CLOSED-LOOP IDENTIFICATION BASED ON MIMO FEEDBACK ERROR LEARNING

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Abstract: Motivated by the recent development of Feedback Error Learning (FEL), this paper proposes a method for closed-loop identification of a Multi Input Multi Output (MIMO) plant. Given a control system roughly designed, a feedforward controller is constructed by learning to achieve desirable responses. Then the learned feedforward controller gives a model of the plant, which will be effective for re-designing the control system to improve the performance. The effectiveness of the method is verified through numerical simulation.

Keywords: Feedback error learning, Closed-loop identification, MIMO system

1. INTRODUCTION

The concept of Feedback Error Learning (FEL) was given by (Kawato *et al.*, 1987), in which they proposed a model of our body acquiring accurate motion. Signal transmission in our neural system is too slow to achieve enough accuracy via feedback alone. Hence our body uses a feedforward mechanism and adapts it by learning from feedback error. Kawato *et al.* applied this mechanism to control system design, which was novel in control literature. They originally adopted artificial neural network as a function generator for the feedforward controller, but it turned out later on that a linear filter is sufficient to this purpose (Miyamura and Kimura, 2002).

Inspired by their pioneering work, a lot of investigation has been done and FEL has become an established method for designing a control system having a good tracking property without extensive system modeling; see (Miyamura and Kimura, 2002; Muramatsu and Watanabe, 2004) for stability proofs of learning, (Alali *et al.*, 2006a; Alali *et al.*, 2006b) for its MIMO generalization, stability

proof, and its application to manipulators, etc. The scheme has a two-degree-of-freedom structure consisting of an adaptive feedforward controller and a fixed feedback controller roughtly designed previously; see Fig. 1. Here, the pre-filter is given to compensate the row relative degree of the plant. If, in particular, $P(s)$ is biproper, then we can take $W(s) = I$.

In this scheme, we tune the parameter in the feedforward controller so that the feedback error converges to zero. An interesting point is that, after convergence, the acquired feedforward controller has enough knowledge about the inverse model of the plant, since its cascade connection with the plant is equal to the prefilter; see Fig. 2. Thus, it is natural to expect that we can identify the plant parameter from this knowledge. In other words, closed-loop identification is achieved by applying FEL to the plant. The obtained system parameters can be used for re-designing the control system to improve the performance.

In this paper, we first summarize the MIMO FEL scheme recently proposed by the present authors
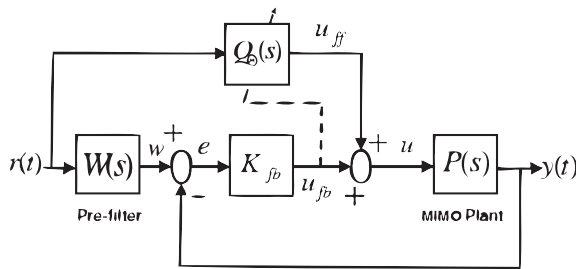
Fig. 1. MIMO Feedback Error Learning Architecture

in (Alali *et al.*, 2006a), and then use the acquired feedforward controller to obtain the system model. The performance of identification is evaluated with some examples in various conditions.

Closed-loop identification is attractive since it gives a model while operation (*i.e.*, on-line), but this is a challenging task due to numerical difficulty. This is the case also in our scheme, but the result turns out to be practical enough. A striking feature is that even an unstable plant can be identified as far as the closed-loop is stable.

