Monitoring and Multilevel Protection of the Integrity of Tightly-Coupled Navigation Systems for Maneuverable Aircraft

A.V. Chernodarov*

Abstract: This paper is devoted to the problem of maintaining the integrity of navigation systems (NSs) of maneuverable aircraft under real noise environment. The proposed solution of this problem relies on the potentialities of hardware and algorithmic redundancy when constructing the loops for primary and secondary signal processing. Such a redundancy permits one to improve the reliability of estimation of NS state parameters under uncertainty and also to ensure the mutual support of NSs if critical situations occur. Hardware redundancy is assumed to be attained by the integration, into a unified navigation-time space, of air data, inertial, and satellite measuring channels. It is also assumed that algorithmic redundancy is achieved by the integration, into a single information space, of procedures for adaptive robust signal processing and combined procedures for detecting and counteracting outliers and failures, too. The results of experimental studies are given, which corroborate the effectiveness of applying the proposed approach in practice.

Keywords: Navigation systems, state estimation, system integrity

1. INTRODUCTION

This paper relating to the domain of noise-immune navigation reveals the possibilities of using the methods of multilevel signal processing for improving the objectivity of monitoring the integrity of functionally bound airborne systems, and also the reliability of state estimation of such systems. The term “integrity”, as applied to a navigation system (NS), reflects the ability of the NS to maintain the required operational characteristics irrespective of its operation conditions. The familiar solutions of this problem rely on the detection of failed components, the elimination of such components from the structure, and on the restoration of serviceability of NSs by means of hardware reconfiguration. The implementation of such an approach calls for a substantial hardware redundancy which seems to be impossible in a number of cases. Redundancy reduction may result both in a breach of continuity of navigational support and in a violation of air navigation safety. One possible approach to the solution of this particular problem is based on the integration of NSs, i.e., when their integrity is ensured by the mutual support of measuring means that are physically different in nature. Such an interaction of the NSs with each other allows one to retain or to reduce gradually their performance qualities if critical situations occur and noise conditions change. In tightly-coupled NSs, the mutual support is implemented at the level of the both secondary and primary signal processing. At present, the theoretical foundation for the integration of NSs is the mathematical apparatus of the extended Kalman filtering (EKF) and decision theory. However, under the conditions of statistical and parametric uncertainty, the realization of integration characteristics of NSs on the basis of such a mathematical apparatus involves a number of difficulties caused by the possible loss in integrity of the signal processing system itself. By the integrity of an integrated data processing (IDP) system is meant the state of this system such that the required estimation reliability of navigational parameters is ensured. The reliability, in its turn, is characterized by the non-divergence condition of the EKF, i.e., by the condition where the estimates obtained fit their predicted mean-square values adequately. This gives grounds to include the following loops intended for the protection of the EKF from its divergence, see Chernodarov et al. (1996):

- a robust-protection loop which minimizes the risk of losing the integrity in circumstances where the actual and “simulated”, i.e., a priori assumed, distribution laws for the generalized state parameters of the IDP system are inconsistent with each other;
- an adaptive-protection loop which provides the parametric tuning, for the actual operation conditions, of the IDP system having a robust risk-oriented architecture.

The technique used to unite, in the interests of counteracting the uncertainty, the above-mentioned loops into an integrated structure relies on the theory of optimization of stochastic systems on the basis of non-classical objective functionals. The purpose of this paper is to justify a unified technology for counteracting the uncertainty and protecting the integrity of NSs at the level of primary and secondary processing.

In tightly-coupled NSs, protection of the integrity is ensured by their mutual support at the level of both secondary (SDP) and preliminary (PDP) data processing. Traditionally, in the SDP loop, the inertial NS is a master NS, and, in the PDP loop, the satellite NS is a master NS. A technology for the monitoring and protection of the NS integrity at the level of secondary signal processing was discussed in Chernodarov et al. (1999). In the present paper we propose that in the primary processing of the signals of sensors, i.e., outside information should be used.
2. STRUCTURE OF A SYSTEM FOR THE PRIMARY PROCESSING OF INERTIAL-SENSOR SIGNALS

