i & 0 & 1\end{bmatrix}\begin{bmatrix} q^i\tilde v^s_{e1} - \tilde y^i\omega^s_{e3} + \tilde z^i\omega^s_{e2}\\ q^i\tilde v^s_{e2} - \quad \omega^s_{e3} - \tilde z^i\omega^s_{e1}\\ q^i\tilde v^s_{e1} + \quad \omega^s_{e2} + \tilde y^i\omega^s_{e1}\end{bmatrix}. \qquad (5)$$

Here

$\tilde y^i = y^i/f_e$ and $\tilde z^i = z^i/f_e$ are the normed focal coordinates of the point indicated, where f_e is the telescope equivalent focal distance;

$q^i = 1-(\tilde c_{21}\tilde y^i + \tilde c_{31}\tilde z^i)/\tilde c_{11}$ is the scalar function, and $\tilde v^s_{ei} = v^s_{ei}(t)/D(t), i = 1, 2, 3$ are the components of the vector of the normed progressive motion velocity.

Based on (5), the desired components of the vector-column $\boldsymbol{\omega}_{\mathrm{e}}^{s}$ are obtained in the explicit form for a frame survey (tracking) and with the use of single numerical integration of the quaternion differential equation during scanning survey. In the last case, for programmed values of the quaternion $\mathbf{\Lambda}(t)$ and vector $\boldsymbol{\omega}(t)$ a high-accuracy interpolation is carried out by a set of 3-degree vector splines.

Actual sequence of the VRF positions with respect to the IRF at observing a terrestrial polygon is carried out by the method of backward dynamical photogrammetric intersection – with applying the precise tie to the time moments t_s appearing at the electronic images of the polygon's reference points on the electronic photo-frame. Here we apply a technique that is similar to the technique for determination of the VRF attitude with respect to the IRF at the ACAC mode presented above.

The measuring information from the star tracker cluster is processed by means of the QUEST algorithm, filtering by the Savitsky – Goley method and extrapolation by vector splines. In this case, the estimation of the angular rate vector $\boldsymbol{\omega}(t)$ of the base $\mathbf{S}$ with respect to the IRF is carried out according to the backward vector kinematic equation for the Rodrigues parameters' vector (3) by applying derivation of a vector spline with respect to explicit analytic relations.

Misalignment between estimated by the STC and actual (in electronic photo of polygon) the VRF motions with respect to the IRF gives the following information:

- the accuracy estimation on restore of the VRF actual attitude during observing a terrestrial part;
- a more accurate definition of a constant correction quaternion for taking into account a reciprocal position of the VRF and virtual reference frame of the star tracker cluster at observing terrestrial part.

After fulfilling an alignment identification on terrestrial polygons, the SC onboard equipment has the possibility for operative solution of tasks on a posteriori restore of actual the VRF attitude and its angular rate vector at any time moment of the Earth's optoelectronic observation.

8 Conclusion

We briefly present the elaborated innovation methods for a high-accuracy definition of the OT actual attitude on the basis of a posteriori processing a measuring information directly aboard a spacecraft. This technology is based on the known methods of smoothing, approximation, filtering the vector measurements and interpolation of filtered results by vector splines.

The new techniques are described for precise in-flight alignment identification of a telescope and astrosystem based on four star trackers, which have an optimal arrangement on a land-survey spacecraft body. Some concrete numeric results are presented, which demonstrate the efficiency of the techniques proposed.

The methods proposed give an opportunity to process preliminary the electronic video-information on a strict given terrestrial part directly aboard a spacecraft, to add its by a service data on actual conditions of a space survey, and to present these results into a space center by radio-line.

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