2. PROBLEM FORMULATION

Consider an n-dimensional controllable and observable linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \ y(t) = Cx(t) + Du(t) \quad (1)$$

with m-inputs and m-outputs (*i.e.*, square system). Assume that the system is minimum phase. Namely, we assume

$$\det \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} \neq 0, \text{ for all } \mathrm{Re}(s) \geq 0. \quad (2)$$

If, further, $\det D \neq 0$, then the system is biproper (invertible with properness) and construction of MIMO-FEL is straightforward. Namely, we can take $W(s) = I$ in Fig. 1, and after learning we
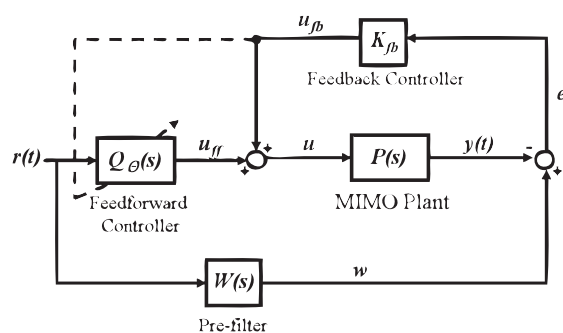

Fig. 2. MIMO-FEL Identification Scheme

have $Q_\Theta(s) = P(s)^{-1}$, so that we attain $y(t) = r(t)$. Then $P(s)$ is readily identified by inverting the estimated transfer matrix $Q_\Theta(s)$.

If, on the other hand, $\det D = 0$, then we need to define an appropriate pre-filter to compensate the delay of the plant as we will explain below. In order to clarify the point, we further assume $D = 0$. Therefore, the transfer matrix is $P(s) = C(sI - A)^{-1}B$. (If $\det D = 0$ and $D \neq 0$, then the argument will be more complicated but our basic idea still holds.)

Also, we assume that A, B, C are unknown, but that

$$\Lambda := \begin{bmatrix} c_1 A^{\mu_1 - 1} B \\ \vdots \\ c_m A^{\mu_m - 1} B \end{bmatrix} \quad (3)$$

is invertible, where c_k is the kth row vector of C and μ_k is the minimal integer such that

$$c_k A^{\mu_k - 1} B \neq 0, \ \mu_k \geq 1, \quad k = 1, \cdots, m. \quad (4)$$

The integers $\mu_1, \mu_2, \cdots, \mu_m$ can be regarded as a generalization of the relative degree to MIMO systems. We assume that these integers are known. Furthermore, they have a close relationship to the following concept (Wolovich and Falb, 1976).

Definition 1. Given $P(s)$, a square polynomial matrix $L(s)$ is called an interactor if

$$\lim_{s \to \infty} L(s)P(s) \quad (5)$$

is a finite matrix with full rank.

Under the assumption that Λ in (3) is invertible, we can set the interactor matrix to a diagonal form

$$L(s) = \begin{bmatrix} l_1(s) & & 0 \\ & \ddots & \\ 0 & & l_m(s) \end{bmatrix} \quad (6)$$

where $l_k(s)$ is an arbitrarily chosen Hurwitz polynomial of degree μ_k for $k = 1, \cdots, m$. Now the above mentioned delay-compensation is attained by taking the pre-filter

$$W(s) = L(s)^{-1} = \begin{bmatrix} \dfrac{1}{l_1(s)} & & 0 \\ & \ddots & \\ 0 & & \dfrac{1}{l_m(s)} \end{bmatrix}. \quad (7)$$

In Fig. 1, the objective of the control is to minimize the error signal between $w(t)$ and the plant output $y(t)$. We put $W(s)$ to compensate the inevitable delay in $P(s)$ due to its strict properness and take it in the diagonal form so that each r_i corresponds y_i without interaction between channels.

Now the following equations can be deduced from Fig.1:

$$\begin{aligned}
y(t) &= P(s)u(t) \\
u(t) &= u_{fb}(t) + u_{ff}(t) \\
u_{fb}(t) &= K_{fb}e(t), \ e(t) = w(t) - y(t) \quad (8) \\
w(t) &= W(s)r(t) \\
u_{ff}(t) &= Q_\Theta(s)r(t),
\end{aligned}$$

where K_{fb} is a feedback gain already designed to stabilize the plant, and Θ is a tunable parameter. Note that the first equation implies

$$y(t) = \pounds^{-1}[P(s)](t) * u(t),$$

where $*$ denotes the time-domain convolution. For the sake of simplicity, we will adopt this kind of abuse in time and frequency domains throughout this paper.