The current state of the art in the monitoring theory is characterized by a wide use of the methods of optimal Kalman filtering, see Gertler (1998). As is known, estimation systems that are Kalman ones in structure include loops intended for parameters prediction and for their updating on the basis of observation processing. When implementing the prediction loop, provision should be made for models that reflect variations in sensor output signals between the sessions where observations are formed. We propose that such models should be constructed, on a real-time basis, from the moving sample of readings of sensor signals by the use of the Chebyshev orthogonal polynomials. In view of the smoothing properties of the Chebyshev polynomials, it is apparently also possible to perform preliminary restoration of the valid signal at the prediction stage. Updating of the predicted signal and estimation of the instrumental drifts of sensors are realized from the processing of observations. As observations, we propose that the residual between the predicted and actual sensor signals should be used, along with the appropriate invariants. The invariants can be “a priori” known physical quantities, such as a change in the rotation angle of an inertial measurement unit, a change in the sensor output signal of the appropriate order, etc. Based on the polynomial and temporal filtering of sensor signals, it is apparently possible to realize monitoring procedures for inertial measurement units according to the combined goodness-of-fit test $\chi^2$, see Chernodarov et al. (2003). The use of such a test permits one to recognize outliers against the background of failures, to improve monitoring reliability, and to ensure strapdown inertial navigation system (SINS) integrity.

In this paper, a loop intended to protect the integrity of signals of an inertial measurement unit (IMU), which is based on fiber-optic gyros (FOG) is dealt with. The necessity of protecting SINSs built around FOGs from discordant signals is connected with the fact that fiber optic tools intended to measure angular-rotation rate are highly sensitive to external disturbances. In the implementation of the above loop, the following parameters are used:

$\hat{\Theta}_i$ are observed readings of the FOG output signal; 
\[ \hat{\Theta}_{i+1} = \sum_{k=0}^{m} q_k P_k(t_i) \] is the predicted value of the FOG output signal; 
$P_k(t_i)$ are the Chebyshev normalized orthogonal polynomials; 
$q_k$ are weight coefficients; 
$\nu_j = z_j - \hat{z}_j$ is the residual between the actual value $z_j$ and the predicted value $\hat{z}_j = H_j m_j$ of observations; 
$m_j$, $\hat{x}_{i+1}$ are the estimates of the error vector $x_i = [\Delta \Theta_j, \Delta \Omega_j]^T$ at the $i$-th step after the $j$-th component and the whole vector $z_j$ of observations are processed, respectively; $z_\Omega$, $z_\Theta$ are the signals of observations; $H_j$ is the row vector of coupling coefficients; $\Delta \Theta_{i+1}$, $\Delta \Omega_i$ are the angular error of the FOG and its instrumental drift at the $i$-th instant of time $t_i$, respectively;

\[ \beta_j = \nu_j/\alpha_j; \quad F_j = \hat{\alpha}_j^2 / \alpha_j^2; \quad \hat{\alpha}_j^2 = \sum_{k=1}^{N} (\nu_k - \hat{\nu}_k)^2 / (N - 1); \]

$\alpha_j$ is a scaling parameter; 
$\bar{\nu}_N = 1/N \sum_{k=1}^{N} \nu_k$; $N$ is the number of FOG signal readings on a moving time interval; 
RKF is a robust modification, see Chernodarov et al. (1996), of the Kalman filter; 
$\eta^2$, $\gamma^2$ are tolerances, see Chernodarov and Patrikeev (2003); $\omega$ is the delay by one bit; 
$\Omega_j \Lambda \mu$ is the rotation angle (an invariant) of the IMU oy-axis in the inertial space over the time $\Delta \mu = t_i - t_{i-1}$ when the base has no motion with reference to the Earth. The parameter $\beta_j$ is formed using the current residual and it reflects the current status of the $j$-th channel of the vector of observations. If it is out of the tolerance $\gamma^2$, this fact may be associated both with outliers and with failures. The parameter $F_j$ is the quotient of the actual and predicted variance of the residual. It is formed over an averaged range of values of the residual on a moving time interval. Therefore, if it is out of the tolerance $\eta^2$, this fact may be associated with a gradual failure. In accordance with the proposed technology, in the absence of discrepancy, the residual $\nu_j$ is processed by an EKF, whereas a failure is counteracted by connecting a redundant channel, and an outlier is counteracted by the robust processing of the residual with the use of the influence function $\Psi(\beta)$, see Chernodarov et al. (1996). This function defines the level of confidence in incoming measurements.