3. OVERVIEW OF MIMO-FEL

We now construct a tunable feedforward controller $Q_\Theta(s)$ in order to generate $u_{ff}(t)$ using the following linear filter:

$$\begin{aligned}
\dot{\xi}_1(t) &= A_f\xi_1(t) + B_f r(t) \\
\dot{\xi}_2(t) &= A_f\xi_2(t) + B_f u_{ff}(t) \\
u_{ff}(t) &= F(t)\xi_1(t) + G(t)\xi_2(t) + H(t)r(t) \\
&= \Theta(t)\xi(t),
\end{aligned}$$

$$(9)$$

where

$$\Theta(t) = [F(t) \ G(t) \ H(t)], \quad \xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ r(t) \end{bmatrix}.$$

$$(10)$$

A_f and B_f are taken in a controllable canonical form as:

$$A_f = \begin{bmatrix}
0 & 1 & & 0 & & & & \\
\vdots & & \ddots & & & & 0 & \\
& & \cdots & 1 & & & & \\
-f_\mu & & & -f_1 & & & & \\
& & & & \ddots & & & \\
& & & & & 0 & 1 & 0 \\
& 0 & & & & \vdots & & \ddots \\
& & & & & & \cdots & 1 \\
& & & & & -f_\mu & & -f_1
\end{bmatrix}$$

$$(11)$$

$$B_f = \begin{bmatrix}
0 & & \\
\vdots & & 0 \\
1 & & \\
& \ddots & \\
& & 0 \\
0 & & \vdots \\
& & 1
\end{bmatrix}, \quad (12)$$

where A_f is a stable matrix and (A_f, B_f) is controllable. Note that the unknown parametric matrices $F(t)$, $G(t)$, $H(t)$ enter linearly in the above setting.

The matrix $\Theta(t)$ is tuned using the following learning law

$$\frac{d\Theta}{dt} = \alpha u_{\text{fb}}(t)\xi^T(t) \quad (13)$$

with a small constant α, called learning factor. The derivation and the stability analysis of the learning law as above have been studied extensively in (Alali $et\ al.$, 2006a). We omit a detail here and simply note the following:

(1) As will be seen later, simulation and experimental results show effectiveness of the learning law.
(2) A striking feature of FEL is to use feedback error to tune the feedforward controller. Hence it is rather transparent to show $e(t) \to 0$.
(3) After learning, $y(t) \to w(t)$ and hence we have $P(s)Q_\Theta(s) = W(s)$ as far as $r(t)$ is a sufficiently rich signal. This motivates us to use FEL for closed-loop identification.

4. MIMO-FEL IDENTIFICATION ANALYSIS

The main objective of FEL as mentioned above is to improve the tracking performance by means of learning. It is then natural to expect that the acquired feedforward controller $Q_\Theta(s)$ has enough knowledge on the plant model, and hence we can identify the plant parameter from this knowledge.

If $P(s)$ was biproper, then we could take $W(s) = I$ so that the plant could be identified as

$$P(s) = Q_\Theta(s)^{-1}, \quad (14)$$

by means of the convergent $\Theta(t)$.

When $P(s)$ is strictly proper (as we have assumed), $P(s)^{-1}$ becomes improper, so that we need to put the pre-compensator $W(s)$. The following arguement is useful to clarify our method.

For the original system (1) and the interactor $L(s)$ in (6), there exists a gain R such that

$$N(s) = C(sI - A + BR)^{-1}B = L(s)^{-1}\Lambda. \quad (15)$$

The gain R can be obtained by the simple formula (Mutoh and Nikiforuk, 1992):

$$R := \Lambda^{-1}[L_0, \cdots, L_\mu]\begin{bmatrix} C \\ CA \\ \vdots \\ CA^\mu \end{bmatrix} \qquad (16)$$

where

$$L(s) = L_0 + L_1 s + \cdots + L_\mu s^\mu, \qquad (17)$$

$$\mu := \max(\mu_1, \cdots, \mu_m). \qquad (18)$$

Based on the above, we have the following lemma.

Lemma 1: If the feedforward controller $Q(s)$ is defined by

$$Q(s) = [I - R(sI - A + BR)^{-1}B]\Lambda^{-1}, \qquad (19)$$

then

$$P(s)Q(s) = W(s). \qquad (20)$$

Proof: see (Mutoh and Nikiforuk, 1992).

Lemma 1 means that if we take

$$u_0(t) = Q(s)r(t), \qquad (21)$$

then we readily have $e \equiv 0$ in Figs. 1 and 2. Hence if $Q_\Theta(s)$ converges to $Q(s)$, then our control objective is achieved, but *not necessarily* vice versa. We will see this below.

We note that the parameterization of (9) can yield an arbitrary transfer matrix from $r(t)$ to $u_{\text{ff}}(t)$. By taking the Laplace transform of (9), we have

$$\begin{aligned}
u_{\text{ff}}(t) &= F(t)(sI - A_f)^{-1}B_f r(t) \\
&\quad + G(t)(sI - A_f)^{-1}B_f u_{\text{ff}}(t) + H(t)r(t) \\
&= \left[I - G(t)(sI - A_f)^{-1}B_f\right]^{-1} \\
&\quad \cdot \left\{H(t) + F(t)(sI - A_f)^{-1}B_f\right\}r(t) \\
&=: Q_\Theta(s)r(t). \qquad (22)
\end{aligned}$$

It is already known that $e(t) \to 0$ as $t \to \infty$ by the learning law of Section 3 (Alali *et al.*, 2006a), if we choose K_{fb} such that

$$M(s) = K_{fb}[(H_0 + F_0(sI - A_f)^{-1}B_f)L(s) + K_{fb}]^{-1} \qquad (23)$$

is strictly positive real. Then we have

$$P(s)Q_\Theta(s)r(t) = W(s)r(t), \qquad (24)$$

at least for the given $r(t)$. If, further, $P(s)Q_\Theta(s) = W(s)$, then one can identify the plant prameter based on this learning feedforward controller. In order to guarantee this, we need to take $r(t)$ sufficiently rich (*i.e.*, satifying the persistent excitation (PE) condition).

If we do not take such $r(t)$ as a reference signal, correct identification is not expected any more, although FEL assures the perfect tracking performance anyway. Even so, we may still be able to estimate the frequency characteristic of $P(s)$ to some extent, because the good tracking is achieved for $r(t)$ with limited frequency components. This is an interesting issue but further analysis remains as our future work.

Thus, the procedure of closed-loop identification based on MIMO-FEL can be summarized as follows:

Step 1: We use the learning law (13) to tune the linear filter parameter in (10).

Step 2: As $\Theta(t)$ converges, we use the parameter to estimate the plant parameter by $P(s) = W(s)Q_\Theta^{-1}(s)$. The inversion is feasible by (22).

Step 3: If $r(t)$ is sufficiently rich, then we obtain the true value. Otherwise we only obtain partial knowledge of $P(s)$.

5. SIMULATION RESULTS

We consider two examples with first and third order systems, respectively. Firstly, to illustrate our idea for closed-loop identification, we perform numerical simulation using a simplest firsr order system.