3. PECULIARITIES BY SECONDARY PROCESSING OF NAVIGATION SIGNALS

During the secondary data processing, see Chernodarov et al. (1999), the parameters are formed, which describe the translational and rotational motion of an object in a navigational coordinate frame. At present, it is considered that, in order for the above-mentioned problem to be solved, the use of dissimilar (in the principle of operation) measuring devices and the unification, into an integrated structure, of these devices on a basis of the procedures of extended Kalman filtering are justified. Therefore, the necessity arose of protecting the integrity of a system for secondary integrated data processing. By the integrity of an SDP system, in this case, is meant the state of the system such that the required estimation reliability of navigational parameters is ensured.

The reliability, in its turn, is characterized by the non-divergence condition of the EKF, see Fitzgerald (1971), i.e., by the condition where the estimates obtained fit their predicted mean-square values adequately. This gives grounds to consider the loops intended for adaptive robust protection.
of the EKF from divergence as a means for the maintenance of the integrity of an SDP system.

4. ROBUST MODIFICATION OF U-D KALMAN FILTER

Following Huber (1981), by «robustness» we shall here mean the non-sensitivity of an estimating filter to small deviations from the assumptions regarding the models of the errors of sensors such as a gyroscope, an accelerometer, a pseudorange sensor, and a pseudo-velocity sensor. As for integrated NSs, this assumption is chiefly that the noise of sensors is Gaussian in character. That is why, a need arises for protecting the filter from outlying observations the errors of which are anomalous with reference to the Gaussian distribution. Present-day approaches to the solution of the above problem rely on the application of the influence function \( \psi \), which defines the level of confidence in incoming observations. Such a function can be formed for the normalized residual \( \beta_j = v_j / \alpha_j \), where the residual itself, i.e., \( v_j = z_j - \hat{z}_j \) is the difference in value between the actual observation \( z_j \) and the predicted observation \( \hat{z}_j = H_m m_j \). For the normalized residual, one can not only form the robust-likelihood function \( \rho(\beta) \) but one can also perform the optimization of the estimates

\[
\hat{x}_j = \arg \min \sum_i \rho(\beta_i),
\]

where \( \rho(\beta) = -\ln f(\beta) \); \( f(\beta) \) is a probability density function. The solution of the problem (1), in view of the constraint

\[
x_j = \Phi x_{j-1} - \Gamma z_{j-1} = 0,
\]

is an algorithm for robust estimation, where \( \Phi \), \( \Gamma \) are transition matrices for the state vector \( x \), and the disturbance vector \( z \), respectively. The above algorithm is available in the Kalman-Joseph form (KJF) in Chernodarov et al. (1994). As an alternative to the KJF, the numerically stable U-D filter finds application, too. See Bierman (1977). The structure of such a filter is formed on a basis of the following representation of the covariance matrix:

\[
S_{ii} = U_{ii} D_{ii} U_{ii}^T,
\]

where \( U_{ii} \) is an upper triangular \( n \times n \) matrix with unity diagonal elements; \( D_{ii} \) is a diagonal matrix. In this case, when the Riccati equation is solved for the matrices \( U_{ii} \) and \( D_{ii} \), the extraction of square roots is not needed. At the same time, the positive properties of the triangular factorization \( S_{ii} = U_{ii} D_{ii}^{1/2} \) are retained. At present, the U-D technology is considered to be basic when onboard EKF modifications are constructed. The mapping of such an algorithm onto the robust KF structure has the following form:

Prediction:

\[
m_j = \hat{x}_{j-1} = \Phi \hat{x}_{j-1} + \Gamma z_{j-1}.
\]