Giving

$$A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 1 \\ 1.5 & 2 \end{bmatrix},$$

we have the following transfer matrix of the plant:

$$P(s) = \begin{bmatrix} \dfrac{1}{s+5} & \dfrac{2}{s+5} \\ \dfrac{3}{s+5} & \dfrac{4}{s+5} \end{bmatrix}.$$

Using (3), we have $\mu_1 = 1$, $\mu_2 = 1$, and

$$\Lambda = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

which is invertible. Then, we select the following interactor

$$L(s) = \begin{bmatrix} s+1 & 0 \\ 0 & s+1 \end{bmatrix}.$$

We now calculate the gain R in order to find the exact $Q(s)$ using equations (16) and (19):

$$R = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix},$$

$$Q(s) = \begin{bmatrix} \dfrac{-2s-10}{s+1} & \dfrac{s+5}{s+1} \\ \dfrac{1.5s+7.5}{s+1} & \dfrac{-0.5s-2.5}{s+1} \end{bmatrix}.$$

It can be verified that $P(s)Q(s) = W(s) = L^{-1}(s)$.

Second, without using A, B, C, or R, we tune $\Theta(t)$ using the learning rule (13) so that we

obtain the inverse of the system. In order that the reference input is sufficiently rich, we take $r(t)$ as a Pseudo-random Binary Signal (PRBS) generated for identification purpose in Matlab, as shown in Fig. 3. Thus, we need to set A_f and B_f based on the upper bound of the relative degree (18) $\mu = 1$ (see (11) and (12)) as

$$A_f = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, B_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We choose the following feedback gain matrix that maintains the closed-loop stability and satisfy SPR condition

$$K_{fb} = \begin{bmatrix} 6 & 4 \\ 7 & 3 \end{bmatrix}.$$

The time evolution of $\Theta(t)$ can be seen in Fig. 4. The convergent value is

$$\Theta = \begin{bmatrix} -1.9929 & 0.9905 & 3.0039 & 0.0042 & -2.0006 & 1.0022 \\ 1.4932 & -0.4994 & 0.0016 & 3.0032 & 1.5016 & -0.4997 \end{bmatrix}.$$

The nominal Θ_0 is

$$\Theta_0 = \begin{bmatrix} -2 & 1 & 3 & 0 & -2 & 1 \\ 1.5 & -0.5 & 0 & 3 & 1.5 & -0.5 \end{bmatrix}.$$

It can be seen clearly that $\Theta(t) \to \Theta_0$. Then we can obtain the learning feedforward controller $Q_{\Theta_0}(s)$ from (23)

$$Q_{\Theta_0}(s) =$$
$$\frac{1}{s^2 + 1.993s + 0.9929} \begin{bmatrix} -2.001s^2 - 11.98s - 9.932 \\ 1.502s^2 + 8.992s + 7.454 \end{bmatrix}$$
$$\begin{bmatrix} 1.002s^2 + 5.996s + 4.973 \\ -0.4997s^2 - 2.994s - 2.48 \end{bmatrix}.$$

Note that if we round off the convergent Θ, then we have $\Theta(t) = \Theta_0$ as $t \to \infty$. As a result, if we substitute Θ_0 in (23), we have $Q_{\Theta_0}(s) = Q(s)$.

Now we estimate the plant using $Q_\Theta(s)$ in (25)

$$\hat{P}(s) = \frac{1}{s^3 + 10.96s^2 + 34.74s + 24.79}$$
$$\times \begin{bmatrix} 0.9891s^2 + 5.927s + 4.91 \\ 2.972s^2 + 17.8s + 14.76 \end{bmatrix}$$
$$\begin{bmatrix} 1.984s^2 + 11.87s + 9.843 \\ 3.96s^2 + 23.72s + 19.66 \end{bmatrix}.$$

The comparison between the estimated plant $\hat{P}(s)$ and the true plant $P(s)$ is shown in Figs. 5 and 6 in terms of the bode plot and step response. The etimated one matches the true one. Further, if we round off the convergent Θ then we obtain $\hat{P}(s) = P(s)$.

In order to check the dependency on signals, we also tested a reference signal which is not

sufficiently rich $(r_c(t) = [\sin(t) \ \cos(t)]^T)$. The time evolution of $\Theta_c(t)$ can be seen in Fig. 7. The convergent value is

$$\Theta_c = \begin{bmatrix} -0.6448 & 0.1530 & 1.0810 & -1.0344 & -2.4260 & 1.3156 \\ 0.5205 & -0.0527 & -0.6753 & 0.7263 & 2.0291 & -0.6936 \end{bmatrix},$$

which is far away from the nominal value Θ_0. However, we have $e(t) \to 0$ as shown in Fig. 8. The obtained feedforward learning controller is:

$$Q_{\Theta_c}(s) = \frac{1}{s^2 + 6.193s + 8.857}$$
$$\times \begin{bmatrix} -2.426s^2 - 20.39s - 42.81 \\ 2.029s^2 + 16.2s + 32.2 \end{bmatrix}$$
$$\begin{bmatrix} 1.316s^2 + 10.44s + 20.65 \\ -0.6936s^2 - 5.741s - 11.91 \end{bmatrix}.$$

The second example (i.e., third order system) is

$$P(s) = \begin{bmatrix} \dfrac{s^2 + 4s + 1}{s^3 + 8s^2 + 19s + 23} & 0 \\ 0 & \dfrac{2s^2 + 9s + 7}{s^3 + 8s^2 + 19s + 23} \end{bmatrix}.$$

Using sufficiently rich reference input $r(t)$ as in Fig. 3, $\Theta(t)$ has converged to the following matrix

$$\Theta = \begin{bmatrix} -0.9358 & -0.0029 & 3.8129 & -0.0181 & 0.7893 & -0.0007 \\ 0.0006 & -1.0457 & -0.0081 & 3.4219 & 0.0150 & 0.4516 \end{bmatrix}.$$

The estimated plant is as follows:

$$\hat{P}(s) = \begin{bmatrix} \dfrac{1.267s^2 + 2.371s + 0.3993}{s^3 + 6.499s^2 + 13.74s + 9.482} \\ \dfrac{-0.04221s^2 - 0.1604s + 0.01846}{s^3 + 6.999s^2 + 15.99s + 11.85} \end{bmatrix}$$
$$\begin{bmatrix} \dfrac{0.001934s^2 + 0.0399s + 0.04784}{s^3 + 6.499s^2 + 13.74s + 9.482} \\ \dfrac{2.214s^2 + 7.512s + 3.6}{s^3 + 6.999s^2 + 15.99s + 11.85} \end{bmatrix}.$$

The comparison between the estimated plant $\hat{P}(s)$ and the true plant $P(s)$ is shown in Figs. 9 and 10 in terms of the bode plot and step response. In Fig. 9, the estimated one matches the true one for the diagonal elements while the off diagonal elements are close to zero in term of the gain (i.e., ≈ -40 dB, -70 dB, respectively). This is a good approximation. Fig. 10 also shows that the estimated one is close to the actual one.

6. CONCLUSION

In this paper we have proposed a closed-loop identification technique by means of MIMO-FEL. If the reference signal is persistantly exiting, the method has turned out to be effective in simulation.

Thus, one can improve the control performance by re-designing the controller with the obtained

plant model. At this moment, the convergence is slow, though this is often the case in adaptation or learning algorithms. It is important to improve this point, and to compare the proposed method with existing methods for closed-loop identification. After submission of the paper, the authors became aware of the papers (Kaneko *et al.*, 2005; Kaneko *et al.*, 2006; Miyachi *et al.*, 2006), where two degree of freedom structure is used as in our approach. Hence relationships to these papers are most interesting.

By the way the auhors have also tested MIMO-FEL with a real experiment for a two-link manipulator. They succeeded in approximating it by a first order system using local angular velocity and angular feedback with relatively high gain. The acquired learning controller has improved the performance significantly as shown in Fig. 11, which means that MIMO-FEL is effective in noisy cases. A detail will be presented elsewhere.