Updating:

\[
f_j = H_j U_{j-1} ; \quad V_j = D_{j-1} f_j^T ;
\]

\[
\tilde{\alpha}_j^2 = f_j^T \psi^* \alpha_j^* ; \quad K_j = U_{j-1} V_j / \tilde{\alpha}_j^2 ;
\]

\[
m_j = m_{j-1} + K_j \beta_j ;
\]

\[
MWGS \quad \left\{ \begin{array}{l}
\tilde{W}_j = [K_j f_j \psi^* U_{j-1} ; K_j] \\
\tilde{D}_j = \text{diag} (D_{j-1}, \alpha_j^2 \psi_j^*)
\end{array} \right\} \rightarrow U_j ; D_j
\]

\[
U_{i,i} = U_j ; D_{i,i} = D_j ; \tilde{\xi}_{ji} = m_i ; j = I^T
\]

where \( m_j \), \( \tilde{\xi}_{ji} \) are the estimates of the state vector \( x \) at the \( i \)-th step after the \( j \)-th component and the whole vector \( z \), of observations are processed, respectively; \( Q \) is a covariance matrix for the disturbance vector. The MWGS is the procedure, see Bierman (1977), intended to transform the aggregate of matrices \( \tilde{W}_j \) and \( \tilde{D}_j \), which are an \( n \times (n + r) \) matrix and an \( (n + r) \times (n + r) \) matrix, respectively, into the aggregate of the \( n \times n \) matrices \( U_j \) and \( D_j \).

\[
\psi_j = \psi(\beta_j) = \frac{\partial \psi(\beta)}{\partial \beta} \bigg| \beta_j ; \quad \psi'_j = \psi'(\beta_j) = \frac{\partial^2 \psi(\beta)}{\partial \beta^2} \bigg| \beta_j
\]

The Joseph-Bierman algorithm presented here has an open architecture that enables extending the capabilities of tools that are designed for protecting the integrity of estimating filters. The functions \( \psi_j \) and \( \psi'_j \) can be formed with due regard for «a priori» assumptions as to the distribution laws of the valid signal and noise. Selection of the values of the above functions relies on necessary conditions, see Mehra (1970), for the filter to be divergence-free, namely:

(a) the generalized parameter has the Gaussian distribution \( \beta \in N(0,1) \);  
(b) the rule of 3\( \sigma \) is fulfilled, see Portenko et al. (1985), for the probability \( P \) that a random variable having the Gaussian distribution will be on the interval \([-3\sigma, 3\sigma]\), i.e.,

\[
P(\| \beta - E(\beta) \| \geq 3\sigma) = 0.0027.
\]

Thus, to the proper functioning of the filter can be put into correspondence the inequality \( \| \beta \| < 3 \) and also the following values of the functions:

\[
f_g(\beta) = (2\pi)^{n/2} \exp(-0.5\| \beta \|^2) ; \quad \rho_g(\beta) = 0.5 \ln(2\pi) + 0.5\| \beta \|^2 ;
\]

\[
\psi_g(\beta_j) = \beta_j ; \quad \psi'_g(\beta_j) = 1.
\]

The violation of inequality (6) may be caused both by disorder in the normal operation of the filter and by the presence of discordant observations. In robust statistic, e.g. Huber (1981), Gaussian random variables having «outliers» are described by the Laplace distribution. The following functions can be made to correspond to such a distribution and to off-design conditions of the filter operation:

\[
f_j(\beta) = 0.5 \exp(-10\| \beta \|) ; \quad \rho_j(\beta) = \ln 2 + 10\| \beta \| ; \quad \psi_j(\beta_j) = 1 ;
\]

\[
\psi'_j(\beta_j) = 10.
\]

The vagueness of boundaries between anomalous and conditioned signals can be taken into account by application
of the mathematical apparatus of fuzzy sets. Such a mathematical apparatus enables one to formalize the uncertainty with the use of fuzzy numbers and their respective membership functions. Specifically, using a symmetric triangular form for description of membership functions, we can allow for the fuzziness of the classification of residuals by means of the appropriate reduction of weight coefficients in the vicinity of the tolerance of $3\sigma$. However, an improved variant of tuning an influence function in the vicinity of the above tolerance can be realized on a basis of the convolution of the Gauss-Laplace PDF: Such a convolution can be performed using the following moment generating functions (MGFs), see Wu (1993):

$$M_{\psi}(T) = M_{\psi}(T) = (1 - T^2)^{-1} \exp(T^2 / 2),$$

where $M_{\psi}(T)$ is an MGF; $T$ is generally a complex number; $K(T) = \ln M_{\psi}(T)$ is a cumulant generating function. Relying on the results of Wu (1993), the following relations can be shown to hold for the normalized residual $\beta_i$:

$$\psi(\beta_i) = T_0 + K^{(3)}(T)/2 \bigg|_{T=T_0},$$

$$\psi'(\beta_i) = [1 + \partial K^{(3)}(T)/\partial T] \bigg|_{T=T_0},$$

where $T_0$ is the value of the argument at a saddle point for which the following equality is valid:

$$K^{(3)}_i(T) - \beta_i = 0$$

(10)

In view of the approximation $\ln(1-T^2) \approx -T^2$ and relations (7)-(10), the parameters $\psi_j$ and $\psi_j'$ take the form

$$\psi_{\beta_i}(\beta_i) = \beta / 3; \quad \psi_{\beta_i}'(\beta_i) = 1 / 3.$$

The influence function and its derivative, which reflect the assumptions considered are shown in Fig.1.

5. ANALYSIS OF THE RESULTS OF STUDIES

The SINS-1000 integrated strapdown inertial satellite navigation system, see Korkishko et al. (2008), which is built around the FOG -1000 fiber-optic gyros designed by the “OPTOLINK” RPC (Zelenograd, Russia) has been the subject of experimental studies. Experiments have been carried out on the ground when the necessary equipment was placed on a test bed and then housed in a mobile laboratory. The timing diagram of SINS operation included the following stages: coarse initial alignment, fine initial alignment, and a navigational mode. At the stage of coarse initial alignment, IMU angular position was approximately determined using sensor output signals. At the stage of fine initial alignment, estimation of and compensation for both the errors of the angular position of IMU sensors and IMU sensor drifts were carried out by the sequentially processing of the observed signals $z_i$ of the following form:

$$z_{\Theta(i)} = C_{\Theta(i)}^T \int_{t_{i-1}}^{t_i} \Theta(\tau) d\tau - [0; 3; \Omega \Delta t_i]^T;$$

$$z_{\lambda(i)} = [\varphi_i \lambda_i]^T_{\text{SINS}} - [\varphi_i \lambda_i]^T_{\text{PIA}};$$

$$z_{\nu(i)} = [V_x V_y V_z]^T_{\text{SINS}}.$$

(11)

(12)

(13)

where PIA stands for the position of the initial alignment; $\varphi_i, \lambda_i$ are the geodetic latitude and longitude of the SINS position; $V = [V_x \ V_y \ V_z]^T$ is the ground velocity vector of IMU motion, given by its components along the axes of the reference navigation frame $\Theta_{\text{SINS}}$; $\Omega$ is the angular velocity of Earth rotation; $\Delta t_i = t_i - t_{i-1}$ is an observation step; $C_{\Theta(i)}$ is the direction cosine matrix, that characterizes the angular position of the IMU-fixed frame $\Theta_{\text{xyz}}$ with respect to the inertial frame $OX_1Y_1Z_1$. In the navigational mode, SINS errors were estimated and compensated for from position and velocity observations, i.e.

$$z_{\Theta(i)} = C_{\Theta(i)}^T [V_x V_y V_z]^T_{\text{SINS}} - [V_x V_y V_z]^T_{\text{GPS}};$$

$$z_{\lambda(i)} = C_{\lambda(i)}^T [V_x V_y V_z]^T_{\text{SINS}} - [V_x V_y V_z]^T_{\text{OPH}};$$

(14)

(15)

where $C_{\lambda(i)}$ is the direction cosine matrix that characterizes the angular position of the reference frame $\Theta_{\text{SINS}}$ with respect to the geodetic frame $\Theta_{\text{OENH}}$. The basic vector $x(t)$ was comprised of 17 parameters, namely: the errors in the reckoning of components of the ground velocity vector; the errors in the reckoning of components of the navigation and orientation quaternions; the angular drifts of FOGs, and the biases of accelerometers. The results of a comparison analysis of SINS operation when using different schemes for the damping of sensor errors were obtained on a basis of the reckoning of motion parameters from the recorded signals of sensors such as the IMU and the GPS.

Certain of the results of a tested experiment on the estimation of accuracy characteristics of the SINS 1000 system are shown in Figs. 2-5. Figure 2 depicts the following signals: the output signal (a light-colored graph, are sec/sec) of the “vertical” gyro; the output signal (a dark-colored graph) of the same gyro, which was smoothed by means of a robust digital filter. In Fig. 3, the following signals are shown: the output signal (a light-colored graph, m/sq.sec) of one of

![Fig.1. Diagram for the control of an estimating filter with an influence function](image)
horizontal accelerometers: the output signal (a dark-colored graph) of the same accelerometer, which was smoothed with the aid of a robust digital filter. The above smoothing has been performed when sensor signals were picked off with a frequency of 1 kHz. Figure 4 depicts the FOG actual instrumental drift (deg/h), which is determined as the mean value of “zero” bias on the time intervals of 10 sec, and its estimate which was obtained both in the processing of observations (11)-(13) with a frequency of 1 Hz during the fine initial alignment (100-600 sec) and when predicting such an estimate in the navigational mode with the aid of algorithm (3). In Fig. 5, an estimate of the accelerometer bias is shown. Beginning with the moment t=600 sec, the SINS-1000 system was functioning in the autonomous inertial mode. Figs. 6-9 show errors in the reckoning of the ground velocity \( \Delta V \) and circular error in the object position \( \Delta S \). Figure 6 reflects the dynamic behavior of the ground velocity error when sensor drifts are damped, and Fig. 7 reflects the above behavior when the sensor drifts are not damped. Figure 8 reflects the dynamic behavior of the circular error in the object position when sensor drifts are damped, and Fig. 9 reflects the dynamic behavior of the circular error in the object position when sensor drifts are not damped, where

\[
\Delta S = \sqrt{\delta \phi^2 + \delta \lambda^2}; \quad \Delta V = \sqrt{\Delta V_E^2 + \Delta V_N^2};
\]

\[
\delta \phi = (\phi_{\text{SINS}} - \phi_{\text{GPS}})R; \quad \delta \lambda = (\lambda_{\text{SINS}} - \lambda_{\text{GPS}})R;
\]

\[
R = a (1 - 0.5e^2 \sin^2 \phi); \quad a = 6378245 \text{ m}; \quad e^2 = 0.0066934; \quad \Delta V_E = V_E(\text{SINS}) - V_E(\text{GPS}); \quad \Delta V_N = V_N(\text{SINS}) - V_N(\text{GPS}).
\]

\( \dot{\phi}, \dot{\lambda}, \text{arc sec/sec} \)

Fig. 2. Output signal of the “vertical” gyro

\( a_x, \text{m/s}^2 \)

Fig. 3. Output signal of one of horizontal accelerometers

\( \Delta \omega_2, \text{arc deg/h} \)

Fig. 4. FOG instrumental drift and its estimate

\( \Delta a_x, \text{m/s}^2 \)

Fig. 5. Accelerometer bias estimate

\( \Delta V, \text{m/sec} \)

Fig. 6. Dynamic behavior of the error of the ground velocity when sensor drifts are damped

\( \Delta S, \text{km} \)

Fig. 8. Circular error of the object position estimate when sensor drifts are not damped

\( \Delta V, \text{m/sec} \)

Fig. 7. Dynamic behavior of the error of the ground velocity when sensor drifts are not damped
Thus, the character of sensor signals necessitates protection of the integrity of the EKF together with the loop intended to damp errors.

6. CONCLUSIONS

The results of our studies corroborate the effectiveness of combination, into a unified technological process, of the following procedures for multilevel processing of sensor signals: adaptive robust polynomial smoothing of the signals; optimal estimation of instrumental sensor drifts with the use of the Kalman filter. With the approach noise components of sensor errors are “suppressed” at the stage of signals smoothing, and auto-correlated (systematic) components of sensor errors are “suppressed” at the stage of drifts estimation with due regard for information about the models of noise and invariants. The technique proposed here for processing redundant measurements makes it possible to detect and to recognize random and gradual failures, and pre-fault conditions; to resist random failures and outliers by means of algorithmic reconfiguration and tuning of the loops that provide adaptive robust protection of the integrity of the data processing systems.

REFERENCES