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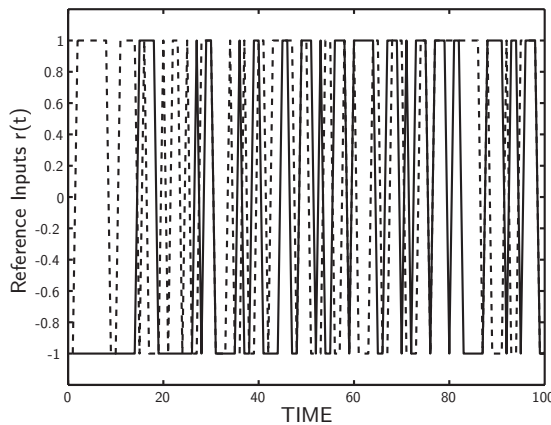
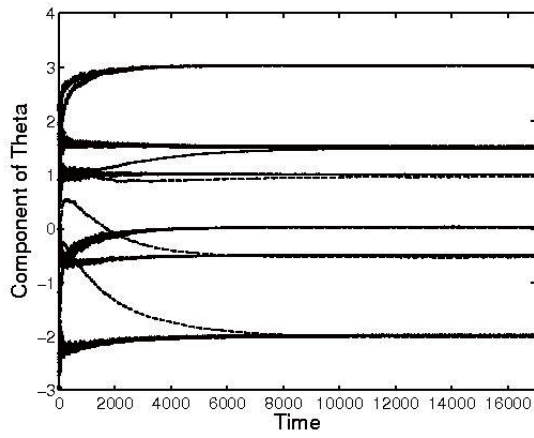Fig. 3. Reference Inputs: solid line: $r_1(t)$, dashed line: $r_2(t)$
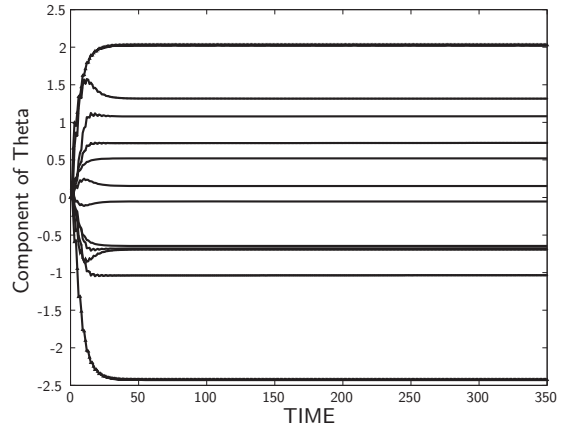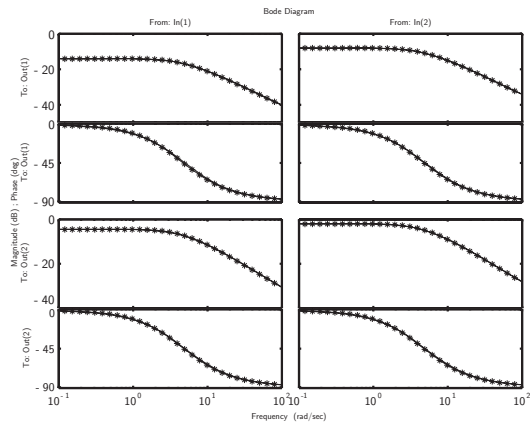
Fig. 4. Time evolution of $\Theta(t)$

Fig. 5. Bode Plot: solid line: true, star line: estimated
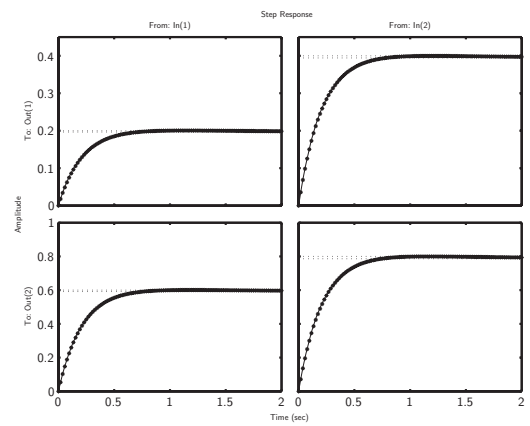
Fig. 6. Step Response: solid line: true, star line: estimated

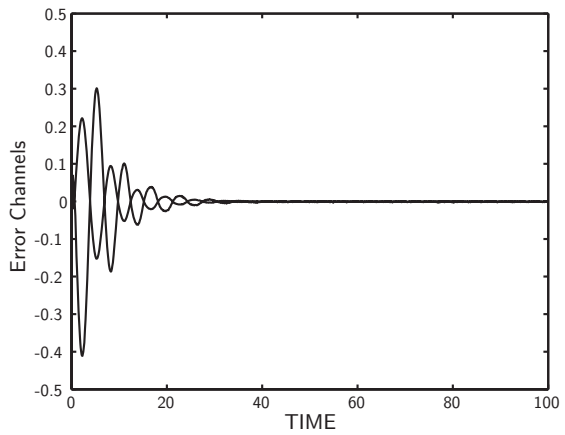Fig. 7. Time evolution of $\Theta_c(t)$

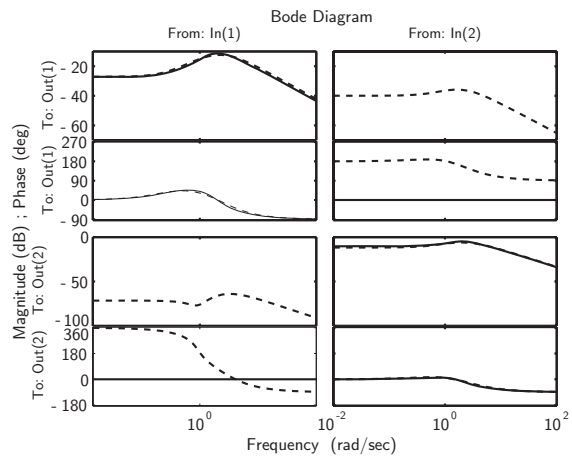Fig. 8. Error Channels: solid line: $e_1(t)$, dashed line: $e_2(t)$

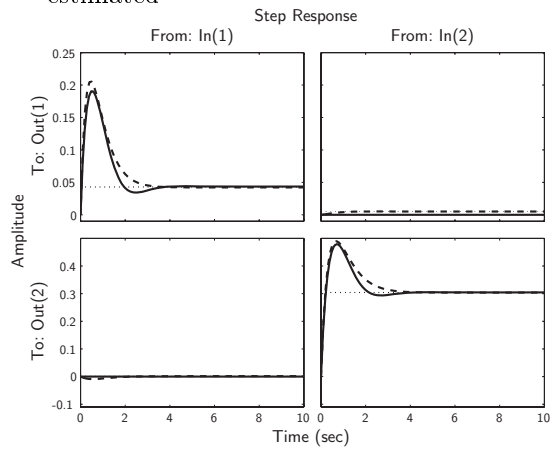Fig. 9. Bode Plot 2: solid line: true, dashed line: estimated
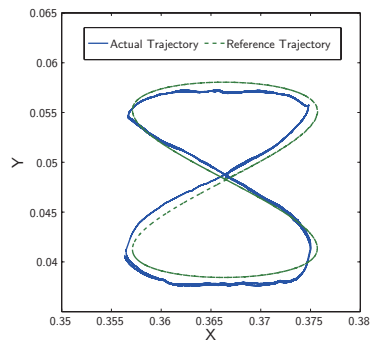

Fig. 10. Step Response 2: solid line: true, dashed line: estimated

Fig. 11. Experimental